

Integers (pages 184–186)



An **integer** is any number from the set $\{\dots, -3, -2, -1, 0, +1, +2, +3, \dots\}$. Integers greater than 0 are **positive integers**. Integers less than 0 are **negative integers**. Zero is neither positive nor negative.

Integers	<ul style="list-style-type: none"> Two numbers are opposites of one another if they are the same distance from zero on a number line, but on opposite sides of zero. The absolute value of an integer is its distance from 0 on a number line. The absolute value of n is written as n.
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EXAMPLES

A Find the opposite of 3.

Three is 3 units to the right of 0 on the number line. The opposite of 3 is -3 because -3 is 3 units to the left of 0.

B Find the absolute value of -4 .

On the number line, -4 is 4 units from 0, so $|-4| = 4$.

Try These Together

1. Write an integer to represent a debt of \$6.
HINT: A debt can be written as a negative integer.

2. Find $|y|$ if $y = -42$.
HINT: What is the distance from -42 to zero, without regard to direction?

PRACTICE

Write an integer for each situation.

- | | |
|--------------------------|----------------------------------|
| 3. to move back 3 spaces | 4. 20°F below zero |
| 5. a loss of 15 yards | 6. a shirt that shrunk 4 inches |

Write the integer represented by the point for each letter. Then find its opposite and its absolute value.



7. *A* 8. *B* 9. *C* 10. *D* 11. *E* 12. *F*

13. Sports Steve Cram was one of the world's top runners in the 1980s. When he was 17 years old, he ran a mile in 3 minutes and 57 seconds. When he broke the world record in 1985, he ran about 11 seconds faster. Express this lowering of his time as an integer.

- 14. Standardized Test Practice** Maggie jumped 4 feet from her trampoline down to the ground. Which integer describes the change in her distance from the ground?
- A** 1 **B** -1 **C** -4 **D** 4

Answers: 1. -6 2. 42 3. -3 4. -20 5. -15 6. -4 7. 1, -1 , 1 8. 6, -6 , 6 9. -10 , 10, 10 10. 2, -2 , 2 11. -4 , 4, 4 12. -7 , 7, 7 13. -11 14. C
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Comparing and Ordering Integers

(pages 188–190)



You can use a number line to order integers.

Ordering Integers On a number line, a number to the left is less than a number to the right.

EXAMPLES

A Which is greater, -4 or 2 ?

On a number line, -4 is to the left of 2 , so $-4 < 2$ or $2 > -4$.

B Order these integers from least to greatest. $3, 0, -5, -1, 2, -3, -6$

Think of these numbers on a number line in order from left (least) to right (greatest). $-6, -5, -3, -1, 0, 2, 3$

Try These Together

Replace each ● with **with** or **to** to make a true sentence.

1. $-12 \bullet -6$

HINT: Which integer is to the left on a number line?

2. $8 \bullet -9$.

HINT: A positive integer is always greater than a negative integer.

PRACTICE

Replace each ● with **with** or **to** to make a true sentence.

3. $-5 \bullet -6$

4. $15 \bullet -2$

5. $17 \bullet -18$

6. $25 \bullet 28$

7. $-16 \bullet -28$

8. $-2 \bullet -8$

9. $-19 \bullet 19$

10. $30 \bullet 26$

11. $-19 \bullet 21$

12. $-45 \bullet -43$

Order the integers from least to greatest.

13. $8, -3, 6, -4, 5$

14. $17, 12, -14, -6, 5, -3, -2$

15. Which is greater, 8 or -8 ?

16. **Weather** The high temperatures for one week in Minneapolis, Minnesota, were $0^\circ, -5^\circ, -2^\circ, 3^\circ, 8^\circ, 10^\circ$, and -16° Fahrenheit. Order the temperatures from least to greatest.



17. **Standardized Test Practice** Order the integers, $-7, 8, -11$, and 14 from greatest to least.

A $14, 8, -7, -11$

B $-7, 8, -11, 14$

C $-11, -7, 8, 14$

D $-7, -11, 8, 14$

Answers: 1. $<$ 2. $<$ 3. $<$ 4. $<$ 5. $<$ 6. $<$ 7. $<$ 8. $<$ 9. $<$ 10. $>$ 11. $<$ 12. $<$ 13. $-4, -3, 5, 6, 8$ 14. $-14, -6, -3, -2, 5, 12, 17$ 15. 8 16. $-16^\circ, -5^\circ, -2^\circ, 0^\circ, 3^\circ, 8^\circ, 10^\circ$ 17. **A**

The Coordinate System

(pages 191–194)



You can graph points in a plane on a **coordinate system**.

<p>Graphing on a Coordinate System</p>	<ul style="list-style-type: none"> • A coordinate system has a horizontal number line (called the x-axis) and a vertical number line (called the y-axis). These lines cross at right angles at a point called the origin. • These lines separate the plane into four quadrants. • You can name any point on a coordinate system using an ordered pair of numbers. • The first number in an ordered pair is the x-coordinate. It tells how far the point is to the right or left of the origin. • The second number in an ordered pair is the y-coordinate. It tells how far the point is up or down from the origin.
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EXAMPLES

A The point at $(4, -3)$ is in which quadrant?

The first number in the ordered pair (4) , tells you to move 4 units to the right from the origin. The second number (-3) tells you then to move down 3 units from there. The point at $(4, -3)$ is in Quadrant IV.

B In which quadrant are both the coordinates negative?

In Quadrant III, the points are to the left and down from the origin. Both coordinates of the ordered pairs in Quadrant III are negative.

Try These Together

1. In the graph below, what is the ordered pair for point K ? In which quadrant is K ?

HINT: How far to the left of the origin is K ? How far down?

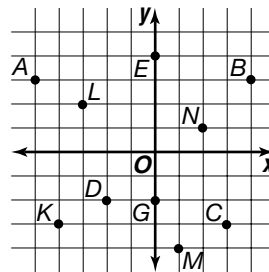
2. In the graph below, what is the x -coordinate of point B ? What is the y -coordinate of B ?

HINT: B is in Quadrant I, so both coordinates are positive.

PRACTICE

Name the ordered pair for each point labeled at the right.

- | | |
|--------|---------|
| 3. D | 4. L |
| 5. A | 6. G |
| 7. C | 8. M |
| 9. N | 10. E |



11. Standardized Test Practice In which quadrant of a coordinate plane would you find $F(-4, -1)$?

- A** Quadrant I **B** Quadrant II **C** Quadrant III **D** Quadrant IV

Answers: 1. $(-4, -1)$, III 2. 4, 3 3. $(-2, -2)$, IV 4. $(-3, 2)$, II 5. $(-5, 3)$, II 6. $(0, -2)$, IV 7. $(3, -3)$, IV 8. $(1, -4)$, III 9. $(2, 1)$, I 10. $(0, 4)$, I 11. C

Adding Integers (pages 197–200)



Two integers that are opposites are called **additive inverses**. The Additive Inverse Property states that the sum of any number and its additive inverse is 0.

$$3 + (-3) = 0 \qquad a + (-a) = 0$$

Adding Integers with the Same Sign	<ul style="list-style-type: none"> • The sum of two positive integers is positive. • The sum of two negative integers is negative.
Adding Integers with Different Signs	<p>To add integers with different signs, subtract their absolute values. The sum is</p> <ul style="list-style-type: none"> • positive if the positive integer has the greater absolute value. • negative if the negative integer has the greater absolute value.

EXAMPLES

A Solve $a = -7 + 3$.

The signs of these two integers are different. Find the absolute value of each.
 $| -7 | = 7, | 3 | = 3$
Notice that -7 has the greater absolute value. Subtract the absolute values: $7 - 3 = 4$. The sum of $-7 + 3$ is -4 . The sum has the sign of the integer that has the greater absolute value.

B Evaluate $x + (-3)$ if $x = -2$.

$x + (-3) = -2 + (-3)$ Replace x with -2 . The signs are the same. The sum of two negative integers is negative.
 $-2 + (-3) = -5$

Try These Together

1. Is $-4 + (-8)$ positive, negative, or 0?

HINT: Examine the signs.

2. Is $7 + (-2)$ positive, negative, or 0?

HINT: Which number has the greater absolute value?

PRACTICE

Tell whether the sum is positive, negative, or zero.

3. $-10 + 12$

4. $-7 + 7$

5. $-6 + (-3)$

6. $-2 + 3$

Solve each equation.

7. $-12 + 8 = x$

8. $y = 4 + 5$

9. $z = 15 + (-5)$

Evaluate each expression if a 2, b 5, and c 4.

10. $a + b$

11. $a + c$

12. $a + (-2)$

13. Money Matters Krishana used a \$5 bill to pay for a hamburger and soda that cost \$2. She used the expression $\$5 + (-\$2)$ to find out how much change she would get back. How much change did Krishana receive?



14. Standardized Test Practice What is the value of $k + j$ if $k = -7$ and $j = -5$?

A -12

B -2

C 12

D 2

Answers: 1. negative 2. positive 3. positive 4. zero 5. negative 6. positive 7. -4 8. 9 9. 9 10. -3 11. -2 12. 0 13. $\$3$ 14. **A**

Subtracting Integers (pages 202–205)



Adding the additive inverse of an integer produces the same result as subtracting the integer.

Subtracting Integers

To subtract an integer, add its additive inverse.
 $8 - 2 = 8 + (-2)$ $a - b = a + (-b)$

EXAMPLES**A** Find $6 - (-3)$.

Rewrite this subtraction problem as adding the inverse.

$$6 - (-3) = 6 + 3$$

$$6 - (-3) = 9$$

B Find $-9 - 4$.

Rewrite this subtraction problem as adding the inverse.

$$-9 - 4 = -9 + (-4)$$

$$-9 - 4 = -13$$

Try These Together1. Find $-4 - (-8)$.

HINT: Rewrite the subtraction as adding the inverse.

2. Solve $g = 5 - (-2)$.

HINT: Rewrite as 5 plus the inverse of -2 .

PRACTICE**Solve each equation.**

3. $h = -8 - 3$

4. $-12 - 6 = k$

5. $-6 - 4 = m$

6. $-2 - (-1) = n$

7. $10 - (-3) = x$

8. $y = -15 - 7$

Evaluate each expression if $r = 2$, $s = 5$, and $t = 4$.

9. $r - 5$

10. $s - t$

11. $t - (-8)$

12. $7 - r$

13. $r - s$

14. $8 - s$

15. $r - (-2)$

16. $s - 6$

17. $14 - t$

18. Sports Oliver and his sister Sarah had a bicycle race. Sarah is younger and cannot ride as fast as Oliver. So Sarah started 2 meters in front of the starting line and Oliver started 5 meters behind the starting line. How many meters behind his sister did Oliver start?

**19. Standardized Test Practice** Find p if $m - (-p) = 5$ and $m = 3$.**A** -4 **B** -5 **C** 3 **D** 2

Answers: 1. 4 2. 7 3. -11 4. -18 5. -10 6. -1 7. 13 8. -22 9. -3 10. -1 11. 4 12. 5 13. 7 14. 13 15. 4 16. -11 17. 18 18. 7 19. D

Multiplying Integers

(pages 207–209)



When you multiply two integers, look to see if they have the same or different signs.

Multiplying Integers with Different Signs	The product of two integers with different signs is negative.
Multiplying Integers with the Same Sign	The product of two integers with the same sign is positive.

EXAMPLES

A Find $(-3) \times (-5)$.

These two integers have the same sign, so their product is positive.

$$(-3) \times (-5) = 15$$

B Find the product of -7 and 3 .

These two integers have different signs, so their product is negative.

$$(-7) \times 3 = -21$$

Try These Together

1. Solve $g = 5(-2)$.

HINT: Are the signs of the factors the same or different?

2. Solve $h = 4(-5)$.

HINT: What sign will the product have?

PRACTICE

Solve each equation.

3. $-10(-3) = j$

4. $2(-6) = k$

5. $9(-5) = m$

6. $-4(-7) = n$

7. $-5(-8) = p$

8. $q = -12(-2)$

9. $r = -15(3)$

10. $t = 7(-3)$

11. $w = -8(-4)$

Evaluate each expression if $a = 3$, $b = 8$, $c = 2$, and $d = 4$.

12. $-3a$

13. $5c$

14. $-2bd$

15. $-15c$

16. $2ab$

17. $-2ad$

18. Game Shows Phil was a contestant on a game show where every time he answered incorrectly, he lost \$500. Phil answered incorrectly 3 times. Write an equation to show how much money Phil lost.



19. Standardized Test Practice A golfer played 3 days in a row in a tournament and was 4 strokes under par each day. What integer represents the number of strokes under par this player was at the end of the tournament?

A -12

B 12

C -7

D 7

Answers: 1. -10 2. -20 3. 30 4. -12 5. -45 6. 28 7. 40 8. 24 9. -45 10. -21 11. 32 12. -9 13. -10 14. 64 15. 30 16. -48 17. -24 18. $d = 3(-\$500)$ 19. **A**

Dividing Integers

(pages 212–214)



The rules for dividing integers are very similar to those for multiplying integers.

Dividing Integers

The quotient of two integers with the same sign is positive.
The quotient of two integers with different signs is negative.

EXAMPLES

A Find the quotient of 15 and -5 .

*The signs of these two integers are different.
The quotient is negative.*

$$15 \div (-5) = -3$$

B Find $-24 \div (-3)$.

*The signs of these two integers are the same.
The quotient is positive.*

$$-24 \div (-3) = 8$$

Try These Together

1. Solve $16 \div (-2) = a$.

HINT: Are the signs the same or different?

2. Solve $b = -21 \div (-3)$.

HINT: Compare the signs.

PRACTICE

Solve each equation.

3. $-8 \div (-4) = c$

4. $24 \div (-6) = d$

5. $-18 \div (-3) = e$

6. $f = -32 \div (-8)$

7. $g = -40 \div 8$

8. $h = -44 \div (-11)$

9. $54 \div (-9) = j$

10. $k = -9 \div 3$

11. $m = -72 \div (-8)$

12. $-5 \div (-5) = n$

Evaluate each expression if $a = 24$, $b = 8$, and $c = 4$.

13. $a \div b$

14. $b \div c$

15. $a \div (-12)$

16. $b \div 2$

17. $a \div (-1)$

18. $-32 \div b$

19. Entertainment Tickets to the zoo for Larissa's family cost \$35. If there are 5 people in her family, what was the cost per person?



20. Standardized Test Practice What is the quotient of -45 and 9 ?

A 36

B -36

C 5

D -5

Answers: 1. -8 2. 7 3. 2 4. -4 5. 6 6. 4 7. -5 8. 4 9. -6 10. -3 11. 9 12. 1 13. -3 14. 2 15. -2 16. -4 17. -24 18. 4 19. \$7 20. D

Graphing Transformations

(pages 215–217)



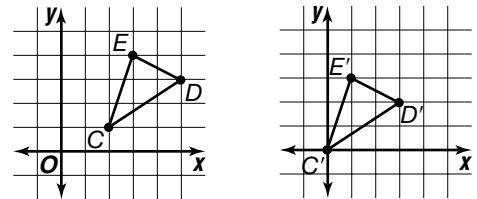
When a geometric figure is moved to a different position, the various kinds of moves are called **transformations**. When you flip a figure over a line, the move is called a **reflection**. When you slide a figure, the move is called a **translation**.

<p>Transforming Figures</p>	<p>Reflections:</p> <ul style="list-style-type: none"> • When you flip a figure over the x-axis, you are changing the up-and-down positions of the vertices, so the y-coordinates change. • When you flip a figure over the y-axis, you are changing the left-and-right positions of the vertices, so the x-coordinates change. <p>Translations:</p> <ul style="list-style-type: none"> • A translation may change one or both of the coordinates of the vertices.
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EXAMPLE

Triangle CDE with vertices $C(2, 1)$, $D(5, 3)$, and $E(3, 4)$ is translated two units left and one unit down. What are the new coordinates of its vertices?

$C(2, 1)$ moves two left and one down so C' is $(0, 0)$.
 To find D' , subtract 2 from the x -coordinate and subtract 1 from the y -coordinate. D' is at $(3, 2)$.
 $E(3, 4)$ translates to $E'(1, 3)$.
 The new coordinates are $C'(0, 0)$, $D'(3, 2)$, and $E'(1, 3)$.



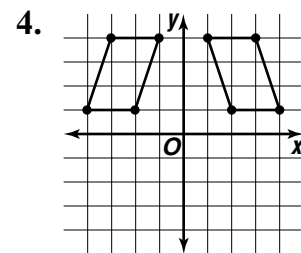
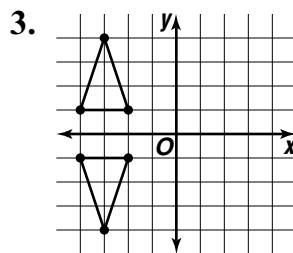
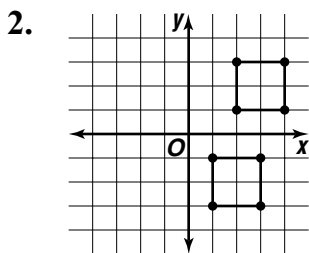
Try This Together

- Triangle CDE from the Example is reflected over the x -axis. What are the new coordinates of the vertices?

HINT: Transform $C(2, 1)$, $D(5, 3)$, and $E(3, 4)$ by multiplying each y -coordinate by -1 .

PRACTICE

Classify each graph as a reflection or a translation.



- Standardized Test Practice** Point $P(5, 3)$ is translated 2 units to the right and 1 unit down to locate P' . What is the ordered pair for P' ?

- A** $(2, 5)$ **B** $(7, 2)$ **C** $(3, 2)$ **D** $(7, 4)$

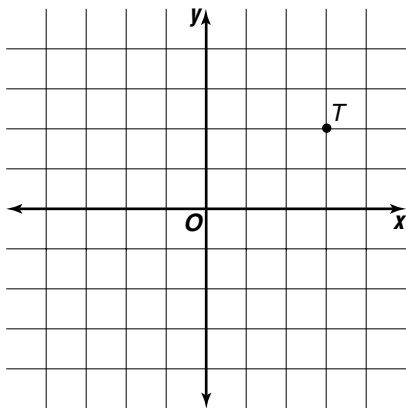
Answers: 1. $C'(2, -1)$, $D'(5, -3)$, and $E'(3, -4)$ 2. translation 3. reflection 4. reflection 5. B

Chapter 5 Review



Coordinate Treasure Hunt

Starting at point T on the coordinate plane below, follow the directions to find the location of a hidden treasure. Record your location at each point.



1. Add 1 to the x -coordinate and two to the y -coordinate. Where are you?
2. Divide both the x - and y -coordinates by -2 .
3. Subtract 2 from the x -coordinate and -5 from the y -coordinate.
4. Add 6 to the x -coordinate and -2 to the y -coordinate.
5. Multiply the x -coordinate by 2 and the y -coordinate by -5 .

What are the coordinates of the hidden treasure?

Answers are located on page 113.