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13-1

The Pythagorean Theorem

(Pages 713–718)

STANDARDS 25.1

You can use the **Pythagorean theorem** to find the length of any side of a right triangle if the lengths of the other two sides are known. A corollary to this theorem can be used to determine whether a triangle is a right triangle.

Pythagorean Theorem	If a and b are the measures of the legs of a right triangle and c is the measure of the hypotenuse, then $c^2 = a^2 + b^2$.
Corollary to the Pythagorean Theorem	If c is the measure of the longest side of a triangle and $c^2 \neq a^2 + b^2$, then the triangle is not a right triangle.

EXAMPLES

A Find the length of leg b of a right triangle if the length of leg a is 24 and the length of the hypotenuse is 30.

$c^2 = a^2 + b^2$ *Pythagorean theorem*
$30^2 = 24^2 + b^2$ *Substitute.*
$900 = 576 + b^2$
$324 = b^2$
$\sqrt{324} = b$
$18 = b$

The length of leg b is 18 units.

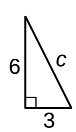
B The lengths of the sides of a triangle are 14 m, 12 m, and 10 m. Is the triangle a right triangle?

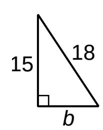
$c^2 = a^2 + b^2$ *Pythagorean theorem*
$14^2 \stackrel{?}{=} 12^2 + 10^2$ *Substitute.*
$196 \stackrel{?}{=} 144 + 100$
$196 \neq 244$

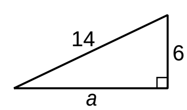
The triangle is not a right triangle.

PRACTICE

Find the length of each missing side. Round to the nearest hundredth.

1. 

2. 

3. 

If c is the measure of the hypotenuse of a right triangle, find each missing measure. Round answers to the nearest hundredth.

4. $a = 12$, $b = 32$, $c = $ __?__

5. $a = 7$, $b = 10$, $c = $ __?__

6. $a = 16$, $c = 52$, $b = $ __?__

7. $a = 2$, $b = 4$, $c = $ __?__

8. $b = 18$, $c = \sqrt{740}$, $a = $ __?__

9. $a = 5$, $b = \sqrt{10}$, $c = $ __?__

10. Art Jessica is making a collage of rectangles for her art project. The largest rectangle is 12 inches long and 8 inches wide. What is the length of a diagonal of the rectangle?

11. Standardized Test Practice Jamal and Gloria start hiking from the same point. After Bill hikes 7 miles due east and Jamal hikes 4 miles due north, how far apart are the two hikers?

A 5.29 mi **B** 5.40 mi **C** 8.06 mi **D** 9.25 mi

Answers: 1. $c = 6.71$ 2. $b = 9.95$ 3. $a = 12.65$ 4. $c = 34.18$ 5. $c = 12.21$ 6. $b = 49.48$ 7. $c = 4.47$ 8. $a = 20.40$ 9. $c = 5.92$ 10. about 14.42 in. 11. C

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13-2 Simplifying Radical Expressions (Pages 719–725)

25.1, 25.2

Product Property of Square Roots	For any numbers a and b, where $a \geq 0$ and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.
Quotient Property of Square Roots	For any numbers a and b where $a \geq 0$ and $b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.
Rationalizing the Denominator	Use the following steps (called **rationalizing the denominator**) to remove a radical from the denominator of a fraction. $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}}$ or $\frac{\sqrt{ab}}{b}$, where $a \geq 0$ and $b > 0$
Conjugates	The binomials $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$ are called **conjugates** of each other. You can use the fact that $\left(a\sqrt{b} + c\sqrt{d}\right)\left(a\sqrt{b} - c\sqrt{d}\right) = a^2b - c^2d$ to produce a product without radicals.
Radicals and Absolute Values	When finding the principal square root of an expression containing variables, be sure that the result is not negative. Use absolute value to ensure nonnegative results where necessary. $\sqrt{x^2} = \lvert x \rvert$ $\sqrt{x^3} = x\sqrt{x}$ $\sqrt{x^4} = x^2$ $\sqrt{x^5} = x^2\sqrt{x}$ $\sqrt{x^6} = \lvert x^3 \rvert$
Simplest Radical Form	A radical expression is in simplest form when the following three conditions have been met. 1. No **radicands** (the expressions under the radical signs) have perfect square factors other than 1. 2. No radicands contain fractions. 3. No radicals appear in the denominator of a fraction.

Try These Together

Simplify. Leave in radical form and use absolute value symbols when necessary.

1. $\sqrt{84}$ **2.** $\sqrt{90}$ **3.** $\sqrt{125x^2}$

HINT: Find the prime factorization of the number under the radical sign, then simplify the perfect squares. For example, $\sqrt{12} = \sqrt{2 \cdot 2 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$.

PRACTICE

Simplify. Leave in radical form and use absolute value symbols when necessary.

4. $\sqrt{156}$ **5.** $\sqrt{270}$ **6.** $\sqrt{800}$ **7.** $\sqrt{44b^5}$

8. $\sqrt{\frac{b^2}{49}}$ **9.** $\sqrt{\frac{36}{z^3}}$ **10.** $\sqrt{\frac{16}{x^2}}$ **11.** $2x\sqrt{8} \cdot 8x\sqrt{7}$

12. **Standardized Test Practice** Simplify $\left(y + \sqrt{5}\right)\left(y - \sqrt{5}\right)$.

A $y^2 - 2\sqrt{5} - 5$ **B** $y^2 - 2\sqrt{5}$ **C** $y^2 - \sqrt{10}$ **D** $y^2 - 5$

Answers: 1. $2\sqrt{21}$ 2. $3\sqrt{10}$ 3. $5\lvert x\rvert\sqrt{5}$ 4. $2\sqrt{39}$ 5. $3\sqrt{30}$ 6. $20\sqrt{2}$ 7. $2b^2\sqrt{11b}$ 8. $\frac{\lvert b\rvert}{7}$ 9. $\frac{6\sqrt{z}}{z^2}$ 10. $\frac{4}{\lvert x\rvert}$ 11. $32x^2\sqrt{14}$ 12. D

13-3

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Operations With Radical Expressions (Pages 727–731)

Radical expressions in which the radicands are alike can be added or subtracted in the same way that monomials are added or subtracted. If the radicals in a radical expression are not in simplest form, simplify them first. Then use the distributive property wherever possible to further simplify the expression. You can also use the FOIL method to multiply radical expressions with different radicands.

EXAMPLES

A Simplify $3\sqrt{11} + 2\sqrt{7} - 5\sqrt{7} + 9\sqrt{11}$.

$3\sqrt{11} + 2\sqrt{7} - 5\sqrt{7} + 9\sqrt{11}$

$= (2 - 5)\sqrt{7} + (3 + 9)\sqrt{11}$

$= -3\sqrt{7} + 12\sqrt{11}$

B Simplify $2\sqrt{12} + 4\sqrt{3}$.

$2\sqrt{12} + 4\sqrt{3} = 2\left(\sqrt{2^2}\sqrt{3}\right) + 4\sqrt{3}$

$= 2\left(2\sqrt{3}\right) + 4\sqrt{3}$

$= 4\sqrt{3} + 4\sqrt{3}$

$= 8\sqrt{3}$

Try These Together

Simplify.

1. $3\sqrt{6} + \sqrt{6}$

2. $14\sqrt{5} - 2\sqrt{5}$

3. $4\sqrt{18} + 2\sqrt{8}$

HINT: Make sure the radicals are in simplest form first, then use the distributive property to further simplify the expression.

PRACTICE

Simplify.

4. $3\sqrt{7} + 4\sqrt{7} - 3\sqrt{7}$

5. $4\sqrt{13} - 2\sqrt{13} + 6\sqrt{13}$

6. $2\sqrt{7x} + 3\sqrt{7x}$

7. $5\sqrt{3a} + 4\sqrt{3a}$

8. $2\sqrt{c} + 6\sqrt{c} - 3\sqrt{c}$

9. $4\sqrt{8} + 3\sqrt{8} + 2\sqrt{8}$

10. $\sqrt{16} + \sqrt{24} + \sqrt{9}$

11. $\sqrt{20} + \sqrt{28} - \sqrt{25}$

12. $\sqrt{30} + \sqrt{40} - \sqrt{12}$

13. $2\sqrt{2} + 2\sqrt{\frac{1}{2}}$

14. $6\sqrt{50} + 3\sqrt{3}$

15. $2\sqrt{72} - 3\sqrt{50}$

16. **Sailing** Before modern navigational tools, old sailing ships would have a small platform on top of the front mast called a crow's nest. Sailors in the crow's nest could see land or other ships that were much farther away than the sailors on deck. The equation $d = \sqrt{\frac{3h}{2}}$ can be used to find the distance d in miles a person h feet high above the water can see. If the deck was 20 feet above the water and the crow's nest was another 32 feet above the deck, about how much farther could sailors in the crow's nest see than those on deck? Round to the nearest tenth of a mile.

17. **Standardized Test Practice** Simplify $6\sqrt{3x} + 4\sqrt{3x} - \sqrt{3x}$.

A $9\sqrt{3x}$ **B** $9\sqrt{x}$ **C** $10\sqrt{3x}$ **D** $27\sqrt{x}$

Answers: 1. $4\sqrt{6}$ 2. $12\sqrt{5}$ 3. $16\sqrt{2}$ 4. $4\sqrt{7}$ 5. $8\sqrt{13}$ 6. $5\sqrt{7x}$ 7. $9\sqrt{3a}$ 8. $5\sqrt{c}$ 9. $18\sqrt{2}$ 10. $7 + 2\sqrt{6}$ 11. $2\sqrt{5} + 2\sqrt{7} - 5$ 12. $\sqrt{30} + 2\sqrt{10} - 2\sqrt{3}$ 13. $3\sqrt{2}$ 14. $30\sqrt{2} + 3\sqrt{3}$ 15. $-3\sqrt{2}$ 16. about 3.4 mi farther 17. A

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13-4 Radical Equations (Pages 732–736)

14.0 STANDARDS

Equations that contain radicals with variables in the radicand are called **radical equations**. To solve a radical equation, first isolate the radical on one side of the equation. Then square each side of the equation to eliminate the radical.

EXAMPLES

A Solve $\sqrt{x} - 4 = -2$.

$\sqrt{x} - 4 = -2$

$\sqrt{x} = 2$ *Add 4 to each side.*

$(\sqrt{x})^2 = 2^2$ *Square each side.*

$x = 4$

Check the solution.

$\sqrt{x} - 4 = -2$

$\sqrt{4} - 4 = -2$

$2 - 4 = -2$

$-2 = -2$

B Solve $\sqrt{2x - 4} = x - 2$.

$\sqrt{2x - 4} = x - 2$

$(\sqrt{2x - 4})^2 = (x - 2)^2$

$2x - 4 = x^2 - 4x + 4$

$0 = x^2 - 6x + 8$

$0 = (x - 4)(x - 2)$ *Factor.*

$x = 4$ or $x = 2$ *Use the zero product property.*

Check your solutions.

$\sqrt{2x - 4} = x - 2$ | $\sqrt{2x - 4} = x - 2$

$\sqrt{2(4) - 4} = 4 - 2$ | $\sqrt{2(2) - 4} = 2 - 2$

$\sqrt{4} = 2$ | $\sqrt{0} = 0$

$2 = 2$ | $0 = 0$

Try These Together

Solve each equation. Check your solution

1. $\sqrt{x} = \sqrt{3}$ **2.** $\sqrt{y} = \sqrt{6}$ **3.** $\sqrt{a} = 3\sqrt{5}$

HINT: Isolate the radical and then square both sides to eliminate the radical.

PRACTICE

Solve each equation. Check your solution

4. $\sqrt{y} - 4 = 0$ **5.** $\sqrt{c} + 4 = 0$ **6.** $\sqrt{s} + 2 = 0$

7. $\sqrt{3t + 1} = 6$ **8.** $\sqrt{2x - 2} = 4$ **9.** $16 - 5\sqrt{2y} = 1$

10. $3 + 2\sqrt{m} = 7$ **11.** $5 + 3\sqrt{4x} = 8$ **12.** $\sqrt{a - 3} = a - 5$

13. $\sqrt{x + 6} = x + 4$ **14.** $3 + \sqrt{a - 3} = 6$ **15.** $15 + \sqrt{y - 12} = 33$

16. Physics The period T of a pendulum is the time it takes to make one complete swing. At the earth's surface, $T = 2\pi\sqrt{\frac{L}{32}}$, where T is measured in seconds and L is the length of the pendulum in feet. To the nearest tenth, how long is a pendulum with a period of 2 seconds?

17. Standardized Test Practice Solve the equation $\sqrt{x + 7} = 2\sqrt{2}$.

A 1 **B** 2 **C** 7 **D** 8

Answers: 1. 3 2. 6 3. 45 4. 16 5. no solution 6. no solution 7. $11\frac{2}{3}$ 8. 9 9. $4\frac{1}{2}$ 10. 4 11. $\frac{1}{4}$ 12. 7 13. −2 14. 12 15. 336 16. 3.2 ft 17. A

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13-5

The Distance Formula (Pages 737–741)

14.0

You can use the distance formula, which is based on the Pythagorean theorem, to find the distance between any two points on the coordinate plane.

The Distance Formula	The distance between any two points with coordinates (x_1, y_1) and (x_2, y_2) s given by the following formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

EXAMPLES

A Find the distance between (2, 3) and (6, 8).

Let $x_1 = 2$, $y_1 = 3$, $x_2 = 6$, and $y_2 = 8$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(6 - 2)^2 + (8 - 3)^2}$

$d = \sqrt{4^2 + 5^2}$

$d = \sqrt{16 + 25}$

$d = \sqrt{41}$ *or about 6.4 units.*

B Find the value of a if $(a, 3)$ and $(2, -1)$ are 5 units apart.

Let $x_1 = a$, $y_1 = 3$, $x_2 = 2$, $y_2 = -1$, and $d = 5$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$5 = \sqrt{(2 - a)^2 + (-1 - 3)^2}$

$5 = \sqrt{(-a + 2)^2 + (-4)^2}$

$5 = \sqrt{a^2 - 4a + 4 + 16}$

$5 = \sqrt{a^2 - 4a + 20}$

$5^2 = \left(\sqrt{a^2 - 4a + 20}\right)^2$

$25 = a^2 - 4a + 20$

$0 = a^2 - 4a - 5$

$0 = (a + 1)(a - 5)$ *Factor.*

$a = -1$ or $a = 5$ *Zero product property*

PRACTICE

Find the distance between each pair of points whose coordinates are given. Express answers in simplest radical form and as decimal approximations rounded to the nearest hundredth if necessary.

1. (4, 6), (1, 5)

2. (15, 4), (10, 10)

3. (−7, −2), (11, 3)

4. (6, 13), (2, 15)

5. (25, 11), (18, 6)

6. $\left(12, 3\sqrt{5}\right), \left(6, 2\sqrt{5}\right)$

Find the value of a if the points with the given coordinates are the indicated distance apart.

7. $(1, 3), (a, -9)$; $d = 13$

8. $(-5, a), (3, -7)$; $d = 10$

9. $(-9, 3), (-2, a)$; $d = \sqrt{74}$

10. **Geometry** Find the perimeter of square $QRST$ if two of the vertices are $Q(5, 9)$ and $R(-4, -3)$.

11. **Standardized Test Practice** Find the distance between the points whose coordinates are $\left(2\sqrt{7}, 4\sqrt{5}\right)$ and $\left(\sqrt{7}, 2\sqrt{20}\right)$.

A $\sqrt{5}$ **B** $\sqrt{7}$ **C** $\sqrt{32}$ **D** $\sqrt{70}$

Answers: 1. $\sqrt{10}$ or 3.16 2. $\sqrt{61}$ or 7.81 3. $\sqrt{349}$ or 18.68 4. $2\sqrt{5}$ or 4.47 5. $\sqrt{74}$ or 8.60 6. $\sqrt{41}$ or 6.40 7. −4 or 6 8. −1 or −13 9. −2 or 8 10. 60 units 11. B

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13-6 Solving Quadratic Equations by Completing the Square

(Pages 743–747)

STANDARDS 14.0, 19.0, 23.0

You can solve some quadratic equations by taking the square root of each side. To do so, the quadratic expression on one side of the equation must be a perfect square. However, few quadratic expressions are perfect squares. To make any quadratic expression a perfect square, use the method called **completing the square**.

Completing the Square	To complete the square for a quadratic expression of the form $x^2 + bx$, follow the steps below. 1. Find $\frac{1}{2}$ of b, the coefficient of x. 2. Square the result of step 1. 3. Add the result of step 2 to $x^2 + bx$, the original expression.

EXAMPLES

A Find the value of c that makes $x^2 + 12x + c$ a perfect square.

1. Find $\frac{1}{2}$ of 12. $\frac{12}{2} = 6$

2. Square the result of step 1. $6^2 = 36$

3. Add the result of step 2 to $x^2 + 12x$. $x^2 + 12x + 36$

So, $c = 36$.
Notice that $x^2 + 12x + 36 = (x + 6)^2$.

B Solve $x^2 + 16x - 10 = 0$ by completing the square.

Notice that $x^2 + 16x - 10$ is not a perfect square.

$x^2 + 16x - 10 = 0$

$x^2 + 16x = 10$ — *Add 10 to each side.*

$x^2 + 16x + 64 = 74$ — *Since $\left(\frac{16}{2}\right)^2$ is 64, add 64 to each side.*

$(x + 8)^2 = 74$ — *Factor $x^2 + 16x + 64$.*

$x + 8 = \pm\sqrt{74}$ — *Take the square root of each side.*

$x = -8 \pm \sqrt{74}$

Solution set: $\{-8 + \sqrt{74}, -8 - \sqrt{74}\}$

PRACTICE

Find the value of c that makes each trinomial a perfect square.

1. $y^2 + 8y + c$ **2.** $a^2 + 6a + c$ **3.** $x^2 + 10x + c$

4. $x^2 + 9x + c$ **5.** $s^2 + 11s + c$ **6.** $z^2 + 7z + c$

Solve each equation by completing the square. Leave irrational roots in simplest radical form.

7. $x^2 + 8x + 12 = 0$ **8.** $y^2 + 6y - 15 = 0$ **9.** $z^2 + 12z - 25 = 0$

10. $a^2 + 14a - 18 = 0$ **11.** $x^2 + 10x + 16 = 0$ **12.** $x^2 + 18x + 17 = 0$

13. **Standardized Test Practice** Which expression shows the solutions of $x^2 + 16x + 32 = 0$?

A $8 + 4\sqrt{2}$ **B** $-8 + 4\sqrt{2}$ **C** $8 \pm 4\sqrt{2}$ **D** $-8 \pm 4\sqrt{2}$

Answers: 1. 16 2. 9 3. 25 4. 20.25 5. 30.25 6. 12.25 7. −2, −6 8. $-3 \pm 2\sqrt{6}$ 9. $-6 \pm \sqrt{61}$ 10. $-7 \pm \sqrt{67}$ 11. −2, −8 12. −1, −17 13. D

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Chapter 13 Review

Radical Roof

The staff at Monsoon High School stores its math textbooks in the storage buildings below. The books are evenly divided among all of the storage buildings. However, the rainy season is fast approaching and some of the storage buildings will leak when it rains. With your parent, help the staff of Monsoon High School find out which roofs will leak before the rains begin. Simplify the expressions on each building. If the simplified expression contains a radical sign (roof), then the storage building will not leak. If the expression does not contain a radical sign, then the building will leak. Mark the leaky buildings with a big X so the staff will know to move the textbooks out of those buildings.

Simplify each expression.

1. $\sqrt{64}$

2. $\sqrt{80x^2}$

3. $\sqrt{72ab^5c^2}$

4. $\dfrac{\sqrt{18}}{\sqrt{2}}$

5. $\dfrac{\sqrt{12}}{\sqrt{2}}$

6. $2\sqrt{5} + 3\sqrt{7} - 5\sqrt{5} + \sqrt{7}$

7. $3\sqrt{2} + \sqrt{5} - \sqrt{2} - \sqrt{5} - 2\sqrt{2}$

8. $\sqrt{3}(2 + \sqrt{3})$

9. $(2\sqrt{5} - 7)(\sqrt{5} + 6) - 5\sqrt{5}$

10. $\dfrac{5}{\sqrt{3} + 6}$

Answers are located on page 113.