

Theoretical Probability

(pages 515–518)



Theoretical probability is the ratio of the number of ways an event can occur to the number of possible outcomes. For example, the theoretical probability of rolling a 1 on a number cube is $\frac{1}{6}$. That's because only one side of a number cube shows a 1, the **event** you are trying to get, while there are six total sides, or possible **outcomes**. The set of all possible outcomes (in this case there are six) is called the **sample space**.

Finding Theoretical Probability	$P(\text{event}) = \frac{\text{number of ways the event can occur}}{\text{number of possible outcomes}}$
Complementary Events	Complementary events are two events in which either one or the other must take place, but they cannot both happen at the same time. The sum of their probabilities is 1. An example of complementary events is rolling an even or odd number when you roll a number cube. $P(\text{event}_1) + P(\text{event}_2) = 1$

EXAMPLE

A student council representative is to be chosen from a class containing 12 boys and 16 girls. What is the probability that a girl will be chosen?

$$\frac{16}{28} \leftarrow \text{number of ways to choose a girl}$$

$$\frac{28}{28} \leftarrow \text{number of possible representatives in the class}$$

Therefore, $P(\text{a girl being chosen to be on the student council}) = \frac{16}{28}$ or $\frac{4}{7}$

Try These Together

There are 5 equally likely outcomes on a spinner, numbered 1, 2, 3, 4, and 5.

- Find $P(\text{even number})$ for the spinner.
HINT: How many outcomes are even numbers, compared to the total number of outcomes?
- Find $P(\text{odd number})$ for the spinner.
HINT: How many outcomes are odd numbers, compared to the total number of outcomes?

PRACTICE

A number cube is marked with 1, 2, 3, 4, 5, and 6 on its faces. Suppose you roll the number cube one time. Find the probability of each event.

- $P(4)$
- $P(4, 5, \text{ or } 6)$
- $P(3 \text{ or } 5)$
- $P(1, 2, \text{ or } 3)$

- Standardized Test Practice** On a science test, 75% of the students got Bs. What is the probability that a particular student did not get a B?

- A** 25% **B** 10% **C** 50% **D** 75%

Answers: 1. $\frac{5}{2}$ 2. $\frac{5}{3}$ 3. $\frac{6}{1}$ 4. $\frac{2}{1}$ 5. $\frac{3}{1}$ 6. $\frac{2}{1}$ 7. A

Making Predictions Using Samples

(pages 522–525)



If you want to make a prediction about a large group of people, you may wish to use a smaller group, or **sample**, from the larger group. The larger group from which you gathered your sample is known as the **population**. To make sure your information represents the population, the sample must be **random**, or drawn by chance from the population. You can then use the information from the sample to make a prediction about the larger population.

EXAMPLES

Kwame found that 20 of the 50 students he surveyed in the lunch line liked enchiladas the best.

- A** What is the probability that any student will like enchiladas the best?

20 out of 50, or $\frac{2}{5}$, like enchiladas.

The probability is $\frac{2}{5}$, or 40%.

- B** There are 520 students at Kwame's middle school. Predict how many like enchiladas the best.

Use a proportion. Let s represent the number of students who like enchiladas. Remember 20 out of 50, or $\frac{2}{5}$ of the students like enchiladas.

$$\frac{2}{5} = \frac{s}{520}$$

1,040 = 5s Multiply to find the cross products.

208 = s Divide each side by 5.

About 208 of 520 students probably like enchiladas.

Try These Together

Kwame found that 10 out of the 50 students liked hamburgers the best.

- 1.** What is the probability that any student will like hamburgers the best?

Hint: Write a ratio.

- 2.** Predict how many of the 520 students will like hamburgers the best.

Hint: Use a proportion.

PRACTICE

- 3. Recreation** Carmelina conducted a survey to find out if students preferred in-line skating or skateboarding. 64 out of 80 students preferred in-line skating. There are 200 students at her school. Predict how many of them prefer in-line skating.



- 4. Standardized Test Practice** A survey was conducted to find out if people preferred cheddar cheese or mozzarella cheese. 5 out of 20 people preferred cheddar cheese. What is the probability that any given person will prefer cheddar cheese?

A $\frac{2}{5}$

B $\frac{1}{5}$

C $\frac{2}{3}$

D $\frac{1}{4}$

Answers: 1. $\frac{5}{1}$ or 20% 2. about 104 3. about 160 4. D

Probability and Area (pages 526–529)



If you've ever played darts, you know that it can be hard to get a bull's-eye. The probability of hitting the bull's-eye is equal to the ratio of the area of the bull's-eye to the total area of the dartboard.

Relationship Between Probability and Area	Suppose you throw a large number of darts at a dartboard. $\frac{\text{number landing in the bull's-eye}}{\text{number landing in the dartboard}} = \frac{\text{area of the bull's-eye}}{\text{total area of the dartboard}}$
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EXAMPLES

A dartboard has three regions, A, B, and C. Region B has an area of 8 in² and regions A and C each have an area of 10 in².

A What is the probability of a randomly thrown dart hitting region B?

$$P(\text{region B}) = \frac{\text{area of region B}}{\text{total area of the dartboard}}$$

$$= \frac{8}{28} \text{ or } \frac{2}{7}$$

B If you threw a dart 105 times, how many times would you expect it to hit region B?

Let $b =$ times the dart lands in region B.

$$\frac{b}{105} = \frac{2}{7}$$

$$7b = 210 \text{ Multiply to find the cross products.}$$

$$b = 30 \text{ Divide each side by 7.}$$
 Out of 105 times, you would expect to hit region B about 30 times.

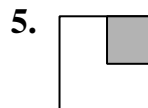
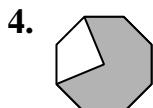
Try These Together

A dartboard has two regions, A and B. Region A has an area of 5 in² and region B has an area of 95 in².

- What is the probability of a randomly-thrown dart hitting region A?
Hint: Write a ratio.
- If you threw a dart 400 times, how many times would you expect it to hit region A?
Hint: Use a proportion.

PRACTICE

Each figure represents a dartboard. It is equally likely that a dart will land anywhere on the dartboard. Find the probability of a randomly-thrown dart landing in the shaded region. How many of 100 darts thrown would hit each shaded region?



6. Standardized Test Practice An apple falls from a tree. About $\frac{2}{3}$ of the ground under the tree is covered with grass. The rest of the ground is covered with dirt. It is equally likely that the apple will fall anywhere on the ground. What is the probability that it will fall on ground covered with dirt?

- A** $\frac{4}{7}$ **B** $\frac{1}{3}$ **C** $\frac{3}{5}$ **D** $\frac{3}{6}$

Answers: 1. $\frac{20}{1}$ 2. about 20 3. $\frac{2}{1}$: about 50 4. $\frac{4}{3}$: about 75 5. $\frac{4}{1}$: about 25 6. B

Finding Outcomes (pages 531–534)

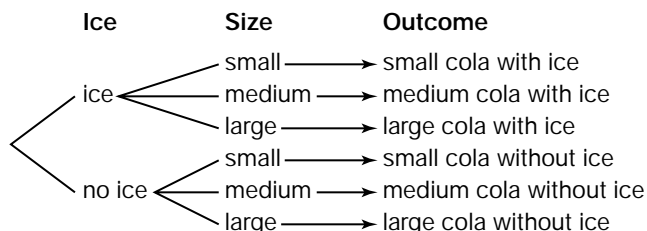


To find outcomes when you are given choices, you can simply list all of the possible outcomes, or use one of the methods below.

Tree Diagram	<table style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Size</th> <th>Topping</th> <th>Outcome</th> </tr> </thead> <tbody> <tr> <td rowspan="2">small</td> <td>none</td> <td>small hot dog without chili</td> </tr> <tr> <td>chili</td> <td>small hot dog with chili</td> </tr> <tr> <td rowspan="2">large</td> <td>none</td> <td>large hot dog without chili</td> </tr> <tr> <td>chili</td> <td>large hot dog with chili</td> </tr> </tbody> </table>	Size	Topping	Outcome	small	none	small hot dog without chili	chili	small hot dog with chili	large	none	large hot dog without chili	chili	large hot dog with chili
Size	Topping	Outcome												
small	none	small hot dog without chili												
	chili	small hot dog with chili												
large	none	large hot dog without chili												
	chili	large hot dog with chili												
Combinations	<p>Combinations are arrangements or listings in which order is not important. To find combinations, make a list. For example, let S stand for a small hot dog, and L stand for a large hot dog, and C stand for chili and N stand for none. Now, list all of the ways you can pair these letters.</p> <p style="text-align: center;">SN, NS, SC, CS, LN, NL, LC, CL</p> <p>Since SN and NS are the same, a small hot dog with no chili, then this arrangement is a combination. The four different combinations are SN, SC, LN, and LC.</p>													

EXAMPLE

At a concession stand, you can order a small, medium, or large cola, with or without ice. Use a tree diagram to find the number of possible outcomes.



Try This Together

- At the school snack bar, you can get apple, grape, or orange juice in a can, bottle, or drink box. Use a tree diagram to find the number of possible outcomes.

PRACTICE

For each situation, draw a tree diagram to show the number of outcomes.

- a choice of black or brown shoes with tan or blue pants
- a choice of grape, apple, or orange juice with a sandwich or slice of pizza



- Standardized Test Practice** The Ramirez family is getting 2 new sofas. They have 6 sofas to choose from. In how many ways can they choose 2 sofas from the 6 sofas?

A 25

B 10

C 15

D 30

Answers: 1. 9 possible outcomes 2. 4 3. 6 For Exercises 2–3, also see students' work to make sure tree diagrams are appropriate. 4. C

Probability of Independent Events

(pages 536–539)



In some board games, you get to repeat your turn if you roll doubles. What is the probability of rolling two 3s? You know that the probability of rolling any given number on one number cube is $\frac{1}{6}$. Since the number that comes up on one number cube does not affect the number that comes up on the second number cube, they are called **independent events**.

Probability of Two Independent Events

The probability of two independent events, A and B, is the product of the probability of event A and the probability of event B.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

EXAMPLES

- A** What is the probability of rolling two 3s in a board game?

$$P(3) = \frac{1}{6}$$

$$P(\text{double } 3s) = P(3) \cdot P(3)$$

$$= \frac{1}{6} \cdot \frac{1}{6}$$

$$= \frac{1}{36} \quad \text{Multiply.}$$

The probability of rolling double 3s is $\frac{1}{36}$.

- B** What is the probability of tossing a coin two times and getting heads both times?

$$P(\text{tossing heads once}) = \frac{1}{2}$$

$$P(\text{tossing heads twice}) = \frac{1}{2} \cdot \frac{1}{2}$$

$$= \frac{1}{4} \quad \text{Multiply.}$$

The probability of tossing a coin two times and getting heads both times is $\frac{1}{4}$.


Try These Together

- You have two bags. Each contains a yellow, blue, green, and red marble. What is the probability of choosing a blue marble from each bag?
Hint: Find the probability of each event. Then multiply.
- With the same bags as Exercise 1, what is the probability of choosing either a yellow or green out of each bag?
Hint: Find the probability of each event. Then multiply.

PRACTICE

One of 4 different colored balls is chosen from a bag and a number cube is rolled. Find the probability of each event.

- $P(\text{red and } 2)$
- $P(\text{green and } 1 \text{ or } 2)$

-  **5. Standardized Test Practice** Danika and Chantal each have identical boxes of crayons that contain eight different crayons each. What is the probability that they will both pick red when they each pull a crayon out of their boxes?

A $\frac{1}{64}$

B $\frac{1}{16}$

C $\frac{1}{6}$

D $\frac{1}{24}$

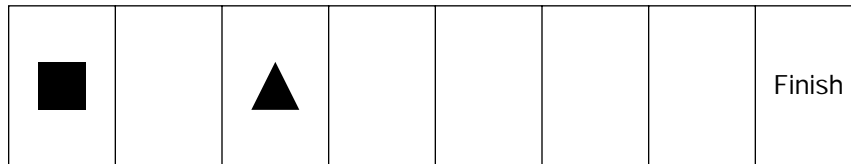
Answers: 1. $\frac{1}{16}$ 2. $\frac{1}{4}$ 3. $\frac{1}{24}$ 4. $\frac{1}{12}$ 5. A

Chapter 13 Review

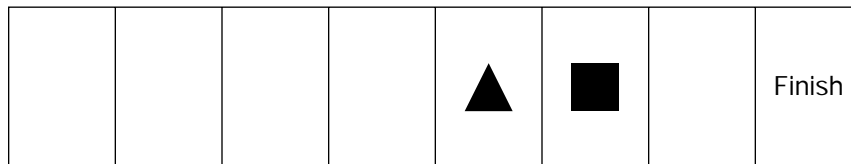


Board Game Probability

Use what you know about probability to help yourself in this board game against a family member. To move your game pieces, you each roll a standard number cube.



- On the board game above, your game piece is represented by the square and your family member's game piece is represented by the triangle. To win the game, you need to land exactly on the finish square. If you and your family member each roll once, which one of you is more likely to land exactly on the finish square? Explain.
- You hold a card that says if you roll a 6 twice in a row you automatically win. What is the probability that you will roll a 6 twice in a row?



- After one roll each, you and your family member are in the spaces above. What is the probability that you both land in the finish square on the next roll?

Answers are located on p. 111.