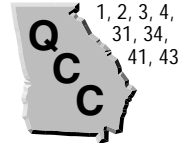


# Divisibility Patterns

(pages 133–136)



A whole number is **divisible** by another whole number if the quotient is also a whole number. For example, 36 is divisible by 12 because the quotient  $36 \div 12$  is the whole number 3. The whole number 3 is a **factor** of 36 since 36 is divisible by 3. Other factors of 36 are 1, 2, 4, 6, 9, 12, 18, and 36.

<b>Determining Divisibility</b>	<p>You can use the following rules to check divisibility.</p> <p>A number is divisible by</p> <ul style="list-style-type: none"> <li>• 2 if the digit in the ones place is even.</li> <li>• 3 if the sum of the digits is divisible by 3.</li> <li>• 4 if the number formed by the last two digits is divisible by 4.</li> <li>• 5 if the digit in the ones place is 5 or 0.</li> <li>• 6 if the number is divisible by both 2 and 3.</li> <li>• 9 if the sum of the digits is divisible by 9.</li> <li>• 10 if the digit in the ones place is 0.</li> </ul>
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## EXAMPLES

**A** Is 474 divisible by 6?

*Test for divisibility by 2. Is the digit in the ones place (4) even? Yes, so 474 is divisible by 2.*  
*Test for divisibility by 3. Is the sum of the digits ( $4 + 7 + 4$ ) divisible by 3? Is 15 divisible by 3? Yes, so 474 is divisible by 3. Since 474 is divisible by both 2 and 3, 474 is divisible by 6.*  
*(Check:  $474 \div 6 = 79$ )*

**B** Is 330 divisible by 2, 3, 4, 5, 6, 9, or 10?

*2: Yes, 0 is even.*  
*3: Yes,  $3 + 3 + 0 = 6$ .*  
*4: No, 30 is not divisible by 4.*  
*5: Yes, ones digit is 0.*  
*6: Yes, it is divisible by 2 and 3.*  
*9: No,  $3 + 3 + 0$ , or 6, is not divisible by 9.*  
*10: Yes, ones digit is 0.*

## Try These Together

1. Is 510 divisible by 10?

*HINT: Look at the last digit.*

2. Is 505 divisible by 2, 3, 4, 5, 6, 9, or 10?

*HINT: Apply each test, one at a time.*

## PRACTICE

**Determine whether the first number is divisible by the second number.**

3. 319; 3

4. 246; 2

5. 315; 3

6. 819; 9

7. 151; 6

8. 128; 4

9. 655; 5

10. 216; 6

**Determine whether each number is divisible by 2, 3, 4, 5, 6, 9, or 10.**

11. 224

12. 460

13. 720

14. 364

15. 231

16. 426

17. 884

18. 855



**19. Standardized Test Practice** Which of the following numbers is divisible by 3 and 4?

**A** 242

**B** 616

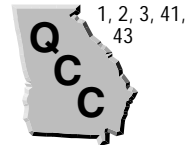
**C** 324

**D** 183

Answers: 1. yes 2. 5 3. no 4. yes 5. yes 6. yes 7. no 8. yes 9. yes 10. yes 11. 2, 4 12. 2, 4, 5, 10 13. 2, 3, 4, 5, 6, 9, 10 14. 2, 4 15. 3 16. 2, 3, 6 17. 2, 4 18. 3, 5, 9 19. C



# Sequences (pages 142–145)



A **sequence** of numbers is a list in a specific order. The numbers in a sequence are called **terms**. If you can always add the same number to the previous term to find the next term, the sequence is an **arithmetic sequence**. If you can always multiply the previous term by the same number to find the next term, the sequence is a **geometric sequence**.

<b>Finding the Pattern in a Sequence</b>	<ul style="list-style-type: none"> <li>To test for an arithmetic sequence, try subtracting the first term from the second. Then test to see if this same number separates each term of the sequence.</li> <li>To test for a geometric sequence, try dividing the second term by the first. Then test to see if this same quotient multiplies each term to give the next term in the sequence.</li> </ul>
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## EXAMPLES

**A** What is the pattern in this sequence?

1, 2, 1, 3, 1, 4, 1, 5, ...

Is it *arithmetic*, *geometric*, or *neither*?

*The pattern is that the first number, and every other number, is 1 and the numbers between are the counting numbers, starting with 2. This sequence is neither arithmetic nor geometric.*

**B** What is the pattern in this sequence?

0, 3, 6, 9, 12, 15, ... What is the next number?

Is this sequence *arithmetic*, *geometric*, or *neither*?

*Each term is equal to the previous term plus 3. The next number is 18. This is an arithmetic sequence.*

## Try These Together

1. Describe the pattern in 1, 3, 5, 7, ... and find the next three terms. Identify the sequence as *arithmetic*, *geometric*, or *neither*.

*HINT: Can you add the same number to 1 to get 3 as you can add to 3 to get 5?*

2. Give four terms of a sequence with this rule: Begin with 1 and multiply each term by 5.

*HINT: The first term is 1 and the second is 5.*

## PRACTICE

**Describe the pattern in each sequence. Identify the sequence as arithmetic, geometric, or neither. Then find the next three terms.**

3. 10, 20, 40, 80, 160, ...

4. 0, 1, 3, 6, 10, ...

5. 30, 33, 36, 39, 42, ...

**Create a sequence using each rule and give four terms beginning with the given number. State whether the sequence is arithmetic, geometric, or neither.**

6. Add 4 to each term; 8.

7. Add 2 to each term; 50.



**8. Standardized Test Practice** What is the missing term in this sequence?

6, 12, 18, ?, 30, 36, 42, ...

**A** 24

**B** 20

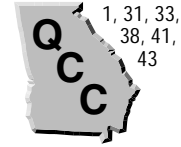
**C** 23

**D** 19

**Answers:** 1. Add 2; 9, 11, 13; arithmetic. 2. 1, 5, 25, 125. 3. Multiply by 2; geometric; 320, 640, 1,280. 4. Add one more to each term than was added to the previous term; neither; 15, 21, 28. 5. Add 3; arithmetic; 45, 48, 51. 6. 8. 12, 16, 20; arithmetic. 7. 50, 52, 54, 56; arithmetic. 8. A

# Greatest Common Factor

(pages 150–153)



The **greatest common factor (GCF)** of two or more numbers is the greatest number that is a factor of each number.

## Finding the Greatest Common Factor

To find the GCF of two or more numbers:

- Method 1: List the factors of each number and then identify the common factors. The greatest of these common factors is the GCF.
- Method 2: Write the prime factorization of each number. Then identify all common prime factors and find their product, which is the GCF.

## EXAMPLES

**A** Find the GCF of 12, 20, and 36 by listing factors.

factors of 12: **1, 2, 3, 4, 6, 12**

factors of 20: **1, 2, 4, 5, 10, 20**

factors of 36: **1, 2, 3, 4, 6, 9, 12, 18, 36**

The greatest of the common factors is 4, which is the GCF of 12, 20, and 36.

**B** Find the GCF of 27 and 90 by using prime factorization.

prime factorization of 27:  $3 \times 3 \times 3$

prime factorization of 90:  $2 \times 3 \times 3 \times 5$

The common prime factors are 3 and 3. Their product is 9. The GCF of 27 and 90 is 9.

## Try These Together

1. Find the GCF of 12 and 16 by listing factors.

*HINT: Circle the factors common to 12 and 16. Then choose the greatest of those circled.*

2. Find the GCF of  $15 = 3 \times 5$  and  $50 = 2 \times 5^2$  by listing common prime factors.

*HINT: There is only one common prime factor.*

## PRACTICE

**Find the GCF of each set of numbers by listing factors.**

3. 54, 81

4. 72, 90

5. 132, 144

6. 20, 36, 44

**Find the GCF of each set of numbers by listing common prime factors.**

7.  $9 = 3^2$   
 $36 = 2^2 \times 3^2$

8.  $45 = 3^2 \times 5$   
 $81 = 3^4$

**Find the GCF of each set of numbers by writing prime factorizations.**

9. 12, 48

10. 36, 54

11. 60, 42

**12. Life Science** The smallest adult male gorillas weigh about 135 kilograms. The smallest adult female gorillas weigh about 70 kilograms. What is the greatest common factor of these two numbers?



**13. Standardized Test Practice** Find the greatest common factor of 96 and 360.

**A** 5

**B** 12

**C** 36

**D** 24

Answers: 1. 4 2. 5 3. 27 4. 18 5. 12 6. 4 7. 9 8. 9 9. 12 10. 18 11. 6 12. 5 13. D

# Simplifying Fractions and Ratios

(pages 154–157)



A **ratio** is a comparison of two numbers by division. You can write the ratio of 3 to 9 as 3:9 or as the fraction  $\frac{3}{9}$ . You can simplify the fraction  $\frac{3}{9}$  by dividing both the numerator and denominator by 3. A fraction is in **simplest form** when the GCF of the numerator and denominator is 1.

<b>Simplifying Fractions</b>	<p>To write a fraction in simplest form:</p> <ul style="list-style-type: none"> <li>• find the GCF of the numerator and denominator,</li> <li>• divide both the numerator and denominator by the GCF, and</li> <li>• write the resulting fraction.</li> </ul>
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## EXAMPLES

**A** Express  $\frac{6}{12}$  in simplest form.

The GCF of 6 and 12 is 6.  
Divide the numerator (6) by 6 to get 1.  
Divide the denominator (12) by 6 to get 2.  
 $\frac{6}{12}$  in simplest form is  $\frac{1}{2}$ .

**B** Express 18:24 in simplest form.

The GCF of 18 and 24 is 6.  
Divide 18 by 6. Divide 24 by 6.  
18:24 in simplest form is 3:4.

## Try These Together

1. Express  $\frac{25}{45}$  in simplest form.

*HINT: The GCF of 25 and 45 is 5.*

2. Express 3:15 in simplest form.

*HINT: What is the GCF of 3 and 15?*

## PRACTICE

**Express each fraction or ratio in simplest form.**

3.  $\frac{82}{94}$

4.  $\frac{54}{63}$

5.  $\frac{48}{16}$

6.  $\frac{55}{105}$

7. 12:60

8. 10:148

9. 14:62

10.  $\frac{8}{72}$

**11. Life Science** There are 2,900 species of jellyfish. They are made up of 2 classes, the hydrozoan and the scyphozoan. There are 200 species of scyphozoan. Express the number of species of scyphozoan to the total number of jellyfish species as a ratio in simplest form.



**12. Standardized Test Practice** Akikta has \$1,200 in his checking account and \$300 in his savings account. Express the amount of money in his checking account to the amount of money in his savings account as a ratio in simplest form.

**A** 4:1

**B** 3:4

**C** 1:4

**D** 3:1

Answers: 1.  $\frac{9}{5}$  2. 1:5 3.  $\frac{47}{41}$  4.  $\frac{7}{6}$  5.  $\frac{1}{3}$  6.  $\frac{21}{11}$  7. 1:5 8. 5:74 9. 7:31 10.  $\frac{9}{1}$  11. 2:29 12. A

## Ratios and Percents

(pages 158–160)



A **percent** is a ratio that compares a number to 100.

<b>Expressing a Ratio as a Percent</b>	$\frac{n}{100} = n\%$ <p>To express a ratio as a percent, first write the ratio as a fraction with a denominator of 100. Then rewrite <math>\frac{n}{100}</math> as <math>n\%</math>.</p>
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### EXAMPLES

**A** Express as a percent: 37 students out of 100.

Write the ratio as a fraction:  $\frac{37}{100}$ .  
 $\frac{37}{100}$  is 37%.

**B** Express \$12.25 per \$100 as a percent.

Write as a fraction:  $\frac{\$12.25}{\$100}$ .  
 \$12.25 per \$100 is 12.25%.

### Try These Together

1. Express as a percent: 32.5 square miles in 100.

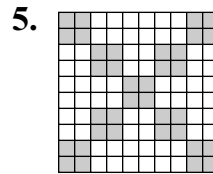
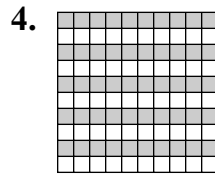
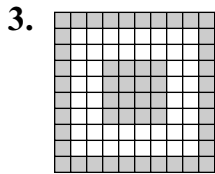
*HINT: Write as a fraction with a denominator of 100.*

2. Express  $33\frac{1}{3}:100$  as a percent.

*HINT: Recall that  $\frac{n}{100} = n\%$ .*

### PRACTICE

Write a percent to represent the shaded area.



Express each ratio as a percent.

6. 62 out of 100

7. 25:100

8.  $\frac{12}{100}$

9. 65.2 to 100

10. \$55 per \$100

11. 38 cars out of 100

12. **Computers** 78 out of 100 computers at Tina's school have CD-ROM drives. Express 78 out of 100 as a percent.



13. **Standardized Test Practice** In Enrique's school, 61 out of every 100 students eat a hot lunch. Express this ratio as a percent.

**A** 3.9%

**B** 6.1%

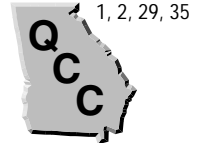
**C** 61%

**D** 39%

<b>Answers:</b> 1. 32.5% 2. $33\frac{1}{3}\%$ 3. 37% 4. 32.5% 5. 36% 6. 62% 7. 25% 8. 12% 9. 65.2% 10. 55% 11. 38% 12. 78% 13. C
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# Fractions, Decimals, and Percents

(pages 161–164)



A percent is a ratio of a number to 100.

<b>Expressing a Fraction as a Percent</b>	To express a fraction as a percent, rewrite the fraction as an equivalent fraction with a denominator of 100. Recall that you can rewrite a fraction as an equivalent fraction by multiplying both numerator and denominator by the same factor.
<b>Expressing a Percent or a Decimal as a Fraction in Simplest Form</b>	To express a percent or decimal as a fraction in simplest form, write the percent or decimal as a fraction with a denominator of 100 and simplify. Recall that $n\% = \frac{n}{100}$ .

## EXAMPLES

**A** Express  $\frac{3}{20}$  as a percent.

To rewrite  $\frac{3}{20}$  as an equivalent fraction with a denominator of 100, multiply both numerator and denominator by 5, since  $100 \div 20$  is 5.

$$\frac{3 \times 5}{20 \times 5} = \frac{15}{100}$$

$$\frac{15}{100} \text{ is } 15\%.$$

$$\frac{3}{20} = 15\%$$

**B** Express 0.34 as a fraction in simplest form.

$$0.34 \text{ is } \frac{34}{100}.$$

The GCF of 34 and 100 is 2.  
Divide both 34 and 100 by 2.

$$\frac{34 \div 2}{100 \div 2} = \frac{17}{50}$$

$$0.34 = \frac{17}{50}$$

## Try These Together

1. Express *fifty-five percent* as a fraction in simplest form.

*HINT: Write as a fraction with a denominator of 100.*

2. Express  $\frac{1}{5}$  as a percent.

*HINT: Write as a fraction with a denominator of 100.*

## PRACTICE

**Express each fraction as a percent.**

3.  $\frac{4}{25}$

4.  $\frac{2}{5}$

5.  $\frac{1}{2}$

6.  $\frac{41}{50}$

**Express each percent or decimal as a fraction in simplest form.**

7. 75%

8. 0.5

9. 0.85

10. 80%



**11. Standardized Test Practice** If  $\frac{3}{10}$  of what a family recycles is cans, what percent is cans?

**A** 10%

**B** 30%

**C** 60%

**D** 15%

Answers: 1.  $\frac{20}{11}$  2. 20% 3. 16% 4. 40% 5. 50% 6. 82% 7.  $\frac{4}{3}$  8.  $\frac{1}{5}$  9.  $\frac{2}{17}$  10.  $\frac{5}{4}$  11. B

# Simple Events (pages 165–168)



An **event** is a specific outcome. Outcomes occur at **random** if each outcome is equally likely to occur.

### Finding Probability

The **probability** of an event is the ratio of the number of ways an event can occur to the number of possible outcomes.

$$P(\text{event}) = \frac{\text{number of ways an event occurs}}{\text{number of possible outcomes}}$$

## EXAMPLE

A certain spinner is equally likely to stop on each of its regions labeled 5, 10, 15, 20, and 25. Find the probability that the spinner will stop on an even number.

$$P(\text{even number}) = \frac{\text{number of ways an even number occurs}}{\text{number of possible outcomes}}$$

Since 2 of the outcomes are even numbers (10 and 20), and there are 5 possible outcomes,  $P(\text{even number}) = \frac{2}{5}$ .

### Try These Together

- What is the probability that a month chosen at random will have 31 days?  
*HINT: How many months out of 12 have 31 days?*
- What is the probability that a day of the week chosen at random has a name that starts with S?  
*HINT: How many days start with S?*

## PRACTICE

**A number cube for a game has six sides numbered 1–6. Find the probability that the number cube will land on each of the following when it is tossed.**

- a 2
- a multiple of 2
- an odd number
- a number greater than 5

**There are 16 colored tennis balls in a bag. Three are blue, 5 are yellow, 4 are green, and 4 are orange. If you reach in the bag and draw one ball at random, what is the probability that you will draw each of the following?**

- a green ball
- a blue ball

- 9. Standardized Test Practice** Ophelia is eating colored candies. There are 80 candies in all and 16 of them are red. What is the probability that she will randomly choose a red candy? Express the fraction in simplest form.

**A**  $\frac{2}{10}$

**B**  $\frac{1}{5}$

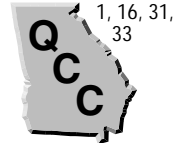
**C**  $\frac{1}{10}$

**D**  $\frac{16}{80}$

Answers: 1.  $\frac{12}{7}$  2.  $\frac{7}{2}$  3.  $\frac{6}{1}$  4.  $\frac{2}{1}$  5.  $\frac{2}{1}$  6.  $\frac{6}{1}$  7.  $\frac{4}{1}$  8.  $\frac{16}{3}$  9. B

# Least Common Multiple

(pages 169–171)



When you multiply a number by the whole numbers 0, 1, 2, 3, 4, and so on, you get **multiples** of the number. The **least common multiple (LCM)** of two or more numbers is the least of their common multiples, other than zero.

<p><b>Finding the Least Common Multiple (LCM)</b></p>	<p>To find the least common multiple of two or more numbers,</p> <ul style="list-style-type: none"> <li>make a list of several multiples of each number. Then identify the common multiples. The least of these is the LCM.</li> </ul> <p>OR</p> <ul style="list-style-type: none"> <li>write the prime factorization of each number. Write each prime factor as a multiplier the <i>greatest</i> number of times it appears in any one of the numbers.</li> </ul>
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## EXAMPLE

Find the LCM of 6, 36, and 40 by writing prime factorizations.

$$6 = 2 \times 3 \quad 36 = 2 \times 2 \times 3 \times 3 \quad 40 = 2 \times 2 \times 2 \times 5$$

Write each prime factor (2, 3, 5) as a multiplier the greatest number of times it appears in any one number. The factor 2 appears three times in 40. The factor 3 appears twice in 36. The factor 5 appears once in 40. The product of  $2 \times 2 \times 2 \times 3 \times 3 \times 5$ , or 360, is the least common multiple of 6, 36, and 40.

## Try These Together

1. Find the LCM of 12 and 30 by listing multiples.

*HINT: Look for the least common multiple in the two lists.*

2. Find the LCM of 12 and 14 by writing prime factorizations.

*HINT: Remember to write each prime factor as a multiplier the greatest number of times it appears in any one of the numbers.*

## PRACTICE

**Find the LCM of each set of numbers by listing multiples.**

3. 3, 10

4. 6, 8

5. 9, 12

6. 3, 5, 6

7. 4, 5, 10

8. 5, 15

9. 3, 8

10. 18, 36

**Find the LCM of each set of numbers by writing prime factorizations.**

11. 6, 9

12. 12, 18

13. 8, 14

14. 10, 36

15. 20, 96

16. 4, 6, 15

17. **Entertainment** Every 10 years, the people of Oberammergau, Germany, put on a special play. Rhonda's family travels to Germany every 3 years. If Rhonda's family was in Germany in the year 2000 and the play was on, what is the next year that the play will be on when Rhonda's family is in Germany?



18. **Standardized Test Practice** What is the least common multiple of 50 and 60?

A 200

B 400

C 300

D 500

Answers: 1. 60 2. 84 3. 30 4. 24 5. 36 6. 30 7. 20 8. 15 9. 24 10. 36 11. 18 12. 36 13. 56 14. 180 15. 480 16. 60 17. 2030 18. C

# Comparing and Ordering Fractions

(pages 172–175)



To compare fractions, rewrite each fraction using the same denominator. Then you only need to compare the numerators.

<p><b>Finding the Least Common Denominator</b></p>	<p>A <b>common denominator</b> is a common multiple of the denominators of two or more fractions. The <b>least common denominator (LCD)</b> is the least common multiple (LCM) of the denominators of two or more fractions.</p> <p>To compare two fractions:</p> <ul style="list-style-type: none"> <li>• find the LCM of the denominators, then</li> <li>• rewrite each fraction using this LCM as the LCD. Compare the numerators.</li> </ul>
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## EXAMPLES

**A** Find the LCD for  $\frac{5}{6}$  and  $\frac{9}{10}$ .

The LCM of 6 and 10 is 30, so the

LCD for  $\frac{5}{6}$  and  $\frac{9}{10}$  is 30.

**B** Is  $\frac{5}{6} <$ ,  $>$ , or  $= \frac{9}{10}$ ?

Rewrite each fraction with the LCD of 30.

Multiply the numerator and denominator of  $\frac{5}{6}$  by 5.

Multiply the numerator and denominator of  $\frac{9}{10}$  by 3.

$$\frac{5}{6} = \frac{25}{30} \quad \frac{9}{10} = \frac{27}{30}$$

Since  $\frac{25}{30} < \frac{27}{30}$ ,  $\frac{5}{6} < \frac{9}{10}$ .

## Try These Together

1. Find the LCD for  $\frac{3}{4}$  and  $\frac{2}{3}$ .

HINT: Find the LCM of 4 and 3.

2. Is  $\frac{5}{8} <$ ,  $>$ , or  $= \frac{2}{3}$ ?

HINT: Rewrite both fractions with the same denominator.

## PRACTICE

Find the LCD for each pair of fractions.

3.  $\frac{5}{12}, \frac{3}{8}$

4.  $\frac{2}{5}, \frac{4}{7}$

5.  $\frac{4}{15}, \frac{1}{3}$

6.  $\frac{1}{6}, \frac{1}{9}$

7.  $\frac{1}{6}, \frac{5}{7}$

8.  $\frac{19}{30}, \frac{7}{10}$

9.  $\frac{9}{16}, \frac{1}{4}$

10.  $\frac{5}{36}, \frac{11}{24}$

Replace each ● with  $,$   $>$ , or  $<$  to make a true sentence.

11.  $\frac{8}{9} \bullet \frac{5}{6}$

12.  $\frac{2}{3} \bullet \frac{8}{13}$

13.  $\frac{5}{6} \bullet \frac{3}{4}$

14.  $\frac{3}{5} \bullet \frac{5}{8}$

15.  $\frac{2}{7} \bullet \frac{1}{4}$

16.  $\frac{7}{10} \bullet \frac{14}{20}$

17.  $\frac{5}{11} \bullet \frac{13}{22}$

18.  $\frac{15}{48} \bullet \frac{3}{8}$



**19. Standardized Test Practice** What is the least common denominator for

$\frac{1}{8}$  and  $\frac{5}{6}$ ?

**A** 36

**B** 24

**C** 18

**D** 45

Answers: 1. 12 2. < 3. 24 4. 35 5. 15 6. 18 7. 42 8. 30 9. 16 10. 72 11. < 12. < 13. > 14. < 15. < 16. = 17. < 18. > 19. B
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# Chapter 4 Review

## Fraction Ladder

Build a ladder out of the following list of fractions. Place the fractions in order from least to greatest on the ladder from bottom to top.

$$\frac{1}{2}$$

$$\frac{19}{20}$$

$$\frac{4}{5}$$

$$\frac{7}{8}$$

$$\frac{3}{4}$$

$$\frac{1}{12}$$

$$\frac{1}{3}$$

$$\frac{3}{10}$$


Answers are located on page 113.