

# A Plan for Problem Solving

(pages 4–7)



You can use a four-step plan to solve problems.

<b>Explore</b>	Determine what information is given in the problem and what you need to find. Do you have all the information you need? Is there too much information?
<b>Plan</b>	Select a strategy for solving the problem. There may be several strategies that you could use. Estimate the answer.
<b>Solve</b>	Solve the problem by carrying out your plan. If your plan does not work, try another, and maybe even a third plan.
<b>Examine</b>	Examine the answer carefully. See if it fits the facts given in the problem. Compare it to your estimate. If your answer is not correct, make a new plan and start again.

## EXAMPLE

Sergio bought a 30-minute long distance phone card for \$4.50. On his home telephone, long distance costs 10 cents per minute. Which is less expensive?

<b>Explore</b>	You need to find out whether a long distance call using the calling card or Sergio's home phone is less expensive.
<b>Plan</b>	You need to find the per-minute rate of the calling card and then compare to the per-minute rate on the home phone. Divide the price of the calling card by the number of minutes, and then compare to the home phone rate. You estimate that the home phone is less expensive.
<b>Solve</b>	$450 \div 30 = 15$ cents per minute. $10 < 15$
<b>Examine</b>	The calling card rate is 15 cents per minute, so the home phone rate is less expensive.

## Try This Together

- The Washington family is going on a 775-mile vacation. Their car gets 31 miles per gallon of gas. If gas costs \$1 per gallon, how much will they spend on gas? *HINT: You need to find the number of gallons of gas the car will use.*

## PRACTICE

Use the four-step plan to solve each problem.

- Hobbies** Tristen is making a quilt with his mother. The quilt has a total of 40 squares. Tristen wants to have an equal number of squares with 8 different colors. How many squares of each color will he have?



- Standardized Test Practice** The school is buying new risers for the choir to stand on during concerts. There are 120 people in the choir and each riser will hold 20 people. How many risers will they need to buy?

**A** 5

**B** 6

**C** 12

**D** 10

Answers: 1. \$25 2. 5 3. B

# Order of Operations (pages 8–10)



When you evaluate an expression, the **order of operations** ensures that the expression always has only one value. The order of operations tells you which operation to use first.

<b>Order of Operations</b>	<ol style="list-style-type: none"> <li>1. Do all operations within grouping symbols first.</li> <li>2. Multiply and divide in order from left to right.</li> <li>3. Add and subtract in order from left to right.</li> </ol>
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## EXAMPLES

**Evaluate each expression.**

**A**  $15 + 6 \div 3$

$$15 + 6 \div 3 = 15 + 2 \quad \text{First divide 6 by 3.}$$

$$= 17 \quad \text{Second, add 15 and 2.}$$

**B**  $(15 + 6) \div 3$

$$(15 + 6) \div 3 = 21 \div 3 \quad \text{First add 15 and 6}$$

$$= 7 \quad \text{within the parentheses.}$$

$$\quad \text{Second, divide 21 by 3.}$$

## Try These Together

**Evaluate each expression.**

1.  $7 \times 4 + 12$

*HINT: Multiply first.*

2.  $(11 - 4) \times 9$

*HINT: Do operations in parentheses first.*

## PRACTICE

**Name the operation that should be done first in each expression.**

3.  $2 \times 8 + 5$

4.  $9 - 2 \times 4$

5.  $22 \div (2 + 9)$

6.  $(4 - 2) \times 5$

**Evaluate each expression**

7.  $4 \div 2 \times 3$

8.  $(10 + 12) \div 11$

9.  $(15 - 8) \times 3$

10.  $36 \times (9 - 9)$

11.  $12 - 4 \div 2$

12.  $54 \div 6 + 18$

13.  $24 \div (3 \times 4)$

14.  $7 - (2 \times 3)$

**Insert parentheses to make each sentence true.**

15.  $12 + 3 - 1 \times 2 = 16$

16.  $1 + 8 + 4 \div 2 = 7$

17.  $16 - 12 \times 3 + 8 = 20$

18.  $7 + 3 \times 8 + 1 = 90$

19. **Shopping** Sonny bought 2 comic books that cost \$3 each, 5 comic books that cost \$2 each and 1 comic book that cost \$4. How much did he spend?

20. **Standardized Test Practice** Carlotta scored a 25 on her math test. Her friend scored twice as many points as she did. When Carlotta retook the test, she scored 4 points less than her friend did the first time. Which expression could you use to find Carlotta's score on her second test?

**A**  $25 - 4 \div 2$

**B**  $(4 \times 25) \div 2$

**C**  $(25 \times 2) - 4$

**D**  $25 - 4 \times 2$

**Answers:** 1. 40 2. 63 3. multiplication 4. multiplication 5. addition 6. subtraction 7. 6 8. 2 9. 21 10. 0 11. 10 12. 27  
 13. 2 14. 1 15. 12 + (3 - 1) \times 2 = 16 16. 1 + (8 + 4) \div 2 = 7 17. (16 - 12) \times 3 + 8 = 20 18. (7 + 3) \times (8 + 1) = 90  
 19. \$20 20. C

## Variables and Expressions

(pages 12–15)



**Variables**, usually letters, are used to represent numbers in some expressions. The branch of mathematics that involves expressions with variables is called **algebra**. **Algebraic expressions** are combinations of variables, numbers, and at least one operation. If you replace variables with numbers, you can **evaluate**, or find the value of, an algebraic expression.

<b>Showing Multiplication in Algebra</b>	$2 \cdot n$ means $2 \times n$
	$2n$ means $2 \times n$
	$np$ means $n \times p$

**EXAMPLES**

**Evaluate each expression if  $b = 12$ .**

**A**  $43 - b$

$$\begin{aligned} 43 - b &= 43 - 12 && \text{Replace } b \text{ with } 12. \\ &= 31 && \text{Subtract } 12 \text{ from } 43. \end{aligned}$$

**B**  $3b + 6$

$$\begin{aligned} 3b + 6 &= 3 \times 12 + 6 && \text{Replace } b \text{ with } 12. \\ &= 36 + 6 && \text{Multiply } 3 \text{ by } 12. \\ &= 42 && \text{Add } 36 \text{ and } 6. \end{aligned}$$

**Try These Together**

**Evaluate each expression if  $r = 8$  and  $s = 5$ .**

1.  $s + r - 2$

*HINT: Replace the variables.*

2.  $9r + s$

*HINT: Replace the variables, then multiply.*

**PRACTICE**

**Evaluate each expression if  $x = 8$ ,  $y = 4$ , and  $z = 2$ .**

- |               |               |               |                  |
|---------------|---------------|---------------|------------------|
| 3. $x - y$    | 4. $y + z$    | 5. $x \div y$ | 6. $x + y + z$   |
| 7. $y - z$    | 8. $xy$       | 9. $yz - x$   | 10. $xz + 2$     |
| 11. $2y - 3z$ | 12. $3x - 10$ | 13. $3yz$     | 14. $x + y - 2z$ |

15. Evaluate  $20x + 3x$  if  $x = 6$ .

16. **Business** Gerod grows tomatoes on his family's farm and sells them at the market every Saturday. He earns \$2 for every pound of tomatoes. Write an algebraic expression to show how much money he earns for  $n$  pounds of tomatoes.



17. **Standardized Test Practice** Marco works at a car wash in the summer. He earns \$2 for each car he washes and \$3 for each car he vacuums. The amount of money he earns is represented by the expression  $2w + 3v$ . If he washes 10 cars and vacuums 20 cars, how much money will he earn?

**A** \$70

**B** \$80

**C** \$50

**D** \$90

Answers: 1. 11 2. 77 3. 4 4. 6 5. 2 6. 14 7. 2 8. 32 9. 0 10. 18 11. 2 12. 14 13. 24 14. 8 15. 138 16.  $2n$  17. B

# Powers and Exponents

(pages 17–20)



When you multiply two or more numbers, each number is called a **factor** of the product. When the same factor is repeated, you can use an exponent to simplify the notation. An **exponent** tells you how many times a number, called the **base**, is used as a factor. A **power** is a number that is expressed using exponents.

<b>Examples of Powers</b>	$4^2$	$4 \times 4$	four to the second power, or four <b>squared</b>
	$2^3$	$2 \times 2 \times 2$	two to the third power, or two <b>cubed</b>
	$5^4$	$5 \times 5 \times 5 \times 5$	five to the fourth power
<b>Order of Operations with Powers</b>	1. Do all operations within grouping symbols first.		
	2. Do all powers before other operations.		
	3. Multiply and divide in order from left to right.		
	4. Add and subtract in order from left to right.		

## EXAMPLES

**A** Write  $8 \cdot 8 \cdot 8$  using exponents.

*The base is 8. Since 8 is a factor three times, the exponent is 3.*

$$8 \cdot 8 \cdot 8 = 8^3$$

**B** Write  $2^5$  as a product, then evaluate.

*The base is 2. The exponent 5 means that 2 is a factor five times.*

$$\begin{aligned} 2^5 &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \\ &= 32 \end{aligned}$$

**C** Evaluate  $5 \cdot 2^4$ .

$$\begin{aligned} 5 \cdot 2^4 &= 5 \cdot 16 \quad \text{Evaluate } 2^4 \text{ first.} \\ &= 80 \quad \text{Multiply 5 and 16.} \end{aligned}$$

## Try These Together

1. Write  $14 \cdot 14 \cdot 14 \cdot 14$  using exponents.

*HINT: How many factors are there?*

2. Evaluate  $17 + 4^3$ .

*HINT: Do all powers before other operations.*

## PRACTICE

**Write each power as a product of the same factor.**

3.  $3^3$

4.  $6^2$

5.  $a^4$

6.  $b^3$

**Write each product using exponents.**

7.  $5 \cdot 5 \cdot 5$

8.  $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

9.  $6 \cdot 6$

10.  $z \cdot z \cdot z \cdot z$

**Evaluate each expression if  $f = 2$ ,  $g = 5$ , and  $h = 6$ .**

11.  $g + f^2$

12.  $h^3 - g^2$

13.  $2f^3$

14.  $gh - f^3$

**15. Standardized Test Practice** Which is equivalent to  $5^3$ ?

**A**  $5 \cdot 5 \cdot 5$

**B**  $5 + 5 + 5$

**C**  $3 \cdot 5$

**D**  $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

Answers: 1.  $14^4$  2. 81 3.  $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$  4.  $6 \cdot 6 \cdot 6$  5.  $a \cdot a \cdot a \cdot a \cdot a$  6.  $b \cdot b \cdot b \cdot b$  7.  $5 \cdot 5 \cdot 5$  8.  $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$  9. 26 10. 24 11. 9 12. 191 13. 16 14. 22 15. A

## Solving Equations (pages 21–23)



In mathematics, an **equation** is a sentence that contains an equals sign, =. You **solve** the equation when you replace the variable with a number that makes the equation true. Any number that makes the equation true is called a **solution**. When you write an equation that represents a real-world problem, you are **modeling** the problem.

## EXAMPLES

**A** Solve  $y + 7 = 10$  mentally.

$$y + 7 = 10$$

$$3 + 7 \stackrel{?}{=} 10 \text{ You know that } 3 + 7 = 10.$$

$$10 = 10 \checkmark$$

The solution is 3.

**B** Solve  $5a = 35$  mentally.

$$5a = 35$$

$$5(7) \stackrel{?}{=} 35 \text{ You know that } 5(7) = 35.$$

$$35 = 35 \checkmark$$

The solution is 7.

## Try These Together

Solve each equation.

1.  $s + 9 = 22$

HINT: What plus 9 equals 22?

2.  $13n = 39$

HINT: 13 times what equals 39?

## PRACTICE

Name the number that is the solution of the given equation.

3.  $17 - x = 15$ ; 2, 3, 4

4.  $12 + y = 17$ ; 3, 4, 5

5.  $2 + z = 10$ ; 7, 8, 9

6.  $m + 5 = 10$ ; 4, 5, 6

7.  $15 \div n = 3$ ; 3, 4, 5

8.  $2j = 6$ ; 1, 2, 3

Solve each equation.

9.  $a + 5 = 11$

10.  $10 - b = 2$

11.  $4 + w = 25$

12.  $p - 30 = 10$

13.  $q = 3 + 6$

14.  $r = 2(9)$

15.  $4s = 8$

16.  $9 - t = 2$

17.  $24 \div f = 6$

18.  $3g = 36$

19.  $h + 23 = 33$

20.  $j = 5 - 2$

21. **Food** If Deepak drinks 28 glasses of milk every week, what is the average number of glasses of milk he drinks each day? Use the equation  $28 = 7m$ , where  $m$  is the number of glasses of milk per day.



22. **Standardized Test Practice** Gabriel has 20 minutes to take a math quiz. The quiz has 10 problems on it. Which equation shows how to find how many minutes Gabriel can spend on each problem?

**A**  $20 \times 10 = p$

**B**  $p = 10 \div 20$

**C**  $p = 20 + 10$

**D**  $20 \div 10 = p$

Answers: 1. 13 2. 3 3. 2 4. 5 5. 8 6. 5 7. 5 8. 3 9. 6 10. 8 11. 21 12. 40 13. 9 14. 18 15. 2 16. 7 17. 4 18. 12 19. 10 20. 3 21. 4 22. D

# Fractals and Other Patterns

(pages 24–27)



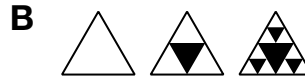
You probably encounter many geometric patterns every day without realizing it. Many quilts and fabrics have alternating patterns of geometric shapes. A **fractal** is a geometric pattern that is made up of smaller and smaller replicas of the same shape.

## EXAMPLES

Identify the pattern. Is the pattern a fractal?



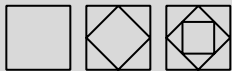
The smaller square moves clockwise to each corner of the larger square. The pattern is not a fractal.



The centers of the sides of each white triangle are connected to form a black triangle. The pattern is made up of smaller and smaller replicas of the same shape, so it is a fractal.

### Try This Together

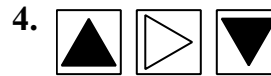
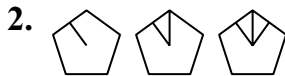
1. Draw the next figure in the pattern.



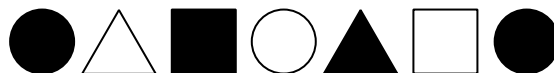
HINT: Insert another square.

## PRACTICE

Draw the next two figures that continue each pattern.



5. **Standardized Test Practice** What is the next figure in the following pattern?




Answers: 1–4. See Answer Key. 5. C

# Area

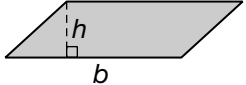
(pages 30–33)



The **area** ( $A$ ) of a closed figure is the number of square units needed to cover its surface.

<b>Area of Rectangles</b>	The area ( $A$ ) of a rectangle equals the product of its length ( $\ell$ ) and width ( $w$ ). $A = \ell w$	
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A **parallelogram** is a four-sided figure whose opposite sides are parallel. One of its sides is called its **base**. The distance from the base to the opposite side is called the **height**. The area of a parallelogram is closely related to the area of a rectangle.

<b>Area of Parallelograms</b>	The area ( $A$ ) of a parallelogram equals the product of its base ( $b$ ) and height ( $h$ ). $A = bh$	
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## EXAMPLES

- A** Find the area of a rectangle with a length of 20 cm and a width of 4 cm.

$$\begin{aligned} A &= \ell w && \text{Write the formula for area.} \\ A &= 20 \times 4 && \text{Replace } \ell \text{ with 20 and } w \text{ with 4.} \\ A &= 80 && \text{Multiply.} \\ &&& \text{The area is } 80 \text{ cm}^2. \end{aligned}$$

- B** Find the area of a parallelogram with a base of 13 m and a height of 5 m.

$$\begin{aligned} A &= bh && \text{Write the formula for area.} \\ A &= 13 \times 5 && \text{Replace } b \text{ with 13 and } h \text{ with 5.} \\ A &= 65 && \text{Multiply.} \\ &&& \text{The area is } 65 \text{ m}^2. \end{aligned}$$

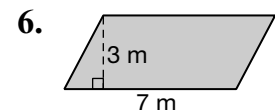
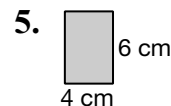
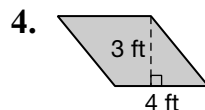
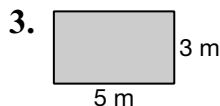
## Try These Together

1. Find the area of a rectangle with a length of 15 in. and a width of 12 in.  
*HINT: The area of a rectangle is length times width.*

2. Find the area of a parallelogram with a base of 24 ft and a height of 11 ft.  
*HINT: The area of a parallelogram is base times height.*

## PRACTICE

Find the area of each rectangle or parallelogram.



7. **Standardized Test Practice** The Angtuaco family is putting sod in their backyard. Their backyard is in the shape of a parallelogram with a base of 60 feet and a height of 100 feet. How many square feet of sod will they need to cover their backyard?

**A** 120 ft<sup>2</sup>

**B** 600 ft<sup>2</sup>

**C** 6,000 ft<sup>2</sup>

**D** 1,200 ft<sup>2</sup>

Answers: 1. 180 in<sup>2</sup> 2. 264 ft<sup>2</sup> 3. 15 m<sup>2</sup> 4. 12 ft<sup>2</sup> 5. 24 cm<sup>2</sup> 6. 21 m<sup>2</sup> 7. C

# Chapter 1 Review



## Treasure Hunt

For the Math Club party, Mitch plans a treasure hunt for the members. Each clue is a math problem. All of the clues together spell out the name of the treasure.

### Find each clue.

- One step in the four-step problem-solving plan involves looking at your answer carefully and seeing if it fits the facts in the problem. What is the name of this step? For Clue 1, use the first letter of this word.
- Evaluate this expression using the order of operations.  

$$20 \div 4 + 1(6 - 1) + 3(4)$$
- Find the area (in square inches) of a parallelogram with a base of 4 inches and a height of 5 inches.

### For Clues 4–6, find the value of each expression.

- $2^4 - 3$
- $5(t - s) + r$  if  $t = 7$ ,  $s = 6$ , and  $r = 4$
- $3^3 - 4^2$
- Solve the equation  $\frac{t}{5} = 3$  mentally.
- Find the next number in this pattern.  
 1 2 3 1 1 2 2 3 3 1 1 1 2 2 2

To discover what the treasure is, make each numbered clue from 2 to 26 into a letter by using the corresponding letter of the alphabet, so 2 → B, 3 → C, and so on, down to 26 → Z. Remember that you found the letter for Clue number 1 in Exercise 1 above.

Write the letters in the blanks that correspond to the numbers of the clues to read the name of the treasure.

**Clue:**    \_\_\_\_\_    \_\_\_\_\_    \_\_\_\_\_    \_\_\_\_\_    \_\_\_\_\_    \_\_\_\_\_    \_\_\_\_\_    \_\_\_\_\_    \_\_\_\_\_    \_\_\_\_\_  
           4        7        2        5        1        3        5        8        6        1        3

Answers are located on page 112.