

# Integers

 (pages 434–436)

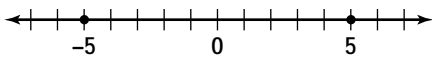

An **integer** is any number from this set of the whole numbers and their opposites:  $\{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$ .

## Writing and Graphing Integers

- Integers that are greater than zero are **positive integers**. You can write positive integers with or without a + sign.
- Integers that are less than zero are **negative integers**. You write negative integers with a – sign.
- Zero is the only integer that is neither positive nor negative.
- Each integer has an **opposite** that is the same distance from zero but in the opposite direction on the number line.

## EXAMPLES

**A** Graph 5 and its opposite on a number line.



A number line always has arrows on both ends, with zero and at least one other number marked to show the size of a unit. Make a dot to show the integers you are graphing.

**B** Write the opposite of  $-12$ .

The opposite of  $-12$  is 12 units from zero, but on the opposite side of zero. Since  $-12$  is at the left of zero, its opposite is  $+12$ , twelve units to the right of zero on a number line.

The opposite of  $-12$  is  $+12$  or 12.

## Try These Together

1. Write an integer to represent a debt of \$9.

*HINT: A debt can be represented by a negative integer.*

2. Write an integer to represent a rise in temperature of 15 degrees.

*HINT: You can represent a rise in temperature with a positive integer.*

## PRACTICE

Draw a number line from  $-10$  to  $10$ . Graph each integer on the number line.

3. 2

4.  $-4$

5.  $-6$

6. 5

7.  $-9$

8.  $-1$

9. 8

10. 3

Write an integer to describe each situation.

11. The water in a lake rises 3 feet.

12. A tennis ball drops 2 meters.

13. The temperature falls 15 degrees.

14. A person climbs 8 meters up a ladder.

Write the opposite of each integer.

15.  $-16$

16.  $-5$

17.  $+36$

18.  $+11$

19. **Standardized Test Practice** Which integer is the opposite of  $-25$ ?

A  $-25$

B 25

C 5

D  $-5$

Answers: 1.  $-\$9$  2.  $+15$  3.  $-10$  4.  $-4$  5.  $-6$  6. 5 7.  $-9$  8.  $-1$  9. 8 10. 3 11. 3 12.  $-2$  13.  $-15$  14. 8 15.  $+16$  16.  $+5$  17.  $-36$  18.  $-11$  19. B

# Comparing and Ordering Integers

(pages 437–439)



You can use a number line to compare numbers.

<b>Comparing Numbers</b>	<ul style="list-style-type: none"> <li>On a number line, the number to the left is always less than the number to the right.</li> <li>Recall that <math>7 &gt; 3</math> means 7 is greater than 3. <math>-7 &lt; 3</math> means that <math>-7</math> is less than 3.</li> <li>To order integers, first graph them on a number line. Then write them in order from left to right, or least to greatest.</li> </ul>
--------------------------	---

## EXAMPLES

**A** Which is greater, 13 or  $-17$ ?

*The integer  $-17$  is farther from zero than 13, but, because it is a negative integer, it is less than any positive integer. Think, "It is better to have \$13 than to have a debt of \$17."*

$$13 > -17$$

**B** Which is greater,  $-7$  or  $-3$ ?

*Think of both of these on a number line. Which integer is to the left? A number to the left is always less than the number to the right.*

$$-7 < -3 \text{ or } -3 > -7$$

## Try These Together

1. Order from least to greatest:

$$2, -2, 5, -5, 0.$$

*HINT: Graph the points on a number line. Then write the integers in order from left to right (least to greatest).*

2. Order from greatest to least:

$$7, -6, 0, -5, 6.$$

*HINT: Graph the points on a number line. Then write in order from right to left (greatest to least).*

## PRACTICE

Replace each  $\bullet$  with  $<$ ,  $>$ , or  $=$  to make a true sentence.

- |                     |                    |                     |                       |
|---------------------|--------------------|---------------------|-----------------------|
| 3. $-3 \bullet 5$   | 4. $8 \bullet 2$   | 5. $4 \bullet -1$   | 6. $-9 \bullet -9$    |
| 7. $-7 \bullet -12$ | 8. $10 \bullet 15$ | 9. $21 \bullet -2$  | 10. $-35 \bullet 5$   |
| 11. $-6 \bullet 3$  | 12. $5 \bullet -4$ | 13. $18 \bullet 21$ | 14. $-47 \bullet -47$ |

15. Which is greatest,  $-10$ ,  $10$ , or  $0$ ?

16. Order  $-5$ ,  $6$ ,  $-9$ , and  $-1$  from least to greatest.

17. Order  $2$ ,  $-3$ , and  $-1$  from greatest to least.

18. Is negative one greater than, less than, or equal to positive one?

19. **Games** Luis and Vilma were playing a game with negative and positive numbers on a number cube to show how many squares to move. Luis rolled a  $+4$  and Vilma rolled a  $-2$ . Who moved farther forward on the game board?



20. **Standardized Test Practice** Which is least,  $-5$ ,  $16$ ,  $-13$ , or  $-27$ ?

A 16

B  $-5$

C  $-13$

D  $-27$

Answers: 1.  $-5$ ,  $-2$ ,  $0$ ,  $2$ ,  $5$  2.  $7$ ,  $6$ ,  $0$ ,  $-5$ ,  $-6$  3.  $<$  4.  $<$  5.  $<$  6.  $=$  7.  $>$  8.  $<$  9.  $>$  10.  $<$  11.  $<$  12.  $>$  13.  $<$  14.  $=$  15.  $10$  16.  $-9$ ,  $-5$ ,  $-1$ ,  $6$  17.  $2$ ,  $-1$ ,  $-3$  18. less than 19. Luis 20. D

# Adding Integers

(pages 441–444)



You can use a number line to add integers.

## Adding Integers

To find the sum of  $5 + (-7)$ , follow these steps.

- Start at zero on the number line.
- Go 5 in the positive direction (right).
- From that point, go 7 in the negative direction (left).
- The point where you end ( $-2$ ) is the sum.

## EXAMPLES

- A** Is this sum positive, negative, or zero?  
 $-3 + 5$

*Which integer is farther from zero?  $+5$ . The sum will have the same sign as the integer that is farther from zero. The sum of  $-3 + 5$  is positive.*

- B** Find the sum of  $-4 + (-3)$ .

*Start at zero on the number line. Go 4 in the negative direction. From that point, go 3 more in the negative direction. You end at the point  $-7$ . The sum of  $-4 + (-3)$  is  $-7$ .*

## Try These Together

1. Is  $8 + (-10)$  positive, negative, or zero?

*HINT: Which integer is farther from zero?*

2. Find the sum of  $-12 + 13$ .

*HINT: Use a number line.*

## PRACTICE

*State whether each sum is positive, negative, or zero.*

3.  $2 + 4$

4.  $5 + (-10)$

5.  $-8 + (-2)$

6.  $-3 + (-3)$

7.  $-1 + 5$

8.  $-4 + (-4)$

9.  $5 + (-3)$

10.  $6 + (-6)$

*Find each sum.*

11.  $-8 + 16$

12.  $15 + (-5)$

13.  $4 + (-3)$

14.  $-7 + 5$

15.  $3 + (-5)$

16.  $-2 + (-2)$

17.  $-6 + 3$

18.  $8 + (-4)$

19. What is 2 plus  $-4$  plus  $-3$ ?

20. Find the sum of  $-14$  and 22.

21. **Football** In a football game, team A was on the 50 yard line. Then they lost 7 yards on the next play. What yard line are they on now?



22. **Standardized Test Practice** What is the sum of  $-8$ , 4, and  $-2$ ?

A  $-6$

B  $-8$

C 4

D 2

Answers: 1. negative 2. 13 3. positive 4. negative 5. negative 6. negative 7. positive 8. negative 9. positive 10. zero 11. 8 12. 10 13. 1 14.  $-2$  15.  $-2$  16.  $-4$  17.  $-3$  18. 4 19.  $-5$  20. 8 21. 43 yard line 22. A

# Subtracting Integers

(pages 445–448)



You can use counters or a number line to subtract integers.

<b>Subtracting Integers</b>	<p>To find the difference <math>4 - (-7)</math>, follow these steps.</p> <ul style="list-style-type: none"> <li>• Place 4 positive counters on a mat.</li> <li>• To subtract <math>-7</math>, you must remove 7 negative counters. To be able to do this, first add 7 zero pairs to the mat.</li> <li>• Remove 7 negative counters. There are 11 positive counters remaining on the mat.</li> <li>• <math>4 - (-7) = 11</math></li> </ul>
-----------------------------	---

## EXAMPLES

**A** Find this difference:  $-3 - (-4)$ .

*Begin with 3 negative counters. Add a zero pair to the mat, then remove 4 negative counters. There is 1 positive counter remaining.*

$$-3 - (-4) = 1$$

**B** Find this difference:  $6 - 8$ .

*Start at zero on the number line and go to 6. From there go 8 in the negative direction (left). You end at  $-2$ .*

$$6 - 8 = -2$$

## Try These Together

1. Find  $6 - (-9)$ .

*HINT: Begin with 6 positive counters, then add 9 zero pairs.*

2. Find  $-9 - 3$ .

*HINT: Start at zero and go 9 in the negative direction. From there, go 3 more in the negative direction.*

## PRACTICE

*Find each difference. Use counters or a number line if necessary.*

- |                 |                 |                |               |
|-----------------|-----------------|----------------|---------------|
| 3. $4 - 2$      | 4. $3 - 5$      | 5. $4 - 7$     | 6. $5 - 1$    |
| 7. $4 - (-5)$   | 8. $-3 - (-3)$  | 9. $6 - 9$     | 10. $-10 - 5$ |
| 11. $-7 - (-2)$ | 12. $14 - (-1)$ | 13. $8 - (-3)$ | 14. $-9 - 4$  |

15. Find  $-3 + 2 - (-6)$ .

16. Find the value of  $x - y$  if  $x = -7$  and  $y = 3$ .

17. **Landscaping** Charlie is a landscaper. He planted a row of flowers 2 feet back from the street. He then planted a row of bushes 4 feet behind the flowers. What negative integer represents how far back from the street the row of bushes is?

18. **Life Science** A dolphin dives 3 feet underwater. It then dives 5 more feet underwater. What negative integer tells how many feet underwater the dolphin is?



19. **Standardized Test Practice** What is the difference  $-15 - (-5)$ ?

A  $-10$

B  $-20$

C  $10$

D  $20$

Answers: 1. 15 2. -12 3. 2 4. -2 5. -3 6. 4 7. 9 8. 0 9. -3 10. -15 11. -5 12. 15 13. 11 14. -13 15. 5 16. -10 17. -6 18. -8 19. A
---

# Multiplying Integers

(pages 449–452)



Remember that multiplication is repeated addition. You can multiply integers by using counters or by using a number line to show repeated addition.

<b>Multiplying Integers</b>	<p><math>4 \times (-3)</math> means to put 4 sets of 3 negative counters on a mat. Then count the counters. There are 12 negative counters, so <math>4 \times (-3) = -12</math>.</p> <p><math>-4 \times (-3)</math> means to remove 4 sets of 3 negative counters. To be able to do this, you must first place 4 sets of 3 zero pairs on the mat. Then remove the 4 sets of 3 negative counters. There are 12 positive counters remaining on the mat, so <math>-4 \times (-3) = 12</math>.</p>
-----------------------------	--

## EXAMPLES

### A Find $-3 \times 5$ .

$-3 \times 5$  means to remove 3 sets of 5 positive counters. Begin with 3 sets of 5 zero pairs on a mat. Then remove the 3 sets of 5 positive counters. There are 15 negative counters remaining, so  $-3 \times 5 = -15$ .

### B Find $2(-11)$ .

You can also use a number line. Begin at zero. Move 11 units to the left, then 11 more units to the left. You end at  $-22$ , so  $2(-11) = -22$ .

## Try These Together

### 1. What is the product of $-4$ and $-8$ ?

*HINT: Begin with 4 sets of 8 zero pairs. Then remove the 4 sets of 8 negative counters.*

### 2. Find the product of 6 and $-2$ .

*HINT: Begin at zero on a number line. Move 2 units to the left 6 times.*

## PRACTICE

Find each product. Use counters or a number line if necessary.

3.  $1 \times (-1)$

4.  $5 \times 4$

5.  $-3 \times (-3)$

6.  $-6 \times 2$

7.  $8 \times (-4)$

8.  $-3 \times (-7)$

9.  $5 \times (-3)$

10.  $-1 \times 9$

11.  $-6(-4)$

12.  $-10(-3)$

13.  $7(-5)$

14.  $8(-9)$

15. Solve  $12(-3) = a$ .

16. What is the product of  $-8$  and  $-2$ ?

17. **Time** In winter, the days get shorter until December 21st. If each day is 2 minutes shorter than the day before, how many minutes will be lost in 5 days?

18. **Game Shows** On a game show, contestants lose \$100 each time they answer a question incorrectly. If a contestant answers 5 questions incorrectly, how much money will he or she lose?



19. **Standardized Test Practice** Find the product of  $-7$  and  $-4$ .

A 3

B  $-11$ 

C 28

D  $-21$ 

Answers: 1. 32 2.  $-12$  3.  $-1$  4. 20 5. 9 6.  $-12$  7.  $-32$  8. 21 9.  $-15$  10.  $-9$  11. 24 12. 30 13.  $-35$  14.  $-72$  15.  $-36$  16. 16 17. 10 minutes 18. \$500 19. C

# Dividing Integers (pages 456–458)



You can model dividing integers with counters or you can use a pattern.

<b>Dividing Integers</b>	<ul style="list-style-type: none"> <li>• When you divide two positive integers, or two negative integers, the quotient is positive.</li> <li>• When you divide a negative integer and a positive integer, the quotient is negative.</li> </ul>
--------------------------	--

## EXAMPLES

**A** Find  $-15 \div (-3)$ .

*The signs are the same. The quotient is positive.*

$$-15 \div (-3) = 5$$

**B** Find  $6 \div (-2)$ .

*The signs are different. The quotient is negative.*

$$6 \div (-2) = -3$$

## Try These Together

1. Find  $-12 \div 3$ .

*HINT: If you divide 12 negative counters into 3 groups, how many negative counters are in each group?*

2. Find  $-20 \div (-5)$ .

*HINT: Do the two integers have the same sign or different signs?*

## PRACTICE

*Find each quotient. Use counters or patterns if necessary.*

- |                   |                    |                   |                    |
|-------------------|--------------------|-------------------|--------------------|
| 3. $8 \div 2$     | 4. $6 \div (-3)$   | 5. $-2 \div 1$    | 6. $10 \div 5$     |
| 7. $14 \div (-7)$ | 8. $-12 \div (-3)$ | 9. $24 \div (-6)$ | 10. $-1 \div (-1)$ |
| 11. $-16 \div 4$  | 12. $-9 \div (-3)$ | 13. $4 \div 2$    | 14. $-5 \div (-1)$ |
15. Find the value of  $-32 \div -16$ .
16. Divide 42 by  $-7$ .
17. **Stock Market** Mr. Jimenez lost \$320 in 4 days in the stock market. How much money did he lose each day?
18. **Plumbing** The Chens' kitchen faucet has a leak. It drips 3 quarts of water every day. How many quarts of water does it drip in one week?



19. **Standardized Test Practice** What is  $-81$  divided by 9?

- A  $-8$                       B 8                      C  $-9$                       D 9

Answers: 1. $-4$ 2. 4 3. 4 4. $-2$ 5. $-2$ 6. 2 7. $-2$ 8. 4 9. $-4$ 10. 1 11. $-4$ 12. 3 13. 2 14. 5 15. 2 16. $-6$ 17. \$80 18. 21 quarts 19. C
---

# The Coordinate System

(pages 459–461)



A **coordinate system** consists of a horizontal line (called the **x-axis**) and a vertical line (called the **y-axis**) that intersect at the **origin**.

- The x-axis and the y-axis divide the plane into four **quadrants**.
- You can name point *P* with an **ordered pair** of numbers. The order makes a difference. The pair (1,3) is not the same as (3,1).
- The first number in the pair tells you how far to move to the right or left of the origin. It is called the *x*-coordinate.
- The second number in the pair tells you how far to move up or down from the *x*-axis. It is called the *y*-coordinate.

## EXAMPLES

**A** Give the ordered pair for the point which is 2 units to the right of the origin and 3 units down.

*Show movements to the right and up with positive integers and movements to the left and down with negative integers. This ordered pair is (2, -3).*

**B** What is the ordered pair for the point 4 units to the left of the origin and 5 units up?

*Since you move to the left, the *x*-coordinate is negative. This ordered pair is (-4, 5).*

## Try These Together

1. What are the coordinates of the origin?

*HINT: How much will you move from zero?*

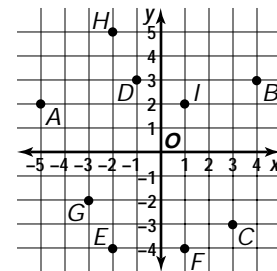
2. What is true of all points in Quadrant III?

*HINT: Which ways do you move from the origin to get to a point in Quadrant III?*

## PRACTICE

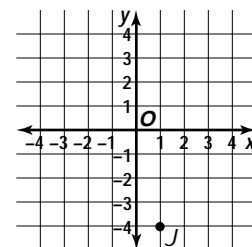
Name the ordered pair for each point.

- |             |              |              |
|-------------|--------------|--------------|
| 3. <i>D</i> | 4. <i>A</i>  | 5. <i>I</i>  |
| 6. <i>C</i> | 7. <i>G</i>  | 8. <i>H</i>  |
| 9. <i>B</i> | 10. <i>F</i> | 11. <i>E</i> |



12. **Standardized Test Practice** In which quadrant is Point *J* located?

- |                       |                      |
|-----------------------|----------------------|
| <b>A</b> Quadrant I   | <b>B</b> Quadrant II |
| <b>C</b> Quadrant III | <b>D</b> Quadrant IV |



Answers: 1. (0, 0) 2. Both the coordinates are negative. 3. (-1, 3) 4. (-5, 2) 5. (1, 2) 6. (3, -3) 7. (-3, -2) 8. (-2, 5) 9. (4, 3) 10. (1, -4) 11. (-2, -4) 12. D

# Graphing Transformations

(pages 464–467)



Translations and reflections are **transformations**, or movements of a figure.

<p><b>Translations and Reflections</b></p>	<ul style="list-style-type: none"> <li>• A <b>translation</b> slides a figure from one location to another. The new figure is called the translation image. The translation of triangle <math>ABC</math> is called triangle <math>A'B'C'</math> (you say “A prime, B prime, C prime”).</li> <li>• A <b>reflection</b> flips a figure over a line to form a reflection image. You can also name the reflection image with <math>A'B'C'</math>.</li> </ul>
--	--

## EXAMPLES

**A** When you translate, or slide, the point  $Q(3, 4)$  to a new position 2 units down, what are the coordinates of  $Q'$ ?

*When you move a point down, you change the y-coordinate.  $Q'$  is  $(3, 2)$ .*

**B** When you reflect, or flip, point  $C(-2, 5)$  over the y-axis, what are the coordinates of  $C'$ ?

*When you flip a point over the y-axis, you change the x-coordinate, because you change whether the point is right or left of the origin.  $C'$  is  $(2, 5)$ .*

## Try These Together

1. What is the translation image of  $S(3, -1)$  that is 2 units left and 4 up?

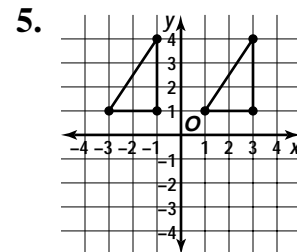
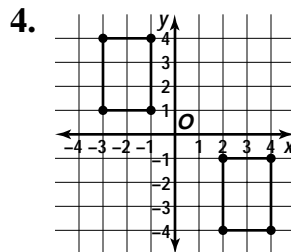
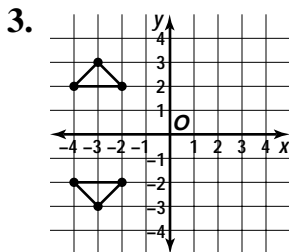
*HINT: Graph  $S$  and move it. Then find the coordinates of  $S'$ .*

2. What is the reflection image of  $T(2, -7)$  when you flip it over the x-axis?

*HINT: A flip over the x-axis changes which side of the x-axis the point is on, so the y-coordinate will change.*

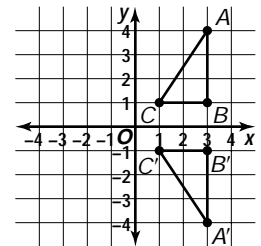
## PRACTICE

Tell whether each transformation is a translation or a reflection.



6. **Standardized Test Practice** What can you say about  $\triangle ABC$  and  $\triangle A'B'C'$ ?

- A  $\triangle A'B'C'$  is a reflection of  $\triangle ABC$ .
- B  $\triangle A'B'C'$  is a translation of  $\triangle ABC$ .
- C  $\triangle A'B'C'$  is not a transformation of  $\triangle ABC$ .
- D They are not related.



Answers: 1.  $S'(1, 3)$  2.  $T'(2, 7)$  3. reflection 4. translation 5. translation 6. A

# Chapter 11 Review



## Up-and-Down Scavenger Hunt

You've entered a haunted house with your friends. The only way you can get out is to find the key to give to the doorkeeper. The key is located under a board on one of the steps on the staircase. You must use your knowledge of integers to find the step where the key is located. All positive integers indicate the number of steps you go up, and negative integers indicate the number of steps you go down.

1. Starting at the bottom of the staircase, go up 5 steps. Then go  $-3$  steps. On which step are you located?
2. From your present location, go to the step that is 3 times the value of your current step. On which step are you now?
3. Subtract  $-11$  steps from your location and go to the corresponding step. Where are you now?
4. First go up one step and then divide the step you are on by  $-3$  to find the number of steps you take next. On which step did you end up?
5. Add  $-8$  steps to your present location and go to the corresponding step. Then multiply the step you are on by 4. The product is the step under which the key is hidden. Which step is it?

Answers are located on p. 110.