

# Ratios and Rates

(pages 312–315)



You can compare two quantities by using a ratio. A common way to express a ratio is as a fraction in simplest form. If the two quantities you are comparing have different units of measure, this kind of ratio is called a **rate**. A rate is in the form of a *unit* rate when the denominator is 1.

### Writing a Rate and a Unit Rate

A rate is a ratio of two measurements that have different units. To write a ratio as a unit rate, divide the numerator and denominator by the same number to rewrite the ratio as a fraction with a denominator of 1.

## EXAMPLES

**A** Write the ratio in three different ways:  
5 sixth-graders out of 15 students.  
Express this ratio as a fraction in simplest form.

As a fraction  $\frac{5}{15}$

As a ratio 5:15

In words 5 to 15

Another way is in the problem: 5 out of 15.

$\frac{5}{15}$  in simplest form is  $\frac{1}{3}$ .

**B** Express the ratio as a unit rate: 15 pencils for \$5. How many pencils can you buy for \$1?

Write the ratio as a fraction.  $\frac{15 \text{ pencils}}{\$5}$

To rewrite the fraction with a denominator of 1, divide numerator and denominator by 5.

$\frac{15 \text{ pencils}}{\$5} = \frac{\frac{15}{5} \text{ pencils}}{\$1}$  or 3 pencils for \$1

## Try These Together

1. Write the ratio in three different ways:  
7 sodas out of 20 are sugar free.

*HINT: Write the numbers in the same order as they appear in the problem.*

2. Express the ratio as a rate: \$14.50 for 5 rides. What is the cost for one ride?

*HINT: Divide numerator and denominator by 5.*

## PRACTICE

**Express each ratio as a fraction in simplest form.**

3. 4 out of 16 papers are typed

4. 5 out of 10 horses are white


5. 7 blue bicycles out of 21 bicycles

6. 4 watermelons out of 10 melons

**Express each ratio as a rate.**

7. \$1.50 for 3 bottles of juice

8. 5 bracelets for \$25.00

 **9. Standardized Test Practice** If milk costs \$5.50 for 2 gallons, how much does it cost per gallon?

A \$11.00

B \$10.50

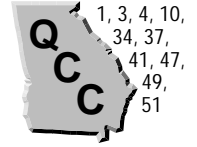
C \$2.75

D \$3.50

Answers: 1. 7:20, 7 to 20;  $\frac{7}{20}$  2.  $\frac{\$14.50}{5 \text{ rides}}$  or \$2.90 per ride 3.  $\frac{4}{16}$  4.  $\frac{5}{10}$  5.  $\frac{3}{1}$  6.  $\frac{2}{5}$  7. \$0.50 per bottle of juice 8. \$5.00 per bracelet 9. C

## Solving Proportions

(pages 317–320)



A **proportion** is an equation that shows that two ratios are equivalent. The general form of a proportion is  $\frac{a}{b} = \frac{c}{d}$ , where neither  $b$  nor  $d$  is equal to zero. The **cross products** of a proportion are  $ad$  and  $bc$ .

<b>Property of Proportions</b>	The cross products of a proportion are equal. If $\frac{a}{b} = \frac{c}{d}$ , then $ad = bc$ .
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## EXAMPLES

**A** Use cross products to find out whether this pair of ratios forms a proportion.

$$\frac{3}{4}, \frac{9}{12}$$

Does  $\frac{3}{4} = \frac{9}{12}$ ? Are the cross products equal?

Does  $3 \times 12 = 4 \times 9$ ? Yes, because  $36 = 36$ .

$\frac{3}{4} = \frac{9}{12}$  is a proportion because the cross products are equal.

**B** Solve the proportion  $\frac{2}{7} = \frac{y}{21}$  for  $y$ .

Write the cross products.

$$2 \times 21 = 7 \times y$$

$$42 = 7y$$

$$\frac{42}{7} = \frac{7y}{7}$$

Divide each side of the equation by 7.

$$6 = y$$

The solution is 6.

## Try These Together

1. Use cross products to determine whether this pair of ratios forms a proportion.  $\frac{0.5}{2}, \frac{0.4}{1.6}$

*HINT: Write the proportion with a ? over the =, and test to see if the cross products are really equal.*

2. Solve the proportion  $\frac{3}{p} = \frac{4}{20}$ .

*HINT: Set the cross products equal to each other and solve for  $p$ .*

## PRACTICE

Determine whether each pair of ratios forms a proportion.

3.  $\frac{1}{2}, \frac{5}{10}$

4.  $\frac{4}{8}, \frac{2}{4}$

5.  $\frac{4}{5}, \frac{1}{8}$

6.  $\frac{8}{13}, \frac{2}{5}$

Solve each proportion.

7.  $\frac{3}{6} = \frac{x}{2}$

8.  $\frac{4}{w} = \frac{2}{11}$

9.  $\frac{9}{12} = \frac{6}{z}$

10.  $\frac{d}{25} = \frac{2}{10}$



11. **Standardized Test Practice** The home economics class is making a casserole. They need 3 eggs for 1 casserole. How many eggs do they need for 4 casseroles?

A 9

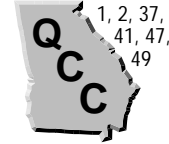
B 12

C 15

D 10

Answers: 1. yes 2. 15 3. yes 4. yes 5. no 6. no 7. 1 8. 22 9. 8 10. 5 11. B

# Scale Drawings

 (pages 324–327)


A **scale drawing** is exactly the same shape as an object, but the drawing may be larger or smaller than the real object. When you know the length in a scale drawing, you can use a proportion to find the actual length,  $\ell$ .

### Reading a Scale Drawing

The scale written on the drawing or model gives the ratio that compares the lengths on the drawing to the actual lengths of the object. Use the scale of the drawing for one of the ratios and the known and unknown lengths for the other ratio. Write a proportion and solve it for the unknown length.

## EXAMPLES

**A** A model car has a scale of 1:16. A window on the model measures  $\frac{1}{32}$  of a meter. What will this same window measure on the real car?

$$\frac{1}{16} = \frac{\frac{1}{32} \text{ meter}}{\ell} \quad \text{Write a proportion.}$$

$$\ell = 16\left(\frac{1}{32} \text{ meter}\right) \quad \text{Find cross products.}$$

$$\ell = \frac{1}{2} \text{ meter} \quad \text{Solve.}$$

The actual window measures  $\frac{1}{2}$  meter.

**B** The doorway of an actual house measures 3 ft wide. How wide will the doorway in a model house be if the scale is 1 ft = 2 in.?

$$\frac{1 \text{ ft}}{2 \text{ in.}} = \frac{3 \text{ ft}}{\ell} \text{ so } 1 \times \ell = 6 \text{ or } \ell = 6.$$

The model doorway will be 6 inches wide.

## Try These Together

- The scale of a map is 1 inch = 25 miles. The distance on the map between two cities is 7 inches. How many miles apart are they?
- A line on a scale drawing of a building measures 15 inches. The same length on the actual building is 5 yards. What is the scale of the drawing in simplest form?

*HINT: Write a proportion using  $\frac{1}{25}$  as one ratio.*

*HINT: One ratio is  $\frac{15}{5}$  and the other is  $\frac{x \text{ inches}}{1 \text{ yard}}$ .*

## PRACTICE

**3. Transportation** The oldest monorail system in the world is in Wuppertal, Germany. Its track is 8.5 miles long. If you wanted to build a model of the track that has a scale of 1 inch = 0.5 miles, how long would the model track be?



**4. Standardized Test Practice** Mavis and Reese want to rearrange the furniture in their living room. Before they move the furniture, they make a model. The scale for the model is 1 inch = 2 feet. If their sofa is actually 6 feet long, how long is the model of the sofa?

**A** 3 inches

**B** 4 inches

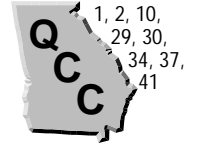
**C** 3 feet

**D** 4 feet

Answers: 1. 175 miles 2. 3 inches to 1 yard 3. 17 inches 4. A

# Percents and Fractions

(pages 330–333)



A **percent** is a ratio that compares a number to 100.

<b>Expressing a Percent as a Fraction</b>	To write a percent as a fraction, follow these steps. <ul style="list-style-type: none"> <li>Express the percent as a fraction with a denominator of 100.</li> <li>Simplify the fraction.</li> </ul>
<b>Expressing a Fraction as a Percent</b>	To write a fraction as a percent, follow these steps. <ul style="list-style-type: none"> <li>Set up a proportion with the fraction as one ratio and <math>\frac{x}{100}</math> as the other.</li> <li>Find the cross products and divide to solve for <math>x</math>. The fraction is equal to <math>x</math> percent.</li> </ul>

## EXAMPLES

**A** Express 75% as a fraction in simplest form.

$$75\% \text{ is } \frac{75}{100}.$$

$$75\% = \frac{75}{100}$$

$$75\% = \frac{3}{4} \quad \text{Divide numerator and denominator by the common factor of 25.}$$

**B** Express the fraction  $\frac{14}{25}$  as a percent.

$$\frac{14}{25} = \frac{x}{100} \quad \text{Write a proportion.}$$

$$1,400 = 25x \quad \text{Find the cross products.}$$

$$\frac{1,400}{25} = x \quad \text{Divide to solve for } x.$$

$$56 = x, \text{ so } \frac{14}{25} = 56\%$$

## Try These Together

1. Express the fraction  $\frac{13}{20}$  as a percent.

*HINT: Write a proportion and solve for  $x$ .*

2. Express 120% as a fraction in simplest form.

*HINT: Begin with the fraction  $\frac{120}{100}$ .*

## PRACTICE

Express each percent as a fraction in simplest form.

3. 25%

4. 10%

5. 30%

6. 45%

7. 60%

8. 95%

9. 16%

10. 58%

Express each fraction as a percent.

11.  $\frac{1}{2}$

12.  $\frac{8}{5}$

13.  $\frac{3}{4}$

14.  $\frac{44}{100}$

15.  $\frac{12}{20}$

16.  $\frac{36}{40}$

17.  $\frac{8}{40}$

18.  $\frac{7}{5}$



19. **Standardized Test Practice** What is 24% expressed as a fraction in simplest form?

A  $\frac{18}{75}$

B  $\frac{12}{50}$

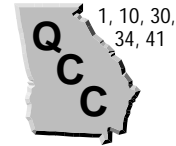
C  $\frac{24}{100}$

D  $\frac{6}{25}$

Answers: 1. 65% 2.  $1\frac{1}{5}$  3.  $\frac{4}{1}$  4.  $\frac{1}{10}$  5.  $\frac{10}{3}$  6.  $\frac{20}{9}$  7.  $\frac{5}{3}$  8.  $\frac{19}{20}$  9.  $\frac{25}{4}$  10.  $\frac{50}{29}$  11. 50% 12. 160% 13. 75% 14. 44% 15. 60% 16. 90% 17. 20% 18. 140% 19. D

# Percents and Decimals

(pages 334–336)



You have seen that percents can be written as fractions. Percents can also be written as decimals, and decimals can be written as percents.

<b>Writing a Percent as a Decimal</b>	To write a percent as a decimal, follow these steps. <ul style="list-style-type: none"> <li>• Rewrite the percent as a fraction with a denominator of 100.</li> <li>• Express the fraction as a decimal.</li> </ul>
<b>Writing a Decimal as a Percent</b>	To write a decimal as a percent, follow these steps. <ul style="list-style-type: none"> <li>• Rewrite the decimal as a fraction with a denominator of 100.</li> <li>• Express the fraction as a percent.</li> </ul>

## EXAMPLES

**A** Express 56% as a decimal.

$$56\% = \frac{56}{100} \text{ which is } 0.56$$

**C** Express 0.35% as a decimal.

$$0.35\% = \frac{0.35}{100} \text{ Multiply by } \frac{100}{100} \text{ to get rid of the decimal in the numerator.}$$

$$= \frac{35}{10,000} \text{ which is } 0.0035$$

**B** Express 0.84 as a percent.

$$0.84 = \frac{84}{100} \text{ which is } 84\%$$

**D** Express 0.103 as a percent.

$$0.103 = \frac{103}{1,000} \text{ Divide numerator and denominator by 10.}$$

$$= \frac{10.3}{100} \text{ which is } 10.3\%$$

## Try These Together

1. Express 0.4% as a decimal.

*HINT: Rewrite as a fraction with a denominator of 100. Then multiply numerator and denominator by 10.*

2. Express 0.09 as a percent.

*HINT: Rewrite as a fraction with a denominator of 100.*

## PRACTICE

*Express each percent as a decimal.*

3. 27%

4. 18%

5. 46%

6. 55%

7. 72%

8. 91%

9. 11%

10. 34.5%

*Express each decimal as a percent.*

11. 0.14

12. 0.87

13. 0.25

14. 0.61

15. 0.59

16. 0.12

17. 0.73

18. 0.063



**19. Standardized Test Practice** In a taste test at a grocery store, people were given a chip with salsa on it and asked if they would buy the salsa. Of those who answered, 67% said “yes.” Express this percent as a decimal.

A 0.22

B 0.67

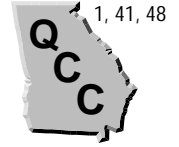
C 0.34

D 0.50

Answers: 1. 0.004 2. 9% 3. 0.27 4. 0.18 5. 0.46 6. 0.55 7. 0.72 8. 0.91 9. 0.11 10. 0.345 11. 14% 12. 87% 13. 25% 14. 61% 15. 59% 16. 12% 17. 73% 18. 6.3% 19. B

# Estimating with Percents

(pages 337–339)



When a problem asks for “about how many,” the word *about* tells you that an exact answer is not needed. You can estimate the answer.

<b>Common Equivalents for Percents and Fractions</b>	Memorizing these common equivalents will help you estimate. Often you can think of money to help you remember these. For example: A quarter is \$0.25 which is $\frac{1}{4}$ of a dollar.
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$20\% = \frac{1}{5}$	$25\% = \frac{1}{4}$	$12\frac{1}{2}\% = \frac{1}{8}$	$16\frac{2}{3}\% = \frac{1}{6}$
$40\% = \frac{2}{5}$	$50\% = \frac{1}{2}$	$37\frac{1}{2}\% = \frac{3}{8}$	$33\frac{1}{3}\% = \frac{1}{3}$
$60\% = \frac{3}{5}$	$75\% = \frac{3}{4}$	$62\frac{1}{2}\% = \frac{5}{8}$	$66\frac{2}{3}\% = \frac{2}{3}$
$80\% = \frac{4}{5}$	$100\% = 1$	$87\frac{1}{2}\% = \frac{7}{8}$	$83\frac{1}{3}\% = \frac{5}{6}$

## EXAMPLES

**A** Estimate 61% of 35.

The table shows that 60% is  $\frac{3}{5}$ . Multiply to estimate.

$$\frac{3}{5} \times 35 = 21. \text{ So } 61\% \text{ of } 35 \text{ is about } 21.$$

**B** Estimate 9% of 415.

10% is  $\frac{1}{10}$ . Multiply to estimate.

$$\frac{1}{10} \times 415 = 41.5. \text{ So } 9\% \text{ of } 415 \text{ is about } 41.$$

## Try These Together

1. Estimate 88% of 64.

HINT: Multiply to find  $\frac{7}{8}$  of 64.

2. Estimate 17% of 24.

HINT: Multiply to find  $\frac{1}{6}$  of 24.

## PRACTICE

*Estimate each percent.*

3. 26% of 40
4. 18% of 10
5. 48% of 30
6. 60% of 21
7. 73% of 104
8. 80% of 51
9. 101% of 41
10. 22% of 80
11. 34% of 9
12. 11% of 80
13. 58% of 25
14. 19% of 45
15. About how much is 48% of 12?
16. Estimate 24% of 200.
17. **School** There are 23 students in Donovan’s class. About 25% of his classmates are older than him. Estimate how many of Donovan’s classmates are older than him.



18. **Standardized Test Practice** Tyler’s family gets a busy signal 21% of the time they try to log on to the Internet. If they tried to log on 10 times in one day, about how many times would they get a busy signal?

**A** 2

**B** 3

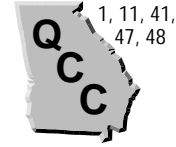
**C** 4

**D** 5

Answers: Sample answers are given. 1. 56 2. 4 3. 10 4. 2 5. 15 6. 12 7. 75 8. 40 9. 41 10. 16 11. 3 12. 8 13. 15 14. 9 15. 6 16. 50 17. 6 18. A

# Percent of a Number

(pages 340–343)



To find the percent of a number, you can change the percent to a fraction or to a decimal, and then multiply by the number. You can also use a calculator.

## Finding the Percent of a Number

- Method 1: Change the percent to a fraction and multiply.
- Method 2: Change the percent to a decimal and multiply.

## EXAMPLES

**A** Find 25% of 56.

$$25\% = \frac{1}{4}$$

$$\frac{1}{4} \times 56 = 14$$

25% of 56 is 14.

**B** Find 103% of 60.

$$103\% = \frac{103}{100} \text{ which is } 1.03$$

$$1.03 \times 60 = 61.8$$

103% of 60 is 61.8.

Notice that when you take a percent greater than 100 of a number, the answer is greater than the number.

## Try These Together

1. Find 0.5% of 30.

*HINT: Rewrite the percent as  $\frac{0.5}{100}$  and*

*then as  $\frac{5}{1,000}$  or 0.005. Then multiply.*

2. Find 7% of 40.

*HINT: Rewrite 7% as  $\frac{7}{100}$  or 0.07.*

## PRACTICE

*Find the percent of each number.*

- 25% of 20
- 40% of 65
- 35% of 80
- 60% of 35
- 80% of 120
- 75% of 64
- 10% of 70
- 20% of 45
- 33% of 300
- 20% of 120
- 50% of 64
- 90% of 60
- What is 90% of 70?
- Find 80% of 80.
- Games** 75% of the games sold at a game store are board games. If the game store sold 256 games in one day, how many of those games were board games?
- Banking** Catalina's mother went to the bank to take out \$40.00. She asked for 50% of the \$40.00 in dollar bills. How much money did she receive in dollar bills?



**19. Standardized Test Practice** What is 30% of 90?

A 27

B 30

C 33

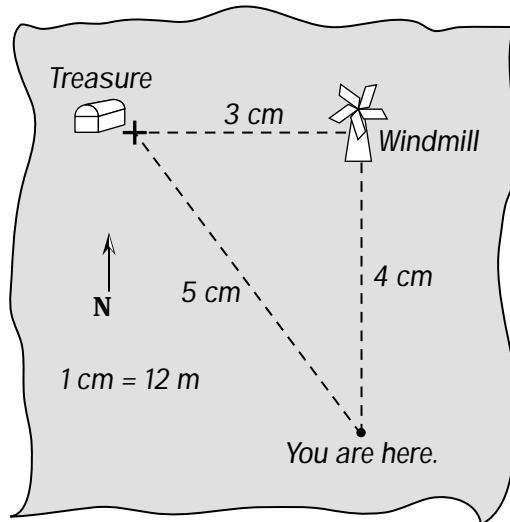
D 24

Answers: 1. 0.15 2. 2.8 3. 5 4. 26 5. 28 6. 21 7. 96 8. 48 9. 7 10. 9 11. 99 12. 24 13. 32 14. 54 15. 63  
16. 94 17. 192 18. \$20.00 19. A

# Chapter 8 Review

## Ratio Treasure

Use the treasure map to answer the following questions.



1. You're using the map to find a hidden treasure. If you walk directly to the treasure, how far will you walk?
2. To make sure you find the treasure, you decide to use a compass to walk north to the windmill first, then east to the treasure. How far are you from the windmill? How far is the windmill from the treasure?
3. Suppose instead that you are 60 meters south of a boulder, and the boulder is 80 meters west of a buried treasure. Draw a treasure map with a scale of 1 cm = 20 m. Be sure to label distances on your map according to the scale.

Answers are located on p. 109.