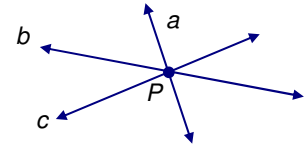


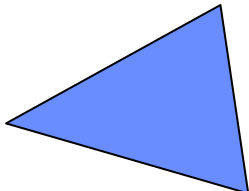
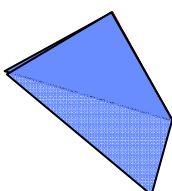
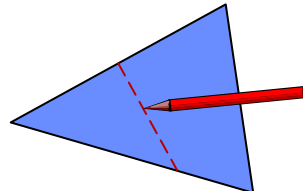
Use Paper Folding to Explore Concurrency

When three or more lines intersect at a common point, the lines are called *concurrent lines*. The point where concurrent lines intersect is called the *point of concurrency*. You can use paper folding to explore points of concurrency in geometric figures such as triangles.



Lines a , b , and c are concurrent at P .

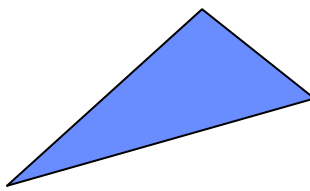
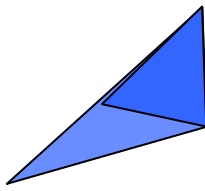
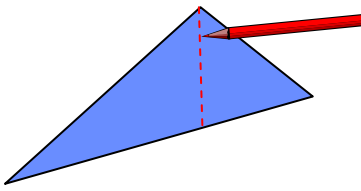
Example 1 Perpendicular Bisectors of a Triangle

<p>Step 1</p>  <p>Draw and cut out a triangle.</p>	<p>Step 2</p>  <p>Fold the triangle so that two vertices meet.</p>	<p>Step 3</p>  <p>Unfold the paper and draw a line along the fold representing the perpendicular bisector of that side.</p>
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Exercises

1. Repeat Steps 1–3 of Example 1 for the remaining two sides and mark the point where each pair of perpendicular bisectors meet. What do you observe?
2. Repeat the activity in Example 1 with two other differently shaped triangles. What do you observe?
3. For each triangle, measure the distance from the point of concurrency of the perpendicular bisectors, called the *circumcenter*, to each of the three vertices. What do you observe?

Example 2 Angle Bisectors of a Triangle

<p>Step 1</p>  <p>Draw and cut out a triangle.</p>	<p>Step 2</p>  <p>Fold the triangle so that two sides meet.</p>	<p>Step 3</p>  <p>Unfold the paper and draw a line along the fold representing the angle bisector of the vertex between the two sides.</p>
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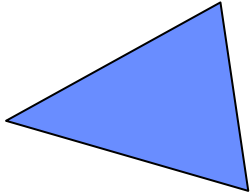
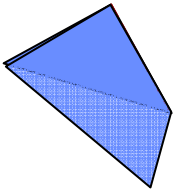
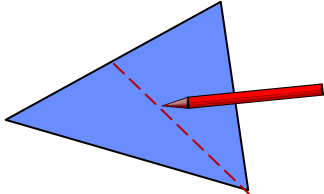
Use Paper Folding to Explore Concurrency

(continued)

Exercises

- Repeat Steps 1–3 of Example 2 for the remaining two angles of the triangle and mark the point where each pair of angle bisectors meet. What do you observe?
- Repeat the activity in Example 2 with two other differently shaped triangles. What do you observe?
- For each triangle, determine the perpendicular distance from the point of concurrency of the angle bisectors, called the *incenter*, to each of the three sides. What do you observe?

Example 3 Explore Relationships with Medians of a Triangle

Step 1	Step 2	Step 3
 <p data-bbox="188 1041 513 1073">Draw and cut out a triangle.</p>	 <p data-bbox="630 1041 954 1098">Fold the triangle so that two vertices meet.</p>	 <p data-bbox="1068 1041 1474 1192">Unfold the paper and draw a line from that side's midpoint to the vertex opposite that side. This line represents a median of the triangle.</p>

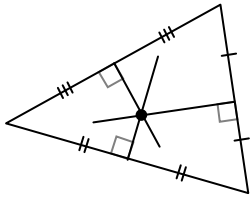
Exercises

- Repeat Steps 1–3 of Example 3 for the remaining two sides and mark the point where each pair of medians meet. What do you observe?
- Repeat the activity in Example 3 with two other differently shaped triangles. What do you observe?
- For each triangle, measure the distance along each median from the point of concurrency P of the medians, called the *centroid*, to the vertex V and from the vertex V to the midpoint M of the opposite side. For each set of measures, write the ratio $\frac{PV}{VM}$. What do you observe?
- Construct three different equilateral triangles on paper, and cut them out. Fold each triangle to locate the circumcenter, incenter, and centroid.
- Using your triangles from Exercise 10, make a conjecture about the relationship among the circumcenter, incenter, and centroid for any equilateral triangle.

Use Paper Folding to Explore Concurrency

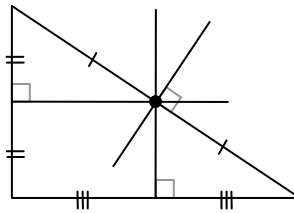
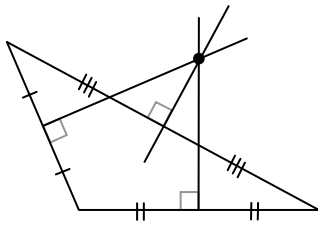
Answers

1.



The perpendicular bisectors of the triangle are concurrent.

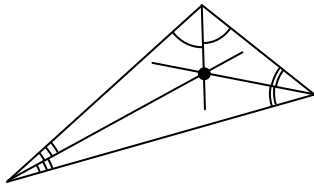
2. Sample answer:



The perpendicular bisectors of each of these triangles are also concurrent.

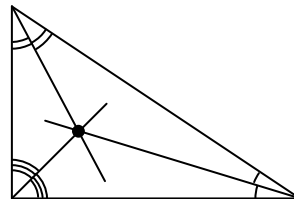
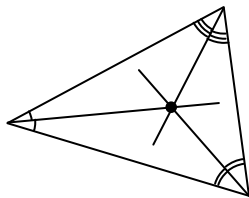
3. The circumcenter of a triangle is the same distance from the vertices of the triangle.

4.



The angle bisectors of the triangle are concurrent.

5. Sample answer:



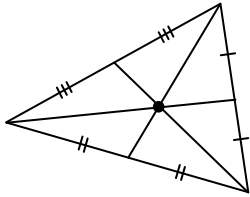
The angle bisectors of each of these triangles are also concurrent.

6. The incenter of a triangle is the same perpendicular distance from each side of the triangle.

Use Paper Folding to Explore Concurrency

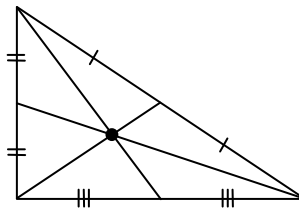
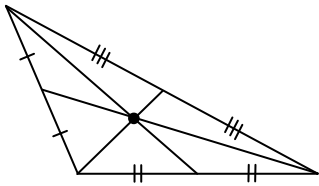
Answers (*continued*)

7.



The medians of the triangle are concurrent.

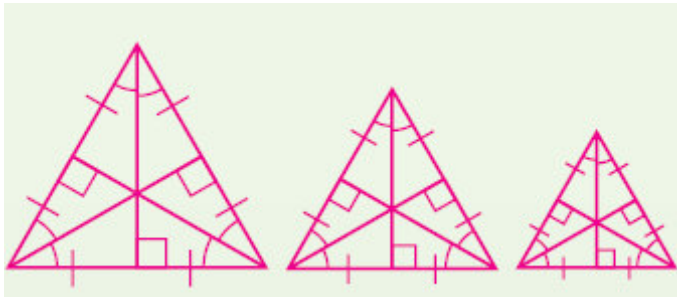
8. Sample answer:



The perpendicular bisectors of each of these triangles are also concurrent.

9. See students' work. Students should observe that the ratios $\frac{PV}{VM}$ are approximately equal to $\frac{2}{3}$.

10.



11. The circumcenter, incenter, and centroid of an equilateral triangle are all the same point.