

Study Guide

Areas of Polygons

The following theorems involve the areas of polygons.

- For any polygon, there is a unique area.
- Congruent polygons have equal areas.
- The area of a given polygon equals the sum of the areas of the nonoverlapping polygons that form the given polygon.

Example: Find the area of the polygon in square units.

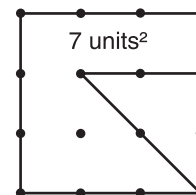
Since the area of each four-dot unit represents 1 square unit, the area of each three-dot unit represents 0.5 square unit.

$A = 6(1) + 2(0.5)$ There are 6 four-dot units and 2 three-dot units.

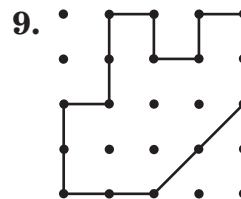
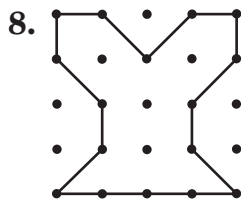
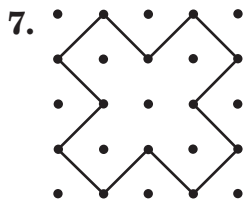
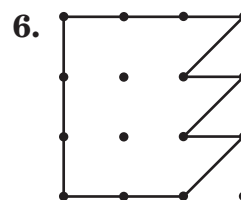
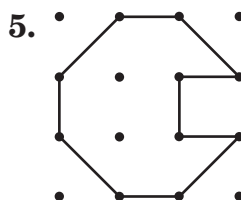
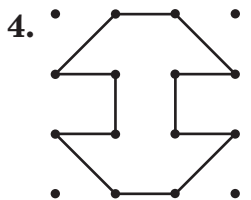
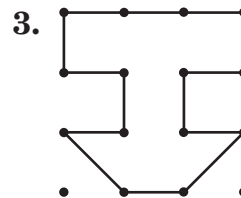
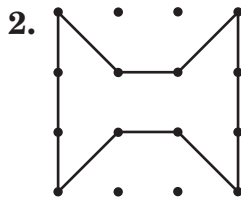
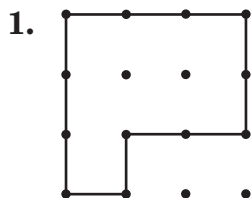
$A = 6 + 1$

$A = 7$

The area of the region is 7 square units, or 7 units².



Find the area of each polygon in square units.



Study Guide

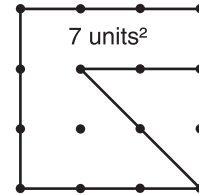
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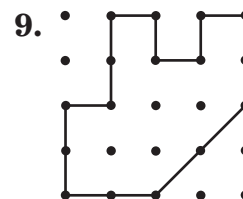
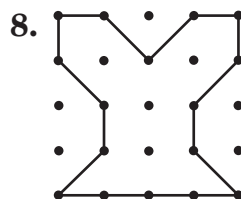
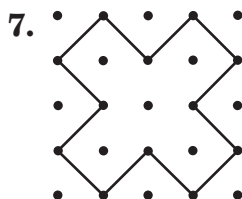
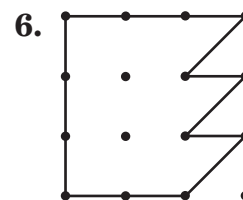
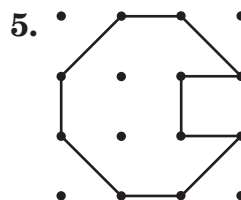
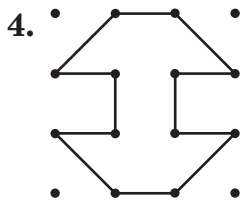
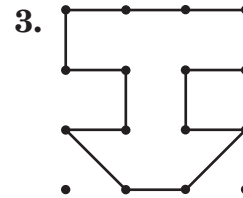
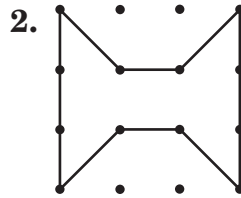
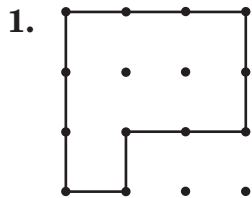
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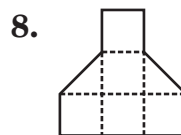
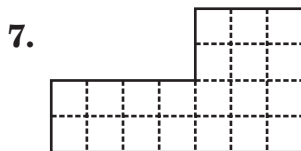
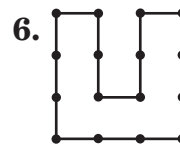
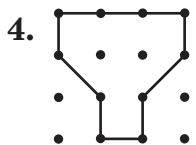
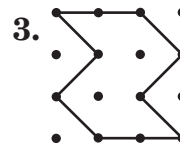
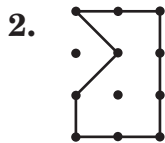
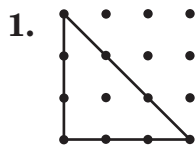
Find the area of each polygon in square units.



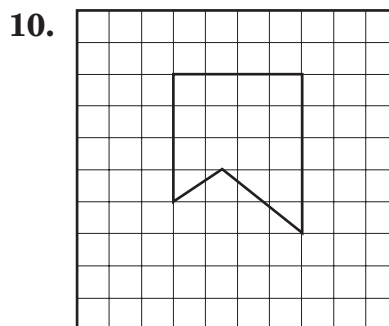
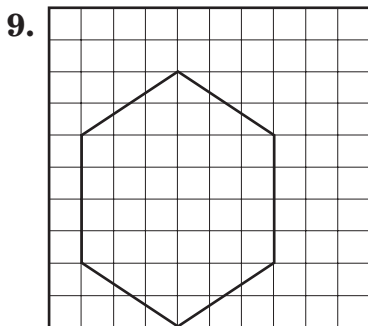
Practice

Areas of Polygons

Find the area of each polygon in square units.



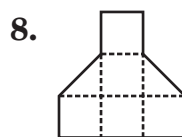
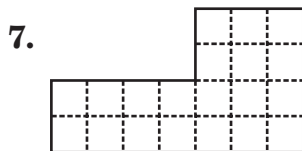
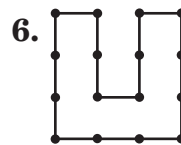
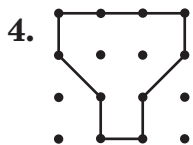
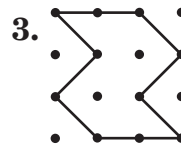
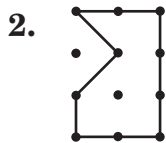
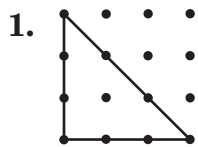
Estimate the area of each polygon in square units.



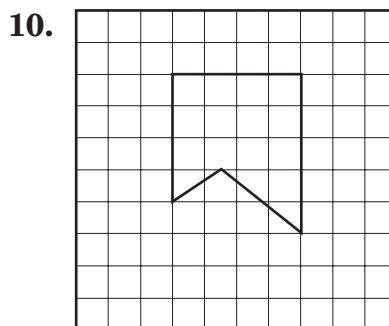
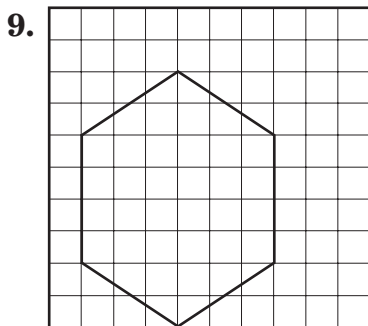
Practice

Areas of Polygons

Find the area of each polygon in square units.



Estimate the area of each polygon in square units.



Hands-On Geometry

Teacher Notes

Area of a Polygon I (Lesson 10–3)

Objective To investigate the areas of polygons using dot paper.

Before students begin drawing polygons, point out that “square units” can be represented literally as squares. Have them represent perfect squares such as 1, 4, 9, and 16 on grid paper. Show them that the square representing 16 square units, for example, consists of 16 unit squares.

You may also wish to review the role of variables in formulas; remind students that they are simply placeholders for numbers.

Answers

Table: $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4

1. See students' work.
2. 9 square units
3. c

Area of a Polygon II (Lesson 10–3)

Objective To investigate the areas of polygons.

This activity extends the investigation of the previous Hands-On Geometry Master to cover the case where there is one dot (rather than no dots) in the interior of a convex polygon drawn on dot paper.

Have students work in pairs to complete this activity. Ask them to take turns drawing the polygons and recording the data in the table. You may wish to have students compare their data with another team before they complete Exercise 1 and again before they complete Exercise 3.

As an extension, ask students to investigate the area of a polygon with two or more interior dots. The formula is $A = \frac{1}{2}x + y - 1$, where x is the number of border dots and y is the number of interior dots.

Answers

Table:

Interior Dots (y)	Border Dots (x)	Area (A)
0	3	0.5
0	4 (square)	1
0	4 (triangle)	1
0	5	1.5
0	6	2
1	4	2
1	5	2.5
1	6	3
1	7	3.5

1. c
2. See students' work.
3. $A = \frac{1}{2}x$; 6 square units

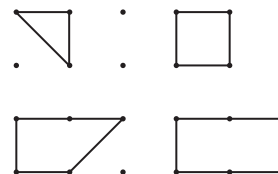
Hands-On Geometry

Area of a Polygon I

Materials

straightedge

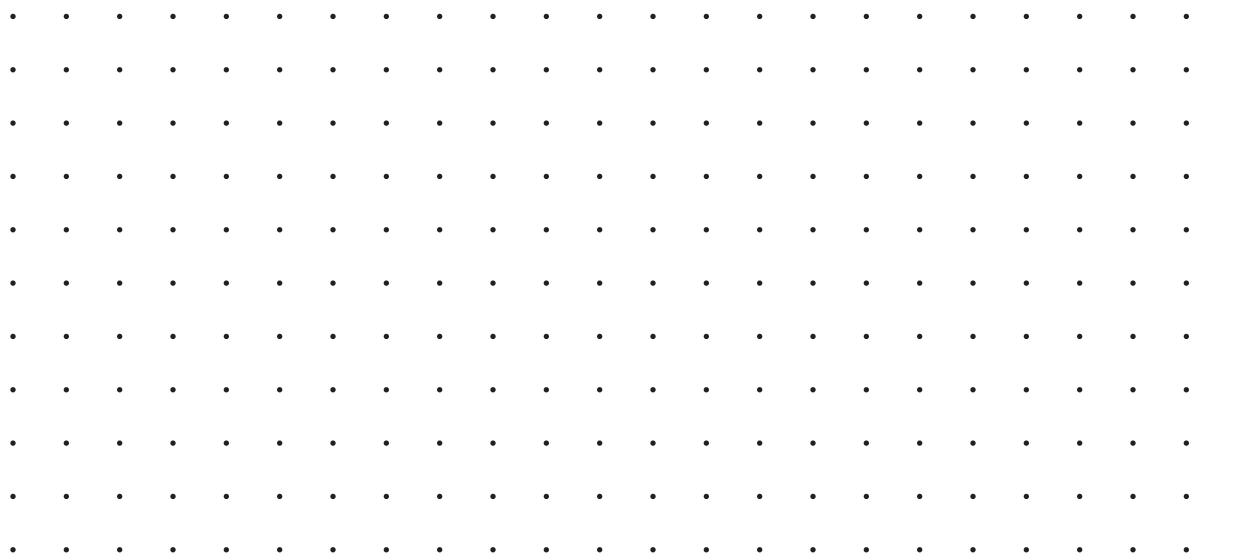
Step 1 On the dot paper in the space below, draw convex polygons that go through 3 dots, 4 dots, 5 dots, and 6 dots, having no dots in the interiors, as shown at the right.



Step 2 Find the areas of the figures you have drawn and write your answers in the appropriate place in the table below. *Note:* The square encloses 1 square unit of area.

Number of Dots on Figure	3	4	5	6	7	8	9	10
Area of Polygon (square units)								

Work Space



Try These

- Draw polygons that go through 7, 8, 9, and 10 dots, having no dots in the interiors. Then complete the table.
 - Predict the area of a figure whose sides go through 20 dots. Verify your answer by drawing the polygon.
-
- Choose the correct relationship that exists between the number of dots on a figure n and its area A . Circle a, b, or c.

a. $A = \frac{n}{2} + 1$

b. $A = \frac{n}{2}$

c. $A = \frac{n}{2} - 1$

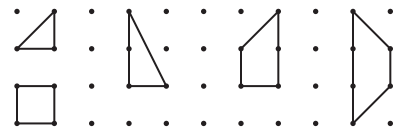
Hands-On Geometry

Area of a Polygon II

Materials

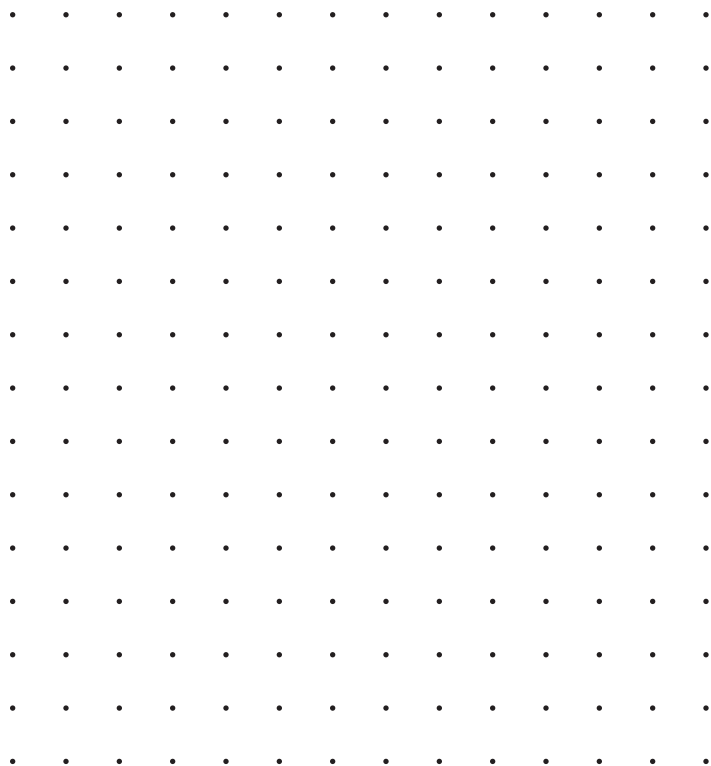
dot paper

Step 1 On the dot paper in the space below, draw the polygons shown at the right.



Step 2 Suppose x represents the number of border dots on the figure and y represents the number of dots in the interior. Complete the first five rows of the table below.

Work Space



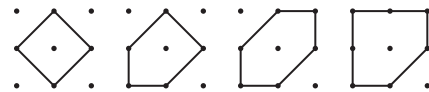
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0	4 (square)	1
0	4 (triangle)	
0	5	
0	6	
1	4	
1	5	
1	6	
1	7	

Try These

1. Which rule shows how to find the measure of the area when the number of border dots x are known and the number of interior dots is 0? Circle a, b, c, or d.

a. $A = x - 3$ b. $A = 2x - 5\frac{1}{2}$ c. $A = \frac{1}{2}x - 1$ d. $A = 4 - \frac{1}{2}x$

2. On the dot paper above, draw the polygons shown at the right. Complete a table like the one above.



3. When $y = 1$, how does the formula for the area change? Write the new formula. Then use it to find the area of a polygon with 12 border dots (x) and 1 interior dot (y).