

Theoretical and Experimental Probability

(pages 530–533)



Theoretical probability is the ratio of the number of ways an event can occur to the total number of possible **outcomes**. For example, the theoretical probability of rolling a 1 on a number cube is $\frac{1}{6}$. That's because only one side of a number cube shows a 1, the event you are trying to get, while there are six total sides, or possible outcomes.

Finding Theoretical Probability	$P(\text{event}) = \frac{\text{number of ways the event can occur}}{\text{number of possible outcomes}}$
Experimental Probability	The experimental probability of an event is the estimated probability based on the number of positive outcomes in an experiment. To find the experimental probability of rolling a 1 on a number cube, you would roll a number cube repeatedly and record the outcomes.

EXAMPLE

A class of 32 students has 18 boys and 14 girls. If one student is chosen to take attendance for the semester, what is the probability that a boy is chosen?

$$\frac{18}{32} \leftarrow \begin{array}{l} \text{number of ways to chose a boy} \\ \text{number of possible students in the class} \end{array}$$

Therefore, $P(\text{a boy being chosen}) = \frac{18}{32}$ or $\frac{9}{16}$.

Try These Together

If you have 12 coins (5 pennies, 4 nickels, 2 dimes, and 1 quarter) in a bag, find the theoretical probability of selecting:

- one quarter in one draw.
- one penny in one draw.

HINT: Think of the ratio of the number of coins in the bag that you want to draw to the total number of coins in the bag.

PRACTICE

Use the same situation for drawing coins as above.

- one dime in one draw
- one nickel in one draw



- 5. Standardized Test Practice** Lavon had a bag of candies. There were 20 candies in the bag: 6 red, 5 orange, 3 brown, 2 yellow, and 4 blue. Without looking, she chose a candy, recorded the color, and returned the candy to the bag. She performed this experiment 100 times and found that she chose an orange candy 22 times. What was the experimental probability of choosing an orange candy?

A $\frac{1}{5}$

B $\frac{1}{4}$

C $\frac{11}{50}$

D $\frac{24}{100}$

Answers: 1. $\frac{1}{12}$ 2. $\frac{12}{1}$ 3. $\frac{5}{5}$ 4. $\frac{3}{1}$ 5. C

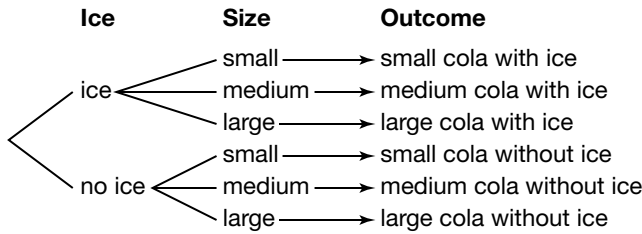
Tree Diagrams (pages 534–536)



One way to find possible outcomes and probability is with a **tree diagram**.

EXAMPLE

At a concession stand, you can order a small, medium, or large cola, with or without ice. Use a tree diagram to find the number of possible outcomes.



Try These Together

For each situation, use a tree diagram to find the total number of outcomes.

- choosing white or rye bread with either ham, turkey, or salami
- going in-line skating or biking to either the library, grocery store, or the mall
- buying a sweater or a shirt in either orange, blue, turquoise, or red

HINT: Each of the two objects in the first set goes with each of the objects in the second set.

PRACTICE

For each situation, use a tree diagram to find the total number of outcomes.

- growing tulips, roses, or daisies in either pink, white, or yellow
- taking a sculpture or woodworking class at either a school, a community center, or a museum
- sitting in a room with a sofa, a chair, a love seat or a recliner, in either a soft, hard, or medium firmness
- Music** You are in charge of music for a party. You bring three CDs: pop, jazz, and country. How many different ways can you play all three CDs so that each one is played exactly once?



8. **Standardized Test Practice** A baseball manager has four possible starting pitchers for a game. He also must decide which of two catchers to put in the starting lineup. How many ways can he choose the players for these two positions?

A 6

B 8

C 9

D 16

Answers: 1. 6 2. 6 3. 8 4. 9 5. 6 6. 12 7. 6 8. B

The Counting Principle (pages 538–541)

In Lesson 13-2, you learned to find outcomes using a tree diagram. In this lesson, you will learn to use the **Counting Principle** to find the number of possible outcomes.

The Counting Principle	If an event M can occur m ways and is followed by an event N that can occur n ways, then the event M followed by N can occur $m \times n$ ways.
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EXAMPLE

Yvette can take her driving test on Monday, Wednesday, or Friday, at 4:00 P.M., 5:00 P.M., or 6:00 P.M. How many different opportunities does she have to take her driving test?

$$\underbrace{\text{number of days the test is given}}_3 \times \underbrace{\text{number of times per day the test is given}}_3 = \underbrace{\text{opportunities to take the test}}_9$$

There are 9 opportunities for Yvette to take her driving test.

Try These Together

Use the Counting Principle to find the total number of outcomes in each situation.

- creating new hybrid flowers with short or long petals in either purple, red, or yellow
- baking a yellow, chocolate, strawberry, or vanilla cake frosted with either vanilla, chocolate, cherry, or strawberry frosting

HINT: Find the number of ways each event occurs, and multiply.

PRACTICE

Use the Counting Principle to find the total number of outcomes in each situation.

- rolling three six-sided number cubes
- making a sandwich with either wheat or pumpernickel bread, and either salami, turkey, or pastrami, and either mustard, mayonnaise, butter, or horseradish
- Automobiles** Each license plate in a given state contains three letters and three numbers. What is the total number of license plates if the first three characters are letters and the last three characters are digits?



6. **Standardized Test Practice** Every Social Security card has a nine-digit identification number. How many possible Social Security numbers are there?

A 100,000

B 1,000,000

C 100,000,000

D 1,000,000,000

Answers: 1. 6 2. 16 3. 216 4. 24 5. 17,576,000 6. D

Independent and Dependent Events

(pages 542–545)



If you roll two number cubes, the number that you roll on the second cube is not affected by the number you rolled on the first cube. These events are called **independent events**. If the result of one event affects the result of a second event, the events are called **dependent events**.

Probability of Independent Events

The probability of two independent events can be found by multiplying the probability of one event by the probability of the second event.

EXAMPLES

- A** Find the probability of tossing a 5 on each of two number cubes.

These are independent events.

$P(5 \text{ on one cube}) = \frac{1}{6}$ because there are six numbers on a cube.

$P(5 \text{ on each cube}) = \frac{1}{6} \times \frac{1}{6}$, or $\frac{1}{36}$.

- B** You have four pennies and four nickels in a bag. What is the probability of drawing two pennies in a row, if you keep the first coin you draw?

These two draws are dependent events.

$P(\text{penny on first draw}) = \frac{4}{8}$ or $\frac{1}{2}$ because there are 4 pennies and 8 coins total.

$P(\text{penny on second draw}) = \frac{3}{7}$ because you removed one penny, leaving 3 pennies and 7 coins total.

$P(\text{two pennies in a row}) = \frac{1}{2} \times \frac{3}{7}$ or $\frac{3}{14}$.

Try These Together

Tell whether each event is independent or dependent.

- tossing a coin twenty times
- choosing two cards from one deck, keeping the first card.

Hint: Does one event affect the other event?

PRACTICE

Find the probability of each event.

- tossing an even number on each of two number cubes
- A bag contains three blue marbles, four red marbles and two clear marbles. Three are drawn without each selection being replaced. Find $P(\text{red, then blue, then clear})$.

- 5. Standardized Test Practice** There are 3 bottles of juice and 4 bottles of water in Nate's ice chest. What is the probability that he will reach into the ice chest without looking and pull out two bottles of water in a row if he does not replace the first bottle?

A $\frac{1}{2}$

B $\frac{4}{7}$

C $\frac{2}{7}$

D $\frac{3}{6}$

Answers: 1. independent 2. dependent 3. $\frac{4}{1}$ 4. $\frac{21}{1}$ 5. C

Permutations (pages 547–549)

Suppose you need to arrange 8 books on a bookshelf in the library. How many ways could you arrange the books? What you're trying to count are **permutations**. You can find the answer to this question by finding $8!$ or eight **factorial**.

Permutation	A permutation is an arrangement, or listing, of objects in which order is important.
Factorial	$n!$ or “ n factorial” is the product of all the counting numbers beginning with n and counting backward to 1. For example, $3! = 3 \times 2 \times 1$, or 6.

EXAMPLE

How many ways can you arrange 8 books on a bookshelf?

Each arrangement is a permutation. Since there are 8 books, there are eight different choices for the first book you place on the shelf. Once the first book is placed, there are seven choices for the second book, six choices for the third book, and so on.

number of permutations = $8!$

number of permutations = $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

number of permutations = 40,320

There are 40,320 ways to arrange the eight books.

Try These Together

Find the value of each expression.

1. $10 \times 9 \times 8 \times 7$

2. $5!$

HINT: For factorials, start with the given number and multiply by each lower number down to one.

PRACTICE

Find the value of each expression.

3. $4!$

4. $6!$

5. $5 \times 4 \times 3$

6. $12 \times 11 \times 10$

7. How many ways could you and two friends line up to run a race?

8. You must select a five-digit password, where each digit must be a number from 0 to 9 without repeating any numbers. How many passwords are there?

9. **Television** There are 51 contestants in a talent pageant each fall. How many ways can first place and runner-up be awarded?



10. **Standardized Test Practice** A television network has six different prime-time slots to fill in one evening. They can choose from ten different shows. How many arrangements of programs could the network show?

A 60

B 151,200

C 200,000

D 310,110

Answers: 1. 5,040 2. 120 3. 24 4. 720 5. 60 6. 1,320 7. 6 8. 30,240 9. 2,560 10. B

Combinations (pages 551–553)

An arrangement, or listing, of objects in which order is not important is called a **combination**. You can find the number of combinations of objects by dividing the number of permutations of the entire set by the number of ways each smaller set can be arranged.

EXAMPLE

How many combinations of two menu items can be chosen from a menu of four items?

There are 4×3 permutations of two items chosen from the menu of four.

There are $2!$ or 2×1 ways to arrange the two items.

$$\frac{4 \times 3}{2 \times 1} = \frac{12}{2} \text{ or } 6$$

There are 6 combinations of two menu items that can be chosen from the menu of four items.

Try These Together

Solve each problem.

1. How many ways can a three-topping pizza be made if the chef must choose from seven ingredients? Assume all three toppings are different.
2. There are four computer-programming jobs to fill from a pool of six applicants. How many ways can four programmers be chosen?

PRACTICE

Solve each problem.

3. In how many ways can 2 flight attendants be selected from a group of 5 to work a flight?
4. Any five Supreme Court justices comprise a majority for the group of nine justices. How many groups of five are there?



5. **Standardized Test Practice** How many ways can a four-member debate team be selected from a group of eight students?

A 60

B 70

C 80

D 90

Answers: 1. 35 2. 15 3. 10 4. 126 5. B

