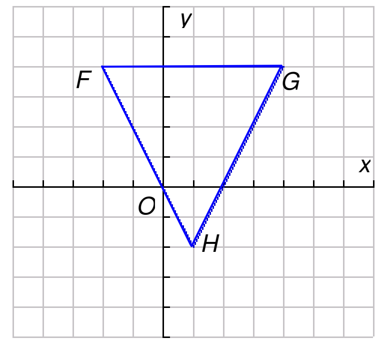
Using Coordinate Geometry to Explore Special Segments

You can use coordinate geometry to investigate geometric relationships of special segments of triangles and other polygons.

Example 1 **Altitudes of a Triangle**

Find equations for the lines containing the altitudes from F to side $\overline{GH}$ and from G to side $\overline{FH}$ of $\triangle FGH$. Then locate the point of intersection of these two lines.



Step 1 Find an equation of the line containing each altitude.

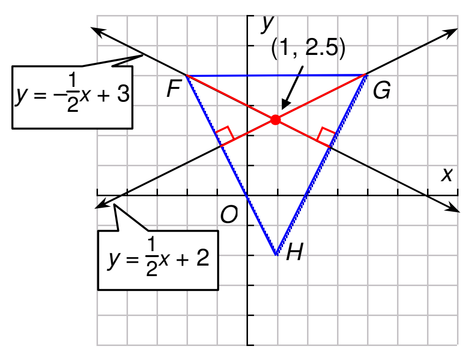
The slope of $\overline{GH}$ is $\frac{4 - (-2)}{4 - 1}$ or 2, so the slope of the altitude from F to $\overline{GH}$, which is perpendicular to $\overline{GH}$, is $-\frac{1}{2}$. Use the coordinates of vertex F and this slope to write an equation of the line containing this altitude.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 4 = -\frac{1}{2}[x - (-2)]$	$m = -\frac{1}{2}$ and $(x_1, y_1) = F(-2, 4)$
$y - 4 = -\frac{1}{2}(x + 2)$	Simplify.
$y - 4 = -\frac{1}{2}x - 1$	Distributive Property
$y = -\frac{1}{2}x + 3$	Add 4 to each side.

The slope of $\overline{FH}$ is $\frac{-2 - 4}{1 - (-2)}$ or -2, so the slope of the altitude from G to $\overline{FH}$ is $\frac{1}{2}$. So, an equation of the line containing this altitude is $y - 4 = \frac{1}{2}(x - 4)$ or $y = \frac{1}{2}x + 2$.

Step 2 Solve the resulting system of equations $\begin{cases} y = -\frac{1}{2}x + 3 \\ y = \frac{1}{2}x + 2 \end{cases}$ to find their point of intersection.

Adding the two equations to eliminate x results in $2y = 5$ or $y = \frac{5}{2}$. Substitute this value for y into one of the two original equations, and solve for x to find that $x = 1$. The coordinates of the point of intersection of these two altitudes of $\triangle FGH$ are $\left(1, \frac{5}{2}\right)$ or $(1, 2.5)$, as shown.



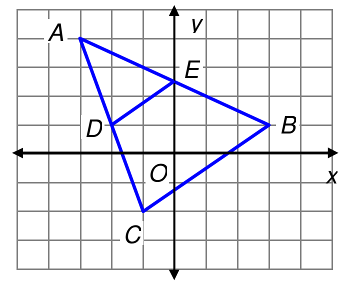
Exercise

1. Use the procedure outlined in Step 1 of Example 1 to find an equation of the line containing the remaining altitude of $\triangle FGH$. Then find the point of intersection of the remaining pairs of altitudes. What do you observe?

Using Coordinate Geometry to Explore Special Segments (continued)

2. Repeat the activity in Example 1 and Exercise 1 with two other triangles with vertices that have integer coordinates. What do you observe?

3. A *midsegment* of a triangle is a segment that connects the midpoints of the two sides of the triangle.

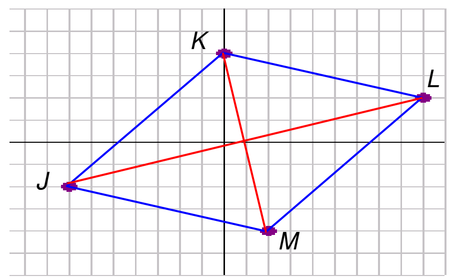


a. Find the endpoints of midsegment $\overline{DE}$ in $\triangle ABC$ shown. Then determine the relationship, if any, between $\overline{DE}$ and side $\overline{CB}$, which is parallel to $\overline{CB}$. Justify your answer.

b. Repeat the activity in part **a** with each of the other two triangle midsegments and the related triangle sides. What do you observe?

Example 2 Diagonals of a Rhombus

Determine the relationship that exists between the diagonals of rhombus *JKLM*.



Looking at the graph, the diagonals of *JKLM* appear to be perpendicular. Use the Slope Formula to confirm this conjecture.

slope of $\overline{KM} = \dfrac{-4 - 4}{2 - 0} = \dfrac{-8}{2}$ or -4

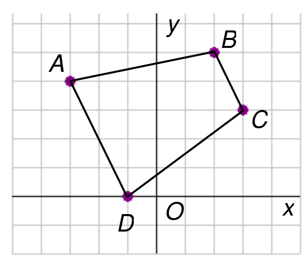
slope of $\overline{JL} = \dfrac{2 - (-2)}{9 - (-7)} = \dfrac{4}{16}$ or $\dfrac{1}{4}$

Since the product of the slopes of the diagonals is -1, the diagonals of this rhombus are perpendicular.

Exercises

4. Repeat the activity in Example 2 with two other rhombi with vertices that have integer coordinates. What do you observe?

5. The *midsegment* of a trapezoid is the segment that connects the midpoints of the legs of the trapezoid. Trapezoid *ABCD* shown has segments $\overline{AB}$ and $\overline{CD}$ as its legs.

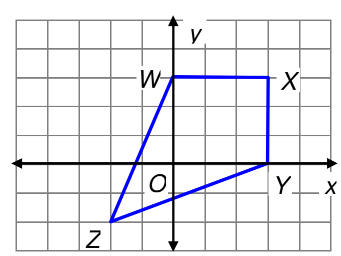


a. Find the coordinates of the midpoint *P* of $\overline{AB}$ and the midpoint *Q* of $\overline{CD}$.

b. Find and compare equations in slope-intercept form for the lines containing $\overline{AD}$, $\overline{BC}$, and $\overline{PQ}$.

c. Repeat the activity in parts **a** and **b** with two other trapezoids with vertices that have integer coordinates. What do you observe?

6. A *kite* is a quadrilateral with exactly two pairs of consecutive congruent sides.



a. Use coordinate geometry to determine the relationship, if any, between the diagonals of kite *WXYZ* shown. Justify your answer.

b. Repeat the activity in part **a** with two other kites with vertices that have integer coordinates. What do you observe?

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Answers

1. $x = 1$; the point of intersection of the remaining pairs of altitudes is the same as that found with the first two pairings, (1, 2.5). That is, the altitudes of the triangle are concurrent.

2. See students' work. The altitudes of each triangle are concurrent.

3a. $D(-2, 1)$ and $E(0, 2.5)$. $DE = 2.5$ and $CB = 5$. Since 2.5 is half of 5, $DE = \frac{1}{2}CB$.

3b. $F(1, -\frac{1}{2})$ is the midpoint of $\overline{CB}$. Midsegment $\overline{DF}$ has endpoints $D(-2, 1)$ and $F(1, -\frac{1}{2})$. $DF = \frac{3\sqrt{5}}{2}$ and $AB = 3\sqrt{5}$. Since $3\sqrt{5}$ is half of $\frac{3\sqrt{5}}{2}$, $DF = \frac{1}{2}AB$.

Midsegment $\overline{EF}$ has endpoints $E(0, \frac{5}{2})$ and $F(1, -\frac{1}{2})$. $EF = \sqrt{10}$ and $AC = 2\sqrt{10}$.

Since $\sqrt{10}$ is half of $2\sqrt{10}$, $EF = \frac{1}{2}AC$.

4. See students' work. The diagonals of these rhombi are also perpendicular.

5a. $P(--0.5, 4.5)$, $Q(1, 1.5)$

5b. $\overline{AD}$: $y = -2x - 2$; $\overline{BC}$:$-2x + 9$; $\overline{PQ}$: $-2x + 3.5$; the lines all have the same slope, indicating that the segments are all parallel.

5c. See students' work. The midsegments of each trapezoid are parallel to the bases of the trapezoid.

6a. An equation of the line that contains diagonal $\overline{WY}$ is $y = -x + 3$ and an equation of the line that contains diagonal $\overline{ZX}$ is $y = x$. These lines are perpendicular since the product of their slopes $1(-1)$ is -1.

6b. See students' work. The diagonals of each kite are also perpendicular.