

# Ratios (pages 317–320)



A **ratio** is a comparison of two numbers by division.

<b>Ratio</b>	<ul style="list-style-type: none"> <li>• Arithmetic: 5 to 1    5:1    <math>\frac{5}{1}</math></li> <li>• Algebra:    <math>a</math> to <math>b</math>    <math>a:b</math>    <math>\frac{a}{b}</math></li> <li>• When you write a ratio as a fraction, write it in simplest form.</li> <li>• Two ratios that have the same value are equivalent ratios.</li> <li>• You can also write ratios as decimals. <math>\frac{1}{4} = 1 \div 4</math>, or 0.25</li> </ul>
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## EXAMPLES

**A** Alfredo gets a hit 2 out of every 10 times he goes to bat. Write this ratio as a fraction in simplest form.

$$\begin{aligned} \frac{\text{hits}}{\text{at bats}} &= \frac{2}{10} \\ &= \frac{2 \div 2}{10 \div 2} \quad \text{The GCF of 2 and 10 is 2.} \\ &= \frac{1}{5} \end{aligned}$$

**B** Dan is 66 inches tall. Joaquin is 6 feet tall. Write the ratio of Dan's height to Joaquin's height in simplest form.

$$\begin{aligned} \frac{66 \text{ in.}}{6 \text{ ft}} &= \frac{66 \text{ in.}}{72 \text{ in.}} \quad \text{Write both measurements in inches. } 6 \text{ ft} = 72 \text{ in.} \\ &= \frac{66 \div 6}{72 \div 6} \quad \text{The GCF of 66 and 72 is 6.} \\ &= \frac{11}{12} \end{aligned}$$

### Try These Together

**Express each ratio as a fraction in simplest form.**

1.  $\frac{12}{15}$

*HINT: Find the GCF and then simplify.*

2. 24 to 36

*HINT: Write as a fraction. Find the GCF and simplify.*

## PRACTICE

**Express each ratio as a fraction in simplest form.**

3. 36:9

4. 45:5

5. 33 to 3

6. 8 to 32

7. 80:10

8. 15 to 45

**Tell whether the ratios are equivalent.**

9. 72:8 and 18:2

10. 48:6 and 52:4

11.  $\frac{60}{12}$  and  $\frac{75}{5}$

12.  $\frac{8}{24}$  and  $\frac{9}{27}$



**13. Standardized Test Practice** Which of the following is 85 tulips to 60 petunias written as a fraction in simplest form?

**A**  $\frac{12}{17}$

**B**  $\frac{17}{5}$

**C**  $\frac{15}{17}$

**D**  $\frac{17}{12}$

Answers: 1.  $\frac{5}{4}$  2.  $\frac{5}{2}$  3.  $\frac{1}{4}$  4.  $\frac{1}{9}$  5.  $\frac{1}{11}$  6.  $\frac{4}{1}$  7.  $\frac{1}{8}$  8.  $\frac{3}{1}$  9. yes 10. no 11. no 12. yes 13. D

# Rates (pages 321–324)



A **rate** is a ratio of two measurements with different units. A **unit rate** is a rate in which the denominator is 1 unit.

## EXAMPLES

- A** In a bike race, Tariq rode 42 km in 2 hours. What was his unit rate?

Write the rate as a fraction. Then find the equivalent rate with a denominator of 1.

$$\begin{aligned} \frac{\text{km}}{\text{hr}} &= \frac{42}{2} \\ &= \frac{42 \div 2}{2 \div 2} \quad \text{The GCF of 42 and 2 is 2.} \\ &= \frac{21}{1} \end{aligned}$$

Tariq rode at a rate of 21 km per hour.

- B** **Population density** is the number of people per square mile. What is the population density of a town with a population of 5,250 and an area of 5 square miles?

$$\begin{aligned} \frac{5,250 \text{ people}}{5 \text{ sq. mi}} &= 5,250 \div 5 \quad \text{Divide.} \\ &= 1,050 \text{ people per square mile} \end{aligned}$$

## Try These Together

Express each rate as a unit rate.

1. \$50 for 10 days

*HINT: Write as a fraction. Then find the equivalent rate with a denominator of 1.*

2. 8 revolutions in 2 minutes

*HINT: Write as a fraction. Then find the equivalent rate with a denominator of 1.*

## PRACTICE

Express each rate as a unit rate.

- 300 tourists in 4 days
- 720 miles in 8 days
- 512 yards in 8 minutes
- \$1.98 for 2 ounces
- \$22.32 for 18 gallons
- 240 miles in 3 hours
- 16 books in 4 days
- 35 people in 5 vans

11. **Sales** A company sold 3,000 popcorn poppers last year. On average, how many popcorn poppers did they sell each month?



12. **Standardized Test Practice** Which of the following gas stations sells gas for the best price per gallon?

**A** \$18.75 for 15 gallons

**B** \$16.64 for 13 gallons

**C** \$26.00 for 20 gallons

**D** \$19.68 for 16 gallons

**Answers:** 1. \$5 per day 2. 4 revolutions per minute 3. 75 tourists per day 4. 90 miles per day 5. 64 yards per minute 6. \$0.99 per ounce 7. \$1.24 per gallon 8. 80 miles per hour 9. 4 books per hour 10. 7 people per van 11. 250 popcorn poppers per month 12. D

## Solving Proportions (pages 325–328)



You can show that two ratios are equivalent with an equation called a **proportion**. When two ratios form a proportion, the **cross products** are equal. You can solve a proportion by using cross products to find a missing term.

**EXAMPLES**

**A** Can you form a proportion with the ratios  $\frac{1}{2}$  and  $\frac{5}{10}$ ?

$$\frac{1}{2} = \frac{5}{10} \quad \text{Set the ratios equal to each other.}$$

$$\begin{array}{c} \textcircled{1} = \textcircled{5} \\ \textcircled{2} = \textcircled{10} \end{array} \quad 1 \times 10 = 10 \text{ and } 2 \times 5 = 10$$

Since the cross products are equal, the ratios form a proportion.

**B** Solve  $\frac{u}{64} = \frac{3}{16}$ .

$$u \times 16 = 64 \times 3 \quad \text{Find the cross products.}$$

$$16u = 192$$

$$\frac{16u}{16} = \frac{192}{16}$$

$$u = 12$$

Divide each side by 16.

The solution is 12.

**Try These Together**

**Solve each proportion.**

1.  $\frac{2}{3} = \frac{x}{9}$

Hint:  $2 \cdot 9 = 3x$ .

2.  $\frac{3}{y} = \frac{1}{4}$

Hint: Find the cross products.

**PRACTICE**

**Solve each proportion.**

3.  $\frac{10}{16} = \frac{5}{n}$

4.  $\frac{j}{2} = \frac{1}{4}$

5.  $\frac{8}{6} = \frac{p}{3}$

6.  $\frac{15}{18} = \frac{5}{k}$

7.  $\frac{z}{40} = \frac{8}{80}$

8.  $\frac{6}{12} = \frac{3}{f}$

9.  $\frac{7}{3} = \frac{q}{21}$

10.  $\frac{r}{6} = \frac{5}{30}$

11. Find the value of  $x$  that makes  $\frac{x}{15} = \frac{8}{20}$  a proportion.

12. **Surveys** A survey at Lincoln Middle School found that 6 of every 10 students prefer math class to science class. If there are 400 students at the school, how many of them would you expect to prefer math class to science class?



13. **Standardized Test Practice** A factory can produce 1,500 cans of juice in 3 hours. How many cans of juice can they produce in 8 hours?

**A** 2,000

**B** 4,000

**C** 8,000

**D** 6,000

Answers: 1. 6 2. 12 3. 8 4. 0.5 5. 4 6. 6 7. 4 8. 6 9. 49 10. 1 11. 6 12. 240 13. B

# Scale Drawings

(pages 332–335)



You can use a **scale drawing** to represent something that is too large or too small for an actual-size drawing. A map is an example of a scale drawing. The **scale** on a map is the ratio of the distance on the map to the actual distance.

## EXAMPLES

- A** Tracy and Tyrone are planning a hiking trip. On the map, their route is 7.5 cm long. The map scale says that 1 cm represents 3 km. What is the actual length of their hike?

Let  $d$  represent the hiking distance. Write and solve a proportion.

$$\frac{\text{map distance}}{\text{actual distance}} \rightarrow \frac{1 \text{ cm}}{3 \text{ km}} = \frac{7.5 \text{ cm}}{d}$$

$$1 \times d = 3 \times 7.5$$

$$d = 22.5$$

The actual length is 22.5 km.

- B** The scale of a blueprint is 1 in. = 4 ft. If the actual width of a porch is 16 ft, what is the width on the blueprint?

Let  $w$  represent the porch width. Write and solve a proportion.

$$\frac{\text{blueprint width}}{\text{actual width}} \rightarrow \frac{1 \text{ in.}}{4 \text{ ft}} = \frac{w}{16 \text{ ft}}$$

$$1 \times 16 = 4 \times w$$

$$16 = 4w$$

$$\frac{16}{4} = \frac{4w}{4}$$

$$4 = w$$

The width on the blueprint is 4 inches.

## Try These Together

**Find the length of each object on a scale drawing with the given scale.**

- a house 75 feet long; 1 inch:1 foot
- a box 3 meters tall; 1 cm:1.5 m

## PRACTICE

**Find the length of each object on a scale drawing with the given scale.**

- a desk 4.5 meters long; 2 centimeters:1 meter
- an airplane with a 54-meter wingspan; 3 centimeters:1 meter
- an automobile that is 8 feet wide; 0.5 inch:1 foot
- a street that is 2 miles long; 5 inches:1 mile
- Find how far it is across the city of Bloomington if it is 2.45 centimeters on a map that has a scale of 1 centimeter:3 kilometers.
- Architecture** The Sears Tower in downtown Chicago is 110 stories high. A scale drawing has a scale of 1.5 centimeters:1 story. How tall is the Sears Tower on the scale drawing?



- 9. Standardized Test Practice** Which is the actual length of a sofa that is 4 inches on a scale drawing if the scale is 2 inches:5 feet?

- A** 10 feet      **B** 20 feet      **C** 15 feet      **D** 30 feet

Answers: 1. 75 in., 2. 2 cm, 3. 9 cm, 4. 162 cm, 5. 4 in., 6. 10 in., 7. 7.35 km, 8. 165 cm, 9. A

# Percents and Fractions (pages 336–338)

You can use a proportion to express a fraction as a percent. To write a percent as a fraction, begin with a fraction that has a denominator of 100.

Then write the fraction in simplest form. Recall that  $\frac{n}{100} = n\%$ .

## EXAMPLES

**A** Express  $\frac{4}{5}$  as a percent.

$$\frac{4}{5} = \frac{n}{100} \quad \text{Write a proportion}$$

$$400 = 5n \quad \text{Multiply to find the cross products.}$$

$$\frac{400}{5} = \frac{5n}{5} \quad \text{Divide each side by 5.}$$

$$80 = n$$

$$\frac{4}{5} = 80\%$$

**B** Express 24% as a fraction in simplest form.

*Estimate: 24% is about 25%, which is  $\frac{1}{4}$ .*

$$24\% = \frac{24}{100}$$

$$= \frac{24 \div 4}{100 \div 4} \quad \text{The GCF is 4.}$$

$$= \frac{6}{25}$$

$$24\% = \frac{6}{25} \quad \text{Compare to the estimate.}$$

## Try These Together

**Express each fraction as a percent.**

1.  $\frac{2}{5}$

2.  $\frac{1}{2}$

*HINT: Solve a proportion in which the fraction is equal to  $\frac{n}{100}$ .*

## PRACTICE

**Express each fraction as a percent.**

3.  $\frac{3}{10}$

4.  $\frac{3}{4}$

5.  $\frac{2}{6}$

6.  $\frac{15}{20}$

7.  $\frac{4}{16}$

8.  $\frac{3}{5}$

**Express each percent as a fraction in simplest form.**

9. 20%

10. 85%

11. 25%

12. 62.5%

13. 87.5%

14. 30%

15. 15%

16. 37.5%

**17. Hobbies** At a hot air balloon festival, 60% of the hot air balloons were partially colored red. Write the percent of hot air balloons that were partially colored red as a fraction in simplest form.



**18. Standardized Test Practice** A store advertised a sale where every item was  $\frac{1}{8}$  off. What is this fraction written as a percent?

**A** 2.5%

**B** 12.5%

**C** 22.5%

**D** 37.5%

Answers: 1. 40% 2. 50% 3. 30% 4. 75% 5.  $33\frac{1}{3}\%$  6. 75% 7. 25% 8. 60% 9.  $\frac{5}{1}$  10.  $\frac{20}{17}$  11.  $\frac{4}{1}$  12.  $\frac{8}{5}$  13.  $\frac{8}{7}$  14.  $\frac{10}{3}$  15.  $\frac{20}{3}$  16.  $\frac{8}{3}$  17.  $\frac{3}{5}$  18. B

# Percents and Decimals

(pages 339–341)



Any decimal can also be written as a fraction. You can use this to express any decimal as a percent.

<b>Writing a Decimal as a Percent</b>	To write 0.32 as a percent, multiply the decimal by 100 and add the percent symbol. So, $0.32 = 0.\overbrace{32} = 32\%$ .
<b>Writing a Percent as a Decimal</b>	To write a percent as a decimal, divide the percent by 100 and remove the percent symbol. $64\% = \overbrace{64} = 0.64$ .

## EXAMPLES

**A** Write 0.72 as a percent.

$0.72 = 72\%$  *Multiply the decimal by 100 and add the percent symbol.*

**B** Write 57% as a decimal.

$57\% = 0.57$  *Divide the percent by 100 and remove the percent symbol.*

### Try These Together

**Express each decimal as a percent.**

1. 0.25

2. 0.76

*HINT: Multiply by 100 and add the percent symbol.*

## PRACTICE

**Express each decimal as a percent.**

3. 0.54

4. 0.67

5. 0.1

6. 0.08

7. 0.42

8. 0.17

**Express each percent as a decimal.**

9. 48%

10. 75%

11. 9%

12. 23%

13. 35%

14. 99.8%

15. 4%

16. 15.1%

17. What decimal is equivalent to 39.5%?

18. Write the percent that is equivalent to 0.652.

19. **Recycling** In a recent year, the aluminum recycling rate was 62.8%. Write this percent as a decimal.

20. **Standardized Test Practice** In the mid-1990s, 48% of the working people in the world were employed in agriculture. How is this percent written as a decimal?

**A** 0.048

**B** 48

**C** 4.8

**D** 0.48

**Answers:** 1. 25% 2. 76% 3. 54% 4. 67% 5. 10% 6. 8% 7. 42% 8. 17% 9. 0.48 10. 0.75 11. 0.09 12. 0.23 13. 0.35 14. 0.998 15. 0.04 16. 0.151 17. 0.395 18. 65.2% 19. 0.628 20. D

# Percents Greater Than 100% and Percents Less Than 1%

(pages 342–345)

When you express a percent greater than 100% as a decimal, the resulting decimal is greater than 1. When you express a percent less than 1% as a decimal, the resulting decimal is less than 0.01.

## EXAMPLES

**A** Write 142% as a decimal.

$142\% = 1.42$  Divide the percent by 100 and remove the percent symbol.

**B** Write 0.00825 as a percent.

$0.00825 = 0.825\%$  Multiply the decimal by 100 and add the percent symbol.

## Try These Together

1. Express 0.682% as a decimal.

*HINT: The resulting decimal is less than 0.01.*

2. Express 3.7 as a percent.

*HINT: The decimal is greater than 1.*

## PRACTICE

**Express each percent as a decimal.**

3. 125%

4. 545%

5. 210%

6. 356%

7. 0.08%

8. 0.85%

**Express each number as a percent.**

9. 7

10. 0.007

11. 1.28

12. 4.5

13. 3.86

14.  $\frac{8}{2,000}$

15. Write 430% as a decimal.

16. Express 0.006 as a percent.

17. **Internet** Lara and Lezlie both use the Internet. Lara's Internet usage is 160% of Lezlie's. By what decimal would you multiply Lezlie's usage to get Lara's usage?



18. **Standardized Test Practice** The area of Indiana is about 0.0098 of the area of the United states. What is 0.0098 written as a percent?

**A** 0.98%

**B** 98%

**C** 9.8%

**D** 0.0098%

Answers: 1. 0.00682 2. 370% 3. 1.25 4. 5.45 5. 2.1 6. 3.56 7. 0.0008 8. 0.0085 9. 700% 10. 0.7% 11. 128% 12. 450% 13. 386% 14. 0.4% 15. 43 16. 0.6% 17. 1.6 18. A

## Percent of a Number (pages 346–348)



Suppose you read in the school newspaper that 62% of students who were surveyed buy their lunches at school. Of the 200 students who were surveyed, how many buy their lunches? You can either use a proportion or multiplication to solve this problem.

## EXAMPLES

**A** Find 62% of 200.

*Method 1: Use a proportion. Let  $s$  represent the number of students who buy their lunches.*

$$\frac{s}{200} = \frac{62}{100}$$

*The ratio of  $s$  students to the 200 who were surveyed equals 62%,*

$$\text{or } \frac{62}{100}.$$

$s \times 100 = 200 \times 62$  *Find the cross products.*

$$\frac{100s}{100} = \frac{12,400}{100}$$

*Divide each side by 100.*

$$s = 124$$

*So, 62% of 200 is 124.*

**B** Find 62% of 200.

*Method 2: Use multiplication.*

*First express the percent as a decimal and then multiply.*

$$62\% \text{ of } 200 = 0.62 \times 200 \quad \text{Remember, } 62\% = 0.62.$$

$$= 124$$

*So, 62% of 200 is 124.*

## Try These Together

**Find each number. Round to the nearest tenth if necessary.**

- Find 50% of 66.
- What number is 20% of 200?

## PRACTICE

**Find each number. Round to the nearest tenth if necessary.**

- Find 80% of 40.
- What number is 12% of 68?
- Find 25% of 160.
- 15% of 74 is what number?
- What number is 18% of 96?
- What number is 75% of 24?
- 85% of 225 is what number?
- Find 33% of 90.
- What number is 37% of 128?
- Find 10% of 100.

- 13. Landscaping** Mr. and Mrs. Morrissey want to replace 60% of the grass in their yard with bushes and plants. If their yard has 6,000 square feet of grass, how many square feet will be replaced with bushes and plants?

- 14. Standardized Test Practice** If 25% of the pieces of a 500-piece puzzle are edge pieces, how many are edge pieces?

**A** 75

**B** 150

**C** 125

**D** 225

Answers: 1. 33 2. 40 3. 32 4. 18 5. 8.2 6. 191.3 7. 40 8. 29.7 9. 11.1 10. 47.4 11. 17.3 12. 10 13. 3,600 14. C

# The Percent Proportion

(pages 349–351)



The **percent proportion** is  $\frac{P}{B} = \frac{r}{100}$ , where  $P$  represents the **percentage**,  $B$  represents the **base**, and  $r$  represents the number per 100. Using the proportion  $\frac{1}{2} = \frac{50}{100}$ , you can see how to use the percent proportion to solve the three basic types of percent equations.

<b>Find the percentage.</b>	What number is 50% of 2? (Percentage)	$\frac{?}{2} = \frac{50}{100}$
<b>Find the rate.</b>	1 is what percent of 2? (Rate)	$\frac{1}{2} = \frac{?}{100}$
<b>Find the base.</b>	1 is 50% of what number? (Base)	$\frac{1}{?} = \frac{50}{100}$

## EXAMPLES

**A** 32 is 40% of what number?

$$\frac{32}{B} = \frac{40}{100}$$

Write the percent proportion replacing  $P$  with 32 and  $r$  with 40.

$$32 \times 100 = B \times 40 \quad \text{Find the cross products.}$$

$$\frac{3,200}{40} = \frac{40B}{40} \quad \text{Divide each side by 40.}$$

$$80 = B \quad \text{So, 32 is 40% of 80.}$$

**B** 30 is what percent of 150?

$$\frac{30}{150} = \frac{r}{100}$$

Write the percent proportion replacing  $P$  with 30 and  $B$  with 150.

$$30 \times 100 = 150 \times r \quad \text{Find the cross products.}$$

$$\frac{3,000}{150} = \frac{150r}{150} \quad \text{Divide each side by 150.}$$

$$20 = r \quad \text{So, 30 is 20% of 150.}$$

## Try These Together

**Find each number. Round to the nearest tenth if necessary.**

1. 6 is what percent of 12?

2. What number is 68% of 90?

## PRACTICE

**Find each number. Round to the nearest tenth if necessary.**

- What percent of 24 is 6?
- 8% of what number is 10?
- What number is 60% of 20?
- What percent of 400 is 60?
- Find 110% of 16.
- What percent of 40 is 8?
- 15 is 30% of what number?
- What percent of 96 is 12?
- What number is 12% of 50?
- 90 is 45% of what number?



**13. Standardized Test Practice** During a flu epidemic, 3 of the 20 students in Marina's class were absent with the flu. What percent of the students in Marina's class were absent?

**A** 10%

**B** 5%

**C** 20%

**D** 15%

Answers: 1. 50% 2. 61.2 3. 25% 4. 20% 5. 125 6. 50 7. 12 8. 12.5% 9. 15% 10. 6 11. 17.6 12. 200 13. D

# Chapter 8 Review



## Commercial Percents

Television stations and networks sell advertising time, called commercials, to pay for the cost of running the station and network, and to pay for producing the programs that you watch. Have you ever wondered how much time you spend watching commercials instead of programs? With a parent, follow the steps below to find out. *Note:* If you or your parent would rather not watch television, you can complete this same exercise by listening to the radio.

1. Decide on a television program to watch. Before the program starts, have this worksheet, a pencil, and a watch with a second hand ready.
2. Using the table below, record the exact time the program begins. Then, using the watch, record the length of time of every commercial break during the program. Networks and television stations usually show four or more commercials in a row, so be sure to record the length of time of each entire commercial break. Also be sure to record the length of time of the commercials after the program, right up until the next program begins.
3. Once you have watched the entire program, find the percent of time you spent watching commercials and the percent of time you spent watching the program. Use the table below to help you.

<b>Program Start Time</b>	
<b>Program End Time</b>	
<b>Total Program Time (minutes)</b>	

<b>Commercial Breaks</b>	<b>Time</b>
<b>Break 1</b>	
<b>Break 2</b>	
<b>Break 3</b>	
<b>Break 4</b>	
<b>Break 5</b>	
<b>(add more breaks if you need them)</b>	
<b>Total Time of Commercials (minutes)</b>	

Use the percent proportion to find out what percent of the program's time was spent on commercials. The total time (in minutes) of the commercials is the percentage ( $P$ ) and the total program time is the base ( $B$ ). You are looking for  $r$ . Round your result to the nearest whole percent.

$$\frac{P}{B} = \frac{r}{100}$$

Percentage of time for commercials:

\_\_\_\_\_

Try this activity with different kinds of programs, with programs of different lengths, and with programs shown at different times of the day. Are the percentages the same? Explain.

Answers are located on page 114.