Exponential Growth and Decay

Exponential growth and decay models apply to any situation where growth is proportional to the initial size of the quantity being considered.

Continuous Exponential Growth or Decay

If an initial quantity N_0 grows or decays at an exponential rate k (as a decimal), then the final amount N after a time t is given by

$$N = N_o e^{kt} \text{ where:}$$

- if k is a *continuous growth rate* then $k > 0$, and
- if k is a *continuous decay rate* then $k < 0$.

Example **Model Exponential Growth**

INTERNET **The table shows the number of hits a new Web site received by the end of January and the end of April of the same year. Write an exponential equation to model the situation.**

Web Site Traffic

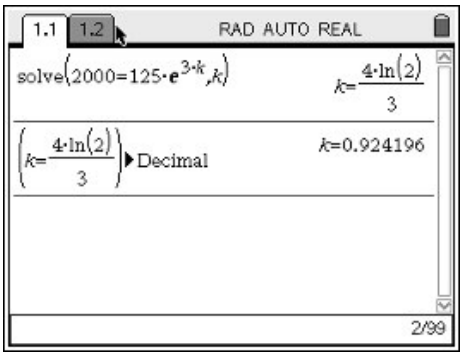
Month	Number of Hits
January	125
April	2,000

Let $N(t)$ represent the number of hits at the end of t months, and assume continuous exponential growth. Then the initial number N_0 is 125 hits, and the number of hits N after a time of 3 months, the number of months from January to April, is 2,000.

$N(t) = N_0 e^{kt}$ Exponential Growth Formula

$2{,}000 = 125e^{k(3)}$ $N(3) = 2{,}000$; $N_0 = 125$; and $t = 3$

Under the Home menu, select **Calculator**. Next, press the menu key, scroll down to **Algebra**, and select **Solve**. Then enter the equation as shown to solve for k. You can convert the answer to decimal form by scrolling down to **Number** under the menu key and selecting **Convert to Decimal**.



The number of hits is increasing at a continuous rate of approximately 92.4% per month. Therefore, an equation modeling this situation is $N(t) = 125e^{0.924t}$.

Exercises

1. **MEMORABILIA** The table shows the value of memorabilia sold by two different vendors during and one week after the World Series. Write an exponential equation to model the situation. Round k to the nearest hundredth.

World Series Memorabilia Sales

Days after Series	Vendor A Sales ($)	Vendor B Sales ($)
0	300,000	200,000
7	37,000	49,000

3. **TECHNOLOGY** A chain of retail computer stores opened 2 stores in its first year of operation. After 8 years of operation, the chain consisted of 206 stores. Write an equation to model the number of stores N as a function years of operation t. Round k to the nearest hundredth. (*Hint*: Use the adjusted exponential growth model $N = N_1 e^{k(t-1)}$).

2. **STOCK** The price per share of a coffee chain's stock was \$0.93 in a month during its first year of trading. During its fifth year of trading, the price per share of stock was \$3.52 during the same month. Write an equation to model the price of stock P as a function of year of trading t. Round k to the nearest hundredth. (*Hint*: Use the adjusted exponential decay function $N = N_1 e^{k(t-1)}$).

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Answers

1. $N = 2e^{0.66(t-1)}$

2. $P = 0.93e^{0.33(t-1)}$

3. vendor A: $N(t) = 300{,}000e^{-0.30t}$; vendor B: $N(t) = 200{,}000e^{-0.20t}$