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Study Guide

Limits and Continuity

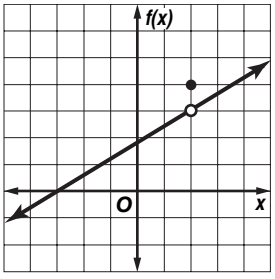
A function f is **continuous** at $x = a$ if and only if

- $f(a)$ is defined,
- $\lim_{x \to a} f(x)$ exists, and
- $\lim_{x \to a} f(x) = f(a)$.

Otherwise, f is **discontinuous** at $x = a$.

If f is continuous at $x = a$ for every real number a, then we say that f is continuous everywhere.

Example 1 **Determine where f is continuous.**



f is not continuous at $x = 2$. $f(2)$ is defined, but $f(2) = 4$. From the graph we can see that $\lim_{x \to 2} f(x) = 3$.

Since $f(2) \neq \lim_{x \to 2} f(x)$, f is discontinuous at $x = 2$. So f is continuous for all real values of x except $x = 2$.

Example 2 **Find a value for a so that f is continuous.**

$$f(x) = \begin{cases} \dfrac{-4x^2 - 12x - 8}{x + 2} \text{ if } x \neq -2 \\ a \text{ if } x = -2 \end{cases}$$

$\dfrac{-4x^2 - 12x - 8}{x + 2} = \dfrac{-4(x + 1)(x + 2)}{x + 2}$ or $-4(x + 1)$ for every value of x except $x = -2$.

So, we can rewrite $f(x)$ as

$$f(x) = \begin{cases} -4(x + 1) \text{ if } x \neq -2. \\ a \text{ if } x = -2 \end{cases}$$

$\lim_{x \to -2} f(x) = \lim_{x \to -2} -4(x + 1) = -4(-2 + 1)$ or 4.

If we let $a = 4$, then $\lim_{x \to -2} f(x) = 4 = f(-2)$. So, for $a = 4$, f is continuous.

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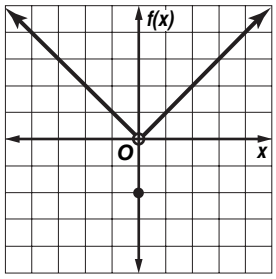
Practice

Limits and Continuity

Use the graph to find f(0) and $\lim_{x \to 0} f(x)$. Is f continuous at x = 0?

1.



Determine if f is continuous.

2. $f(x) = \begin{cases} \dfrac{x^3 + 2x^2 - 4x - 3}{x^2 + x - 6} \text{ if } x \neq 2 \text{ and } x \neq -3 \\ 1 \text{ if } x = 2 \\ -5 \text{ if } x = -3 \end{cases}$

3. $f(x) = \begin{cases} \dfrac{1}{2} \sin x \text{ if } x > 0 \\ 5(1 - \cos x) \text{ if } x \leq 0 \end{cases}$

Find the value of a for which f is continuous.

4. $f(x) = \begin{cases} \dfrac{-4x^2 - 12x - 8}{x + 2} \text{ if } x \neq -2 \\ a \text{ if } x = -2 \end{cases}$

5. $f(x) = \begin{cases} \dfrac{3x^2 + 6x - 24}{2x - 4} \text{ if } x \neq 2 \\ a \text{ if } x = 2 \end{cases}$

6. $f(x) = \begin{cases} \dfrac{\sin x}{x} \text{ if } x \neq 0 \\ a \text{ if } x = 0 \end{cases}$

7. $f(x) = \begin{cases} x^2 + a \text{ if } x \leq 2 \\ -x + 3a \text{ if } x > 2 \end{cases}$

Find any values of x for which f is discontinuous. Explain why f is discontinuous for each value of x that you found.

8. $f(x) = \dfrac{|x|}{x}$

9. $f(x) = \begin{cases} x^3 - 4x^2 + 12x - 5 \text{ if } x \neq 1 \\ 4 \text{ if } x = 1 \end{cases}$

10. $f(x) = \dfrac{\cos x - 1}{\sin x + 1}$

11. $f(x) = \begin{cases} \sqrt{x - 7} \text{ if } x > 7 \\ -\sqrt{7 - x} \text{ if } x < 7 \end{cases}$