Includes:
- Virginia Standards of Learning for Algebra II Correlated to Glencoe Algebra 2
- Algebra II Formula Sheet
- Student Recording Chart
- Diagnostic Test
- Numerous Practice Questions for Each SOL
- Full-Size Sample Test
Test-Taking Tips

• Go to bed early the night before the test. You will think more clearly after a good night's rest.

• Read each problem carefully and think about ways to solve the problem before you try to answer the question.

• Relax. Most people get nervous when taking a test. It's natural. Just do your best.

• Answer questions you are sure about first. If you do not know the answer to a question, skip it and go back to that question later.

• Think positively. Some problems may seem hard to you, but you may be able to figure out what to do if you read each question carefully.

• If no figure is provided, draw one. If one is furnished, mark it up to help you solve the problem.

• When you have finished each problem, reread it to make sure your answer is reasonable.

• Become familiar with a variety of formulas and when they should be used.

• Make sure that the number of the question on the answer sheet matches the number of the question on which you are working in your test booklet.
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Overview

The material in this booklet is designed to help you prepare for the Virginia Algebra II Standards of Learning Assessment (SOL).

It contains:

• a Student Recording Chart,
• the Algebra II Standards of Learning correlated to *Glencoe Algebra 2*,
• a Diagnostic Test,
• practice for each SOL, and
• a Sample Test.

How to Use This Book

**Diagnostic Test**  This test will help you identify any weaknesses you may have as you prepare to take the SOL. Once you’ve taken the test and it’s been graded, complete the Student Recording Chart that is found on page iv. Mark an × in the square for each question that you answered *incorrectly*.

**Practice**  If you missed one or two of the questions for a particular SOL, you could probably use some extra practice with that standard. The Student Recording Chart lists practice pages for each SOL. Complete the appropriate practice pages. If you are unsure about how to do some of the problems, you may want to refer to your mathematics book.

**Sample Test**  After you have completed your practice worksheet(s), take the Sample Test on pages 41 to 50.
**Student Recording Chart**

**Directions** Mark an × by each question from the Diagnostic Test that you answered *incorrectly*. If there are one or two ×s marked for a SOL, write *Yes* in the *Need Practice?* box. Then complete the practice pages for that standard.

<table>
<thead>
<tr>
<th>Standard</th>
<th>All.1</th>
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<td>8 □ 32 □</td>
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<td>Practice Pages</td>
<td>11–12</td>
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<td>11 □ 26 □ 46 □</td>
<td>33 □ 47 □</td>
<td>9 □ 25 □ 45 □</td>
<td>14 □ 17 □ 39 □</td>
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<td>Practice Pages</td>
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<td>20–21</td>
<td>21–22</td>
<td>23–24</td>
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<td>7 □ 40 □</td>
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<td>5 □ 19 □ 22 □ 48 □</td>
<td>34 □ 41 □ 43 □ 49 □</td>
<td>13 □ 29 □</td>
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### Virginia Standards of Learning, Algebra II, Correlated to *Glencoe Algebra 2*

Lessons in which the standards are a primary focus are indicated in **bold**.

<table>
<thead>
<tr>
<th>Standards of Learning</th>
<th>Student Edition Lesson(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AII.1</td>
<td>The student will identify field properties, axioms of equality and inequality, and properties of order that are valid for the set of real numbers and its subsets, complex numbers, and matrices.</td>
</tr>
<tr>
<td>AII.2</td>
<td>The student will add, subtract, multiply, divide, and simplify rational expressions, including complex fractions.</td>
</tr>
<tr>
<td>AII.3a</td>
<td>The student will add, subtract, multiply, divide, and simplify radical expressions containing positive rational numbers and variables and expressions containing rational exponents; and</td>
</tr>
<tr>
<td>AII.3b</td>
<td>The student will write radical expressions as expressions containing rational exponents and vice versa.</td>
</tr>
<tr>
<td>AII.4</td>
<td>The student will solve absolute value equations and inequalities graphically and algebraically. Graphing calculators will be used as a primary method of solution and to verify algebraic solutions.</td>
</tr>
<tr>
<td>AII.5</td>
<td>The student will identify and factor completely polynomials representing the difference of squares, perfect square trinomials, the sum and difference of cubes, and general trinomials.</td>
</tr>
<tr>
<td>AII.6</td>
<td>The student will select, justify, and apply a technique to solve a quadratic equation over the set of complex numbers. Graphing calculators will be used for solving and for confirming the algebraic solutions.</td>
</tr>
<tr>
<td>AII.7</td>
<td>The student will solve equations containing rational expressions and equations containing radical expressions algebraically and graphically. Graphing calculators will be used for solving and for confirming the algebraic solutions.</td>
</tr>
<tr>
<td>AII.8</td>
<td>The student will recognize multiple representations of functions (linear, quadratic, absolute value, step, and exponential functions) and convert between a graph, a table, and symbolic form. A transformational approach to graphing will be employed through the use of graphing calculators.</td>
</tr>
<tr>
<td>AII.9</td>
<td>The student will find the domain, range, zeros, and inverse of a function; the value of a function for a given element in its domain; and the composition of multiple functions. Functions will include exponential, logarithmic, and those that have domains and ranges that are limited and/or discontinuous. The graphing calculator will be used as a tool to assist in investigation of functions.</td>
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P = Preview Lesson, F = Follow-Up Lesson
<table>
<thead>
<tr>
<th>Standards of Learning</th>
<th>Student Edition Lesson(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>AII.10</strong> The student will investigate and describe through the use of graphs the relationships between the solution of an equation, zero of a function, x-intercept of a graph, and factors of a polynomial expression.</td>
<td>7-2, 7-2F, 7-3, 7-4, 7-5, 7-6</td>
</tr>
<tr>
<td><strong>AII.11</strong> The student will use matrix multiplication to solve practical problems. Graphing calculators or computer programs with matrix capabilities will be used to find the product.</td>
<td>4-3, 4-4, 4-7, 4-8</td>
</tr>
<tr>
<td><strong>AII.12</strong> The student will represent problem situations with a system of linear equations and solve the system, using the inverse matrix method. Graphing calculators or computer programs with matrix capability will be used to perform computations.</td>
<td>4-8</td>
</tr>
<tr>
<td><strong>AII.13</strong> The student will solve practical problems, using systems of linear inequalities and linear programming, and describe the results both orally and in writing. A graphing calculator will be used to facilitate solutions to linear programming problems.</td>
<td>3-4</td>
</tr>
<tr>
<td><strong>AII.14</strong> The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions.</td>
<td>8-7</td>
</tr>
<tr>
<td><strong>AII.15</strong> The student will recognize the general shape of polynomial, exponential, and logarithmic functions. The graphing calculator will be used as a tool to investigate the shape and behavior of these functions.</td>
<td>6-1, 6-2, 6-2F, 6-6P, 6-6, 7-1, 7-2, 7-2F, 7-5, 7-6, 8-2, 10-1P, 10-1, 10-2, 10-2F, 10-6</td>
</tr>
<tr>
<td><strong>AII.16</strong> The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve practical problems, including writing the first n terms, finding the nth term, and evaluating summation formulas. Notation will include Σ and an.</td>
<td>11-1, 11-2, 11-3, 11-4P, 11-4, 11-5, 11-6P, 11-6, 11-6F, 11-7</td>
</tr>
<tr>
<td><strong>AII.17</strong> The student will perform operations on complex numbers and express the results in simplest form. Simplifying results will involve using patterns of the powers of i.</td>
<td>5-9</td>
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<tr>
<td><strong>AII.18</strong> The student will identify conic sections (circle, ellipse, parabola, and hyperbola) from his/her equations. Given the equations in (h, k) form, the student will sketch graphs of conic sections, using transformations.</td>
<td>8-2, 8-3, 8-4P, 8-4, 8-5, 8-6, 8-6F</td>
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<tr>
<td><strong>AII.19</strong> The student will collect and analyze data to make predictions and solve practical problems. Graphing calculators will be used to investigate scatterplots and to determine the equation for a curve of best fit. Models will include linear, quadratic, exponential, and logarithmic functions.</td>
<td>2-4, 2-5, 2-5F, 10-1, 10-2, 10-6, 12-7, 12-8F</td>
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<tr>
<td><strong>AII.20</strong> The student will identify, create, and solve practical problems involving inverse variation and a combination of direct and inverse variations.</td>
<td>9-4</td>
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Formulas

Abbreviations

<table>
<thead>
<tr>
<th>Unit</th>
<th>Abbreviation</th>
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<td>ounce</td>
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<td>pound</td>
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<tr>
<td>square centimeter</td>
<td>cm²</td>
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<tr>
<td>cubic centimeter</td>
<td>cm³</td>
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Geometric Figures

Quadratic Formula

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ \pi \approx 3.14 \]

\[ \pi \approx \frac{22}{7} \]
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

For this test you may assume that the value of the denominator of a rational expression is not zero.

1 Which is equivalent to \( \frac{z + 1}{z - 2} - \frac{z - 1}{z + 2} ? \) \( \text{All.2} \)
   \[ \text{A} \frac{6z}{(z - 2)(z + 2)} \quad \text{B} \frac{6z + 4}{(z - 2)(z + 2)} \quad \text{C} \frac{2}{z} \quad \text{D} \frac{2}{(z - 2)(z + 2)} \]

2 Vicki works in a store at the Valley View Mall in Roanoke. She started her job on November 1, 2002, at an hourly wage of $6, and receives a 3% raise every 6 months. If she keeps this job, what will be her hourly wage on July 1, 2006? \( \text{All.16} \)
   \[ \text{F}$6.96 \quad \text{G}$7.16 \quad \text{H}$7.38 \quad \text{J}$7.60 \]

3 If \( a, b, \) and \( c \) are positive real numbers, which equation could be represented by this graph? \( \text{All.15} \)
   \[ \text{A} y = a(x - b)(x - c) \quad \text{B} y = -a(x + b)^2(x - c) \quad \text{C} y = a(x + b)^2(x - c) \quad \text{D} y = -a(x - b)^2(x + c) \]

4 Which equation has solution set \( \{-3 \pm i\sqrt{3}\} ? \) \( \text{All.6} \)
   \[ \text{F} x^2 + 6x + 12 = 0 \quad \text{G} x^2 - 6x + 12 = 0 \quad \text{H} x^2 + 6x + 6 = 0 \quad \text{J} x^2 - 6x - 6 = 0 \]

5 Which could be an equation for this graph? \( \text{All.18} \)
   \[ \text{A} x + 1 = (y - 2)^2 \quad \text{B} y - 2 = (x + 1)^2 \quad \text{C} x - 1 = (y + 2)^2 \quad \text{D} y + 2 = (x - 1)^2 \]

6 Which is equivalent to \( (5 - 2\sqrt{3})^2 ? \) \( \text{All.3a} \)
   \[ \text{F} 13 - 20\sqrt{3} \quad \text{G} 37 - 20\sqrt{3} \quad \text{H} 13 \quad \text{J} 37 - 10\sqrt{3} \]
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

7 What is the solution set for this system of equations? All.14  
\[
\begin{align*}
\begin{align*}
y &= x^2 - 5x + 4 \\
x - y &= 1
\end{align*}
\end{align*}
\]
A \(\{1, 5\}\)  
B \(\{0, 4\}\)  
C \(\{(0, 1), (4, 5)\}\)  
D \(\{(1, 0), (5, 4)\}\)

8 Which is the completely factored form of \(8r^3 - 125s^3\)? All.5  
F \((2r + 5s)(4r^2 - 10rs + 25s^2)\)  
G \((2r - 5s)^3\)  
H \((2r - 5s)(4r^2 + 10rs + 25s^2)\)  
J \((2r - 5s)(4r^2 + 20rs + 25s^2)\)

9 Which function is the inverse of \(f(x) = 2x^3 - 5\)? All.9  
A \(g(x) = \frac{1}{2x^3 - 5}\)  
B \(h(x) = \sqrt[3]{\frac{x + 5}{2}}\)  
C \(j(x) = -2x^3 + 5\)  
D \(k(x) = \frac{\sqrt[3]{x + 5}}{2}\)

10 Matrix \(P\) shows the admission prices at the Science Museum of Virginia in Richmond.  
\[
P = \begin{bmatrix}
\text{Exhibits & Film} & \text{Youth} & \text{Adult} & \text{Senior} & \text{Child} \\
\text{Exhibits Only} & $11.50 & $12.50 & $12.00 & $0 \\
\text{Film Only} & $6.00 & $7.00 & $6.50 & $0 \\
\end{bmatrix}
\]
Suppose matrix \(F\) gives the number of people in each age category for two family groups that are visiting the museum today.  
\[
F = \begin{bmatrix}
\text{Youth} & 3 & 2 \\
\text{Adult} & 2 & 3 \\
\text{Senior} & 0 & 2 \\
\text{Child} & 1 & 0 \\
\end{bmatrix}
\]
If the Jones family bought tickets to see the exhibits only and the McKay family bought tickets to see both the exhibits and the film, what is the total amount that these two families spent on admission? All.11  
F \$78.00  
G \$116.50  
H \$126.50  
J \$144.00

11 How many real solutions does \(\sqrt{2x - 1} = x - 4\) have? All.7  
A 0  
B 1  
C 2  
D 3
Diabetic Test (continued)

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

12 Which equation has no real solutions? **All.6**
   - F \( x^2 - 25 = 0 \)
   - G \( x^2 - 2x - 3 = 0 \)
   - H \( x^2 + x = 0 \)
   - J \( x^2 + 2x + 3 = 0 \)

13 The time needed to remove litter from a park varies directly as the area of the park and inversely as the number of workers. If it takes 4 hours for 5 workers to remove litter from a 120-acre park, how long will it take for 8 workers to remove litter from a 200-acre park? **All.20**
   - A 2 h 30 min
   - B 4 h 10 min
   - C 5 h 36 min
   - D 6 h 40 min

14 Which is the graph of a polynomial function with no real solutions? **All.10**

15 Which graph shows the solution set for \( |3x - 2| = 7 \)? **All.4**

16 What is the value of \( \sum_{n=1}^{6}(n^2 - n) \)? **All.16**
   - F 110
   - G 85
   - H 70
   - J 50
Diagnosic Test (continued)

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

17 Which function has a graph with x-intercepts $\frac{-5}{3}$ and 4?  All.10  17 _________
A $y = 5x^2 - 17x - 12$  
B $y = 3x^2 + 7x - 20$  
C $y = 5x^2 + 17x - 12$  
D $y = 3x^2 - 7x - 20$

18 Which matrix equation is equivalent to this system of equations?  All.12  18 _________
\[
\begin{align*}
3x - 5y &= 1 \\
-4x + y &= 0
\end{align*}
\]
F $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{4} \\ -\frac{5}{12} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$  
G $\begin{bmatrix} \frac{3}{4} \\ -\frac{5}{12} \end{bmatrix} [x \\ y] = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$  
H $\begin{bmatrix} \frac{3}{4} \\ -\frac{5}{12} \end{bmatrix} [x \\ y] = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$  
J $\begin{bmatrix} \frac{3}{4} \\ -\frac{5}{12} \end{bmatrix} [x \\ y] = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

19 Which could be the graph of $\frac{(x+3)^2}{4} + \frac{(y-1)^2}{4} = 1$?  All.18  19 _________
A  
B  
C  
D

20 Which represents $\frac{2}{3 - 4i}$ in the standard form $a + bi$?  All.17  20 _________
F $\frac{6}{25} - \frac{8}{25}i$  
G $\frac{6}{25} + \frac{8}{25}i$  
H $\frac{6}{7} + \frac{8}{7}i$  
J $\frac{6}{5} - \frac{8}{5}i$

21 If $P = \begin{bmatrix} 3 \\ 8 \\ 0 \end{bmatrix}$, $R = \begin{bmatrix} 9 \\ 6 \end{bmatrix}$, and $S = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, which product is not possible?  All.11  21 _________
A $P \times R$  
B $S \times P$  
C $P \times P$  
D $R \times P$
Diagnosis Test (continued)

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

22 Which equation describes a hyperbola with vertices (0, ±3) and asymptotes \( y = \pm \frac{3}{2}x \)?  
\[ \text{AII.18} \]

- **F** \( \frac{x^2}{4} + \frac{y^2}{9} = 1 \)
- **G** \( \frac{x^2}{4} - \frac{y^2}{9} = 1 \)
- **H** \( \frac{y^2}{9} - \frac{x^2}{4} = 1 \)
- **J** \( \frac{y^2}{3} - \frac{x^2}{2} = 1 \)

23 What property is illustrated by the following statement?  
\[ \text{AII.1} \]

If \( x = y \) and \( y = z \), then \( x = z \).

- **A** Distributive Property
- **B** Reflexive Property of Equality
- **C** Symmetric Property of Equality
- **D** Transitive Property of Equality

24 Which inequality has the empty set as its solution set?  
\[ \text{AII.4} \]

- **F** \( |5x + 2| < 0 \)
- **G** \( |5x + 2| = 0 \)
- **H** \( |5x + 2| \leq 0 \)
- **J** \( |5x + 2| > 0 \)

25 If the domain of \( f(x) = x^2 - 6x + 9 \) is \( \{-3, -1, 1, 3\} \), what is the range?  
\[ \text{AII.9} \]

- **A** \( \{16, 36\} \)
- **B** \( \{4, 16, 36\} \)
- **C** \( \{0, 4, 16, 36\} \)
- **D** \( \{0, 4, 14, 18\} \)

26 By simply examining \( \frac{4}{x} - \frac{8}{x^2 - 25} = 12 \), which set includes all of the numbers that cannot be solutions of the equation?  
\[ \text{AII.7} \]

- **F** \( \{-5, 0, 5\} \)
- **G** \( \{-5, 5\} \)
- **H** \( \{0\} \)
- **J** \( \{5\} \)

27 What is the simplest form of \( \frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x} - \frac{1}{y}} \)?  
\[ \text{AII.2} \]

- **A** \( \frac{y - x}{y + x} \)
- **B** \( \frac{y + x}{y - x} \)
- **C** \( \frac{y + x}{xy} \)
- **D** \( -1 \)
28 Which graph shows the solution set for $|4x - 3| \leq 5$? AII.4

29 Which equation shows that $f$ is directly proportional to the square of $d$ and inversely proportional to the square root of $e$? AII.20

30 Ashley and Marita went shopping at the Spotsylvania Mall in Fredericksburg. Their favorite store was having a sale where all sweaters were one price and all jeans were another price. Not including sales tax, Ashley paid $102 for 3 sweaters and 2 pairs of jeans, while Marita paid $96 for 4 sweaters and 1 pair of jeans. If $x$ represents the price of a sweater and $y$ represents the price of a pair of jeans, which matrix equation can be used to find the price for each sweater and the price for each pair of jeans? AII.12

31 What is the solution set for $2x^2 - 6x - 7 = 0$? AII.6

32 Which is a factor of $8x^2 + 10x - 3$? AII.5
Diagnostic Test (continued)

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

33 A parking garage charges $5 for the first hour and $1.75 for each additional half-hour or portion of a half-hour. Which graph best represents this situation?  

A II.8

B

C

D

34 Which type of function would best fit the data in this scatterplot?  

AII.19

F exponential

G linear

H logarithmic

J quadratic

35 Which system of inequalities best represents this graph?  

AII.13

A \[
\begin{align*}
25x + 15y &\leq 375 \\
0 &\leq y \leq 10 \\
x &\geq 0
\end{align*}
\]

B \[
\begin{align*}
15x + 25y &\leq 375 \\
0 &\leq y \leq 10 \\
x &\geq 0
\end{align*}
\]

C \[
\begin{align*}
15x + 25y &\leq 375 \\
y &\leq 10 \\
x &\leq 0
\end{align*}
\]

D \[
\begin{align*}
15x - 25y &\leq 375 \\
0 &\leq y \leq 10 \\
x &\geq 0
\end{align*}
\]
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

36 Which is equal to \(i^{-7}\)? \(\text{All.17}\)

- \(F\) 1
- \(G\) \(-1\)
- \(H\) \(i\)
- \(J\) \(-i\)

37 What is another way to write \(\frac{2}{5} \cdot \frac{1}{3} \cdot \frac{5}{10}\)? \(\text{All.3b}\)

- \(A\) \(\sqrt[3]{a^4b^3c^5}\)
- \(B\) \(\sqrt[5]{(abc)^5}\)
- \(C\) \(\sqrt[12]{(abc)^{12}}\)
- \(D\) \(\sqrt[18]{a^3b^6c^{18}}\)

38 If an object is dropped from a height of \(h_0\) feet, its height \(h\) (in feet) after \(t\) seconds is given by the equation \(h = -16t^2 + h_0\). The SunTrust building in Richmond is 400 feet tall. If a ball is dropped from the top of this building, how long will it take the ball to hit the ground? \(\text{All.6}\)

- \(F\) 25.0 s
- \(G\) 5.0 s
- \(H\) 2.5 s
- \(J\) 2.2 s

39 This is the graph of a polynomial function \(h\). Which appears to be the solution set of \(h(x) = 0\)? \(\text{All.10}\)

- \(A\) \{-2\}
- \(B\) \{-2, 3\}
- \(C\) \{3\}
- \(D\) \(\emptyset\)

40 The figure shows the graph of a system of equations. How many real-number solutions does the system have? \(\text{All.14}\)

- \(F\) 0
- \(G\) 1
- \(H\) 2
- \(J\) 4

41 Since 2000, Virginia’s population has been increasing by about 1.3% per year. What type of function best models this situation? \(\text{All.19}\)

- \(A\) exponential
- \(B\) linear
- \(C\) logarithmic
- \(D\) quadratic
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

42 If \(a, b, c,\) and \(d\) are distinct real numbers, which matrix is the additive inverse of \(M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\)?

- **F** \(\begin{bmatrix} d & c \\ b & a \end{bmatrix}\)
- **G** \(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\)
- **H** \(\begin{bmatrix} -d & -c \\ -b & -a \end{bmatrix}\)
- **J** \(\begin{bmatrix} -a & -b \\ c & d \end{bmatrix}\)

43 The chart shows the number of students enrolled in mathematics classes at Northwest High School in September for several recent years. Assuming a linear relationship, which is the best estimate of the number of students who were enrolled in mathematics classes in September, 2004?

<table>
<thead>
<tr>
<th>Year</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>1,250</td>
<td>1,322</td>
<td>1,405</td>
<td>1,483</td>
</tr>
</tbody>
</table>

- **A** 1,170 students
- **B** 1,200 students
- **C** 1,510 students
- **D** 1,560 students

44 Which type of function is shown here?

- **F** exponential
- **G** logarithmic
- **H** quadratic
- **J** rational

45 If \(f(x) = 3x - 2\) and \(g(x) = -x^2 + 5\), which is \([g \circ f](x)\)?

- **A** \(-9x^2 + 12x + 1\)
- **B** \(-3x^2 + 13\)
- **C** \(-3x^3 + 2x^2 + 15x - 10\)
- **D** \(9x^2 - 12x + 9\)

46 The function \(C(x) = \frac{150x}{100 - x}\) models the cost \(C\), in thousands of dollars, to remove \(x\)% of the pollutants from a lake. What percentage of the lake’s pollutants can be removed if $50,000 is budgeted?

- **F** 25%
- **G** 30%
- **H** 50%
- **J** 75%
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

47 Which function is represented by the graph?  
- A \( y = 2x + 1 \)  
- B \( y = -|x + 2| + 3 \)  
- C \( y = -|x - 2| + 1 \)  
- D \( y = -|x - 2| + 3 \)  

48 Which describes the graph of \( \frac{(x - 2)^2}{4} + \frac{(y + 3)^2}{9} = 1 \)?  
- F circle  
- G ellipse  
- H hyperbola  
- J parabola  

49 Which equation most closely fits the data in this scatterplot?  
- A \( y = 90 - x \)  
- B \( y = \frac{90}{x} \)  
- C \( y = -1.6x + 90 \)  
- D \( y = 0.1(x - 40)^2 \)  

50 The VHR Company makes replicas of historic sites in Virginia. Its two best selling replicas are of Mount Vernon and Monticello. The company makes a profit of $8 on each replica of Mount Vernon and $10 on each replica of Monticello. To use linear programming to maximize profit, the company’s finance officer developed this feasible region from the constraints on the company’s resources and the pattern of demand for these replicas. The number of Mount Vernon replicas to be made each week is represented by \( x \) and \( y \) represents the number of Monticello replicas to be made each week. How many of each replica should the company make each week in order to maximize the profit?  
- F 200 Mount Vernon replicas, 0 Monticello replicas  
- G 150 Mount Vernon replicas, 50 Monticello replicas  
- H 100 Mount Vernon replicas, 100 Monticello replicas  
- J 50 Mount Vernon replicas, 100 Monticello replicas
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

**OBJECTIVE AII.1** Identify field properties, axioms of equality and inequality, and properties of order that are valid for the set of real numbers and its subsets, complex numbers, and matrices.

1. What property is illustrated by $15 + (35 + 12) = 15 + (12 + 35)$?
   - A  Associative Property of Addition
   - B  Commutative Property of Addition
   - C  Distributive Property
   - D  Transitive Property of Equality

2. Let $M$ be the set of all $2 \times 2$ matrices. Which property does *not* hold for set $M$?
   - F  Associative Property of Addition
   - G  Commutative Property of Addition
   - H  Commutative Property of Multiplication
   - J  Identity Property of Multiplication

3. If $x$, $y$, and $r$ are real numbers with $x > y$ and $r < 0$, which of the following inequalities will be true?
   - A  $x + r < y + r$
   - B  $x - r < y - r$
   - C  $\frac{x}{r} > \frac{y}{r}$
   - D  $xr < yr$

4. What is the multiplicative identity element for the set of $3 \times 3$ matrices?
   - F  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
   - G  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
   - H  $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
   - J  $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

5. Which statement is an example of the Symmetric Property of Equality?
   - A  If $a = b$, then $b = a$.
   - B  $c = c$
   - C  If $a = b$ and $b = c$, then $a = c$.
   - D  If $a = b$, then $a + c = b + c$. 
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

OBJECTIVE AII.1  (continued)

6 In which set of numbers does every nonzero number have a multiplicative inverse in the set?
   F  integers
   G  natural numbers
   H  rational numbers
   J  whole numbers

7 If \( z = a + bi \) and \( w = c + di \) are complex numbers, with \( a, b, c, \) and \( d \) all nonzero real numbers, which of the following must be a real number?
   A  \( 2z \)
   B  \( z + w \)
   C  \( z \cdot (a - bi) \)
   D  \( zw \)

8 If \( A \) and \( B \) are \( 2 \times 2 \) matrices, with \( A \neq B \), which statement must be true?
   F  \( AB \) and \( BA \) both exist, and \( AB = BA \).
   G  \( A^{-1} \) exists and is a \( 2 \times 2 \) matrix.
   H  \( -B \) exists and is a \( 2 \times 2 \) matrix.
   J  \( A - B \) and \( B - A \) both exist, and \( A - B = B - A \).

OBJECTIVE AII.2  Add, subtract, multiply, divide, and simplify rational expressions, including complex fractions.

1 Which expression is equal to \( \frac{15m^3n^2 - 25m^4n + 5mn}{5mn} \)?
   A  \( 10m^2n - 20m^3 \)
   C  \( 3m^2n - 5m^3 \)
   B  \( -2m^7n^3 \)
   D  \( 3m^2n - 5m^3 + 1 \)

2 Which is equivalent to \( \frac{8x}{5} - \frac{7y}{3} \)?
   F  \( \frac{24x - 35y}{15} \)
   H  \( 24x - 35y \)
   G  \( \frac{8x - 7y}{15} \)
   J  \( -\frac{56xy}{15} \)

3 Which is equivalent to \( \frac{x^2 - 6x + 9}{x^2 + 4x - 21} \)?
   A  \( \frac{x - 3}{x + 7} \)
   C  \( \frac{x + 3}{x - 7} \)
   B  \( \frac{x + 3}{x + 7} \)
   D  \( \frac{x - 3}{x - 7} \)
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

OBJECTIVE AII.2 (continued)

4 Which is the simplest form of the product \( \frac{2x - 8}{x^2 - 16} \cdot \frac{x^2 + 6x + 8}{4x + 8} \)?

F \( \frac{x + 4}{2(x - 4)} \)

G \( \frac{x + 4}{2(x + 2)} \)

H \( \frac{1}{2} \)

J 2

5 Which is equivalent to \( \frac{2}{x^2 - y^2} \div \frac{6}{x + y} \)?

A \( \frac{3}{x - y} \)

B \( \frac{1}{3(x - y)} \)

C \( \frac{12}{(x^2 - y^2)(x + y)} \)

D \( \frac{x - y}{3(x + y)} \)

6 What is the simplest form of the expression \( \frac{1}{z + 1} + \frac{1}{z - 1} \)?

F \(-2\)

G 2

H \(2z\)

J \(2z^2\)

7 Which expression is equal to \( \frac{5r}{r^2 + 3r + 2} - \frac{3r}{r^2 + r - 2} \)?

A \( \frac{2r}{(r + 2)(r + 1)(r - 1)} \)

B \( 2r^2 - 8r \)

C \( \frac{2r^2 - 8r}{(r^2 + 3r + 2)(r^2 + r - 1)} \)

D \( \frac{2r(r - 4)}{(r + 2)(r + 1)(r - 1)} \)

8 Which is equivalent to \( \frac{x^3 - 1}{x^2 - 1} \)?

F \( x + 1 \)

G \( \frac{x^2 + x + 1}{x + 1} \)

H \( \frac{x^2 + 2x + 1}{x + 1} \)

J \( \frac{x^2 - x + 1}{x + 1} \)

9 Which is the simplest form of the expression \( \frac{\frac{8}{3x + 2y}}{4} \)?

A \( 6x - 4y \)

B \( 2(3x - 2y)^2 \)

C \( 6x + 4y \)

D \( \frac{32}{(3x + 2y)^2(3x - 2y)} \)
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

OBJECTIVE AII.3 Add, subtract, multiply, divide, and simplify radical expressions containing positive rational numbers and variables and expressions containing rational exponents; write radical expressions as expressions containing rational exponents and vice versa.

1 Which is equivalent to \((-\sqrt{5})^3\)?
   A \(-125\)   B \(-5\sqrt{5}\)   C \(-5\sqrt[3]{5}\)   D \(5\sqrt{5}\)

2 Which is the simplest form of the product \((\sqrt[3]{11} - \sqrt[3]{3})(\sqrt[3]{11} + \sqrt[3]{3})\)?
   F \(112\)   G \(8\)   H \(\sqrt[3]{14}\)   J \(\sqrt[3]{8}\)

3 Which expresses the sum \(\sqrt[3]{16} + \sqrt[3]{54}\) as a single term?
   A \(2\sqrt[3]{5}\)   B \(\sqrt[3]{70}\)   C \(\sqrt[3]{70}\)   D \(5\sqrt[2]{2}\)

4 Which is equivalent to \(\frac{6}{3 + \sqrt{7}}\)?
   F \(9 + 3\sqrt{7}\)   G \(9 - \sqrt{7}\)   H \(9 - 3\sqrt{7}\)   J \(\frac{\sqrt{7} - 9}{20}\)

5 What is another way to write \(\sqrt{m^2n^9p^{12}}\)?
   A \(m^2n^3p^3\)   B \(m^2n^9p^3\)   C \(m^2n^{-5}p^{-7}\)   D \((mn)^{23}\)

6 Which is equivalent to \((\sqrt{7} - \sqrt{5})^2\)?
   F \(2 - \sqrt{35}\)   G \(12 - 2\sqrt{35}\)   H \(2\)   J \(12 - \sqrt{35}\)

7 What is another way to write \(\frac{2}{3}b^3\)?
   A \(\sqrt[3]{a^2b^4c^{15}}\)   B \(\sqrt[3]{a^2b^8c^{10}}\)   C \(\sqrt[3]{(abc)^{21}}\)   D \(\frac{3}{a^2b^4c^5}\)

8 Which is equivalent to \(\frac{5x}{\sqrt{5x^2}}\)?
   F \(5\sqrt{x}\)   G \(5\sqrt{x^2}\)   H \(\sqrt{5x^2}\)   J \(\sqrt{25x}\)
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

OBJECTIVE AII.3 (continued)

9 Which is equivalent to \( \left( \frac{2}{a^3} \right)^{-\frac{3}{4}} \) ?

A \( \frac{b^9}{a^8} \)  
B \( a^8b^9 \)  
C \( a^9b^8 \)  
D 1

10 Which is another way to write \( \left( \frac{8x^9}{27x^{-6}} \right)^{\frac{2}{3}} ? \)

F \( \frac{2x^3}{3} \)  
G \( \frac{4x^2}{9} \)  
H \( \frac{4}{9x^{10}} \)  
J \( \frac{4x^{10}}{9} \)

OBJECTIVE AII.4 Solve absolute value equations and inequalities graphically and algebraically. Graphing calculators will be used as a primary method of solution and to verify algebraic solutions.

1 Which graph shows the solution set for \(|2x + 1| = 9|? \)

A  
B  
C  
D

2 What are the solutions of \(|3x - 5| = 12|? \)

F \( -\frac{7}{3} \) and \( \frac{10}{3} \)  
G \( \frac{17}{3} \) and \( -\frac{7}{3} \)  
H \( -\frac{17}{3} \) and \( -\frac{7}{3} \)  
J \( \frac{10}{3} \) and \( -\frac{17}{3} \)

3 Which inequality has the solution set graphed below?

A \( |x + 3| \geq -3 \)  
B \( |x| \geq 3 \)  
C \( |x| \leq 3 \)  
D \( |x| < 3 \)

4 Which inequality can be solved using the shaded region of this graph?

F \( |x - 2| \leq y \leq 3 \)  
G \( |x - 2| \geq y \geq 3 \)  
H \( |x| - 2 \leq y \leq 3 \)  
J \( |x| + 2 \geq y \geq 3 \)
5 Which is the solution set of $|2x - 7| \leq 8$?

A $\{x | \frac{-15}{2} \leq x \leq \frac{15}{2} \}$  
B $\{x | \frac{-15}{2} \leq x \leq \frac{1}{2} \}$  
C $\{x | x \leq \frac{-1}{2} \text{ or } x \geq \frac{15}{2} \}$  
D $\{x | x \leq \frac{-15}{2} \text{ or } x \geq \frac{1}{2} \}$  

6 What are the solutions of $|3x - 1| = |x + 4|$?

F $\frac{3}{4}$ and $-\frac{5}{2}$  
G $-4$ and $\frac{1}{3}$  
H $\frac{5}{4}$ and $\frac{3}{2}$  
J $-\frac{3}{4}$ and $\frac{5}{2}$  

7 Using the graphs of $y = |x + 3|$ and $y = -|x + 3| + 2$ shown, what is the solution set of $|x + 3| = -|x + 3| + 2$?

A $\{0, 2\}$  
B $\{1\}$  
C $\{-3\}$  
D $\{-4, -2\}$  

8 Which inequality has the empty set as its solution set?

F $|2x - 1| < 5$  
G $|2x - 1| < -3$  
H $|2x - 1| > 3$  
J $|2x - 1| \leq 0$  

9 Which inequality has the set of all real numbers as its solution set?

A $|3 - x| \geq -1$  
B $|3 - x| \geq 1$  
C $|3 - x| < 1$  
D $|3 - x| < 0$  

10 Which graph shows the solution set for $|3x - 2| + 4 \geq 9$?
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

**OBJECTIVE AII.5** Identify and factor completely polynomials representing the difference of squares, perfect square trinomials, the sum and difference of cubes, and general trinomials.

1 Which is the completely factored form of $3x^2 - 6x - 45$?  
A $(3x - 15)(x + 3)$  
B $(x - 5)(3x + 9)$  
C $3(x + 5)(x - 3)$  
D $3(x - 5)(x + 3)$

2 Which is a factor of $8z^3 + 1$?  
F $(2z - 1)$  
G $(2z + 1)$  
H $(2z^2 - 1)$  
J $8z^3$

3 Which is a factored form of $81x^2 - 49y^4$?  
A $(9x + 7y)(9x - 7y)$  
B $(9x - 7y)^2$  
C $(9x + 7y^2)(9x - 7y^2)$  
D $(9x - 7y^2)^2$

4 Which is the completely factored form of $m^3 - 27n^3$?  
F $(m - 3n)(m^2 + 3mn + 9n^2)$  
G $(m + 3n)(m^2 - 3mn + 9n^2)$  
H $(m - 3n)(m^2 - 6mn + 9n^2)$  
J $(m + 3n)(m - 3n)^2$

5 Which is a factor of $6x^2 - x - 2$?  
A $(3x + 2)$  
B $(3x - 2)$  
C $(3x + 1)$  
D $(2x - 1)$

6 Which is the completely factored form of $16x^2 - 24xy + 9y^2$?  
F $(4x + 3y)(4x - 3y)$  
G $(4x + 3y)^2$  
H $(16x + 9y)(x + y)$  
J $(4x - 3y)^2$

7 If $4x^2 + 12x - 160$ is factored completely, which will be one of the factors?  
A $(2x - 10)$  
B $(x + 5)$  
C $(x - 8)$  
D $(x + 8)$

8 Which is a factored form of $x^6 + 2x^3 + 1$?  
F $(x^3 + 1)^2$  
G $(x^3 + 1)(x^3 - 1)$  
H $(x^3 - 1)^2$  
J $x^2(x + 1)^2$
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

OBJECTIVE AII.5 (continued)

9 Which is a factor of $15x^2 - 4x - 32$?
   A (5x - 8)  B (3x + 8)
   C (5x + 4)  D (3x - 4)

10 If $x^8 - 1$ is factored completely, which will not be one of the factors?
   F $(x^2 + 1)$  G $(x - 1)$
   H $(x^2 - 1)$  J $(x^4 + 1)$

OBJECTIVE AII.6 Select, justify, and apply a technique to solve a quadratic equation over the set of complex numbers. Graphing calculators will be used for solving and for confirming the algebraic solutions.

1 What is the solution set for $x^2 - 11x + 24 = 0$?
   A {3, 8}  B {3, -8}
   C {-3, 8}  D {-3, -8}

2 Which equation has no real solutions?
   F $x^2 + 6x + 9 = 0$  G $x^2 - 5 = 0$
   H $x^2 + 9 = 0$  J $x^2 - x = 0$

3 What is the solution set for $x^2 + 2x + 4 = 0$?
   A $\{-1 \pm \sqrt{3}\}$  B $\{-1 \pm i\sqrt{3}\}$
   C {±2}  D $\{-1 \pm 2i\sqrt{3}\}$

4 What are the solutions of $5x^2 - 16 = 0$?
   F $\pm \frac{\sqrt{5}}{5}$  G $\pm \frac{4\sqrt{5}}{5}$
   H $\pm \frac{\sqrt{5}}{4}$  J $\pm \frac{16}{5}$

5 Which equation has exactly one real solution?
   A $x^2 - 4 = 0$  B $x^2 + 4 = 0$
   C $4x^2 - 4x + 1 = 0$  D $x^2 + 3x - 4 = 0$

6 Which is the solution set for $x(x - 4) = 20$?
   F $\{2 \pm 2\sqrt{6}\}$  G $\{2 \pm 6\sqrt{2}\}$
   H {20, 24}  J {0, 4}
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

OBJECTIVE AII.6 (continued)

7 Which is a solution of $2x^2 - 3x + 4 = 0$?

A $\frac{3 + \sqrt{23}}{4}$
B $\frac{3 + \sqrt{41}}{4}$
C $\frac{3 - i\sqrt{41}}{4}$
D $\frac{3 - i\sqrt{23}}{4}$

8 Which equation can be solved by factoring?

F $x^2 - 2x + 4 = 0$
G $x^2 - 7x + 10 = 0$
H $x^2 - x - 5 = 0$
J $x^2 + 3x + 1 = 0$

9 Which equation has solution set \{1 ± i\}?

A $x^2 + 2x + 2 = 0$
B $x^2 - x + 1 = 0$
C $x^2 - 2x + 2 = 0$
D $x^2 + 2x + 1 = 0$

10 Which equation has two real, irrational solutions?

F $(x + 6)^2 = 20$
G $(x - 8)^2 = 0$
H $(x + 3)^2 = 49$
J $(x - 5)^2 = -8$

11 If an object is dropped from a height of $h_0$ meters, its height $h$ (in meters) after $t$ seconds is given by the equation $h = -4.9t^2 + h_0$. If a ball is dropped from a height of 19.6 meters, how long will it take the ball to hit the ground?

A 4.0 s
B 3.8 s
C 2.5 s
D 2.0 s

12 If an object is thrown upward from a height of $h_0$ feet at an initial velocity of $v_0$ feet per second, its height $h$ (in feet) after $t$ seconds is given by the equation $h = -16t^2 + v_0t + h_0$. The James Monroe Building in Richmond is 450 feet tall. If a ball is thrown upward from the top of this building at an initial velocity of 60 feet per second, how long will it take the ball to hit the ground?

F 3.75 s
G 5.3 s
H 5.92 s
J 7.5 s
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

**OBJECTIVE AII.7** Solve equations containing rational expressions and equations containing radical expressions algebraically and graphically. Graphing calculators will be used for solving and for confirming the algebraic solutions.

1. For which value of \( x \) does \( \frac{2x - 1}{8} = \frac{5x + 2}{12} \)?
   - A \( \frac{-7}{4} \)
   - B \( \frac{-7}{16} \)
   - C \( \frac{1}{4} \)
   - D \( \frac{7}{4} \)

2. By simply examining \( \frac{4}{x^3 + 3x^2} - \frac{2}{x^2 - 9} = 1 \), which are all of the numbers that cannot be solutions of the equation?
   - F 3
   - G \(-3\) and 3
   - H 9 and 0
   - J \(-3\), 0, and 3

3. For which value of \( x \) does \( \sqrt{x - 5} + 2 = 6 \)?
   - A 69
   - B 37
   - C 21
   - D 16

4. Which is the solution set for \( \sqrt{3 - 4x} = 2x \)?
   - F \( \left\{ \frac{1}{2} \right\} \)
   - G \( \left\{ \frac{-3}{2}, \frac{1}{2} \right\} \)
   - H \( \left\{ \frac{-3}{2}, 2 \right\} \)
   - J \( \emptyset \)

5. Which is the solution set for \( \frac{1}{x + 5} - \frac{3}{x - 2} = \frac{-7}{x^2 + 3x - 10} \)?
   - A \( \left\{ -5 \right\} \)
   - B \( \left\{ -5, 2 \right\} \)
   - C \( \left\{ -12 \right\} \)
   - D \( \emptyset \)

6. How many real solutions does \( \sqrt{x^2 + 3x + 2} = \sqrt{x^2 - 3x + 6} \) have?
   - F 3
   - G 2
   - H 1
   - J 0

7. How many rational solutions does \( \frac{x}{x + 3} - \frac{2x}{x - 4} = 1 \) have?
   - A 0
   - B 1
   - C 2
   - D 3

*Virginia SOL, Algebra II*
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

**OBJECTIVE AII.7** (continued)

8 The formula \( \frac{1}{f} = \frac{1}{p} + \frac{1}{q} \) describes the relationship between the focal length \( f \) of a thin lens, the distance \( p \) between the object and the lens, and the distance \( q \) between the lens and the image. If an object is placed 30 centimeters from a lens with focal length of 12 centimeters, how far will the image of the object be from the lens?

F 0.05 cm  G 10 cm  H 18 cm  J 20 cm

9 For a certain species of tree, the height \( h \), in meters, is approximated by the diameter \( d \), in centimeters, using the formula \( h = 1.75 \sqrt[5]{d^4} \). Which is closest to the diameter of a 45-meter tall tree of this species?

A 79 cm  B 58 cm  C 52 cm  D 26 cm

**OBJECTIVE AII.8** Recognize multiple representations of functions (linear, quadratic, absolute value, step, and exponential functions) and convert between a graph, a table, and symbolic form. A transformational approach to graphing will be employed through the use of graphing calculators.

1 Which function includes the values in the table?

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

A \( y = x - 3 \)  B \( y = |x - 4| \)  C \( y = |x + 4| - 1 \)  D \( y = |x - 4| + 1 \)

2 Which function is represented by the graph?

F \( f(x) = 2^{-x} \)  G \( f(x) = 2^x \)  H \( f(x) = x^2 \)  J \( f(x) = -\sqrt{x} \)

3 Which function includes the values in the table?

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-31</td>
<td>-7</td>
<td>1</td>
<td>-7</td>
<td>-31</td>
</tr>
</tbody>
</table>

A \( f(x) = 2x^2 + 1 \)  B \( f(x) = -2x^2 - 1 \)  C \( f(x) = -2x^2 + 1 \)  D \( f(x) = 8x + 1 \)
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

OBJECTIVE AII.8 (continued)

4 Which function is represented by the graph?
   - F \( f(x) = \frac{1}{2}x + 2 \)
   - G \( f(x) = \frac{1}{2}x - 4 \)
   - H \( f(x) = -\frac{1}{2}x + 4 \)
   - J \( f(x) = 2x + 2 \)

5 In 2004, the cost for mailing a first-class letter within the United States was 37 cents for the first ounce (or part of an ounce), plus 23 cents for each additional ounce or fraction of an ounce. Which graph best represents this situation?

   - A
   - B
   - C
   - D

6 The height \( h \) above the ground of an object dropped from the top of a 100-foot tall building after \( t \) seconds can be modeled by the function \( h(t) = 100 - 16t^2 \). Which graph best represents this function?

   - F
   - G
   - H
   - J
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

OBJECTIVE AII.9 Find the domain, range, zeros, and inverse of a function, the value of a function for a given element in its domain, and the composition of multiple functions. The graphing calculator will be used as a tool to assist in investigation of functions.

1. If the domain of $f(x) = 4 - x^2$ is $\{-2, -1, 0, 1, 2\}$, what is the range?
   - A $\{-4, -3, 0\}$
   - B $\{0, 3, 4\}$
   - C $\{2, 3, 4, 5, 6\}$
   - D $\{4, 9, 16, 25, 36\}$

2. What are the zeros of $f(x) = 2x^2 + 5x - 3$?
   - F $-3$ and $\frac{1}{2}$
   - G $-\frac{1}{2}$ and 3
   - H $-3$ and 2
   - J 2 and 3

3. If the domain of each function is the set of all real numbers, which function has an inverse relation that is also a function?
   - A $f(x) = x^2$
   - B $f(x) = |x|$
   - C $f(x) = x^3$
   - D $f(x) = x^4$

4. What is the domain of $f(x) = \log_2 x$?
   - F all real numbers
   - G $\{x| x \geq 0\}$
   - H $\{x| x > 0\}$
   - J $\{x| x < 0\}$

5. Which function is the inverse of $f(x) = 3x - 6$?
   - A $g(x) = -3x + 6$
   - B $h(x) = \frac{1}{3}x - 2$
   - C $j(x) = \frac{1}{3}x + 6$
   - D $k(x) = \frac{1}{3}x + 2$

6. If $f(x) = 4x + 5$ and $g(x) = -x^2 - 1$, which is the value of $g[f(-2)]$?
   - F $-15$
   - G $-10$
   - H $-3$
   - J 3

7. Which number does not belong to the range of $f$?
   - $f(x) = \begin{cases} 
   -x + 4 & \text{if } x < 2 \\
   1 & \text{if } 2 \leq x \leq 4 \\
   2x - 6 & \text{if } x > 4 
   \end{cases}$
   - A 10
   - B 3
   - C 2
   - D 1
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

**OBJECTIVE AII.9 (continued)**

8 If \( h(z) = 2z^2 + 1 \) and \( k(z) = z - 5 \), which is correct?

- **F** \( [h \circ k](z) = 2z^2 - 4 \)
- **G** \( [h \circ k](z) = 2z^2 - 20z + 51 \)
- **H** \( [h \circ k](z) = 2z^2 - 9z - 5 \)
- **J** \( [h \circ k](z) = 2z^3 - 10z^2 + z - 5 \)

9 The table lists the eight Presidents of the United States who were born in Virginia and the age of each man at the time of his inauguration. Courtney is a high school student in Roanoke. Her math teacher asked her to express the information in this table as a function. How should she write the range of this function?

<table>
<thead>
<tr>
<th>President</th>
<th>Age at Inauguration</th>
</tr>
</thead>
<tbody>
<tr>
<td>George Washington</td>
<td>57</td>
</tr>
<tr>
<td>Thomas Jefferson</td>
<td>57</td>
</tr>
<tr>
<td>James Madison</td>
<td>57</td>
</tr>
<tr>
<td>James Monroe</td>
<td>58</td>
</tr>
<tr>
<td>William Henry Harrison</td>
<td>68</td>
</tr>
<tr>
<td>John Tyler</td>
<td>51</td>
</tr>
<tr>
<td>Zachary Taylor</td>
<td>64</td>
</tr>
<tr>
<td>Woodrow Wilson</td>
<td>56</td>
</tr>
</tbody>
</table>

- **A** 17
- **B** \( \{x \mid 51 \leq x \leq 68\} \)
- **C** \( \{51, 56, 57, 58, 64, 68\} \)
- **D** \( \{57, 57, 57, 58, 68, 51, 64, 56\} \)

**OBJECTIVE AII.10** Investigate and describe through the use of graphs the relationships between the solution of an equation, zero of a function, \( x \)-intercept of a graph, and factors of a polynomial expression.

1 If \( f(x) = 3x(x - 5)(x + 4) \), what is the solution set of \( f(x) = 0 \)?

- **A** \( \{-5, 4\} \)
- **B** \( \{-5, 0, 3, 4\} \)
- **C** \( \{-4, 5\} \)
- **D** \( \{-4, 0, 5\} \)

2 Which function has a graph with \( x \)-intercepts \( -\frac{3}{4} \) and \( \frac{1}{2} \)?

- **F** \( y = 8x^2 + 2x - 3 \)
- **G** \( y = 3x^2 - 2x - 8 \)
- **H** \( y = 8x^2 - 2x - 3 \)
- **J** \( y = 3x^2 + 2x - 8 \)
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

**OBJECTIVE AII.10** (continued)

3. Which is the graph of a quadratic function with exactly one real zero?
   - A
   - B
   - C
   - D

4. This is the graph of a polynomial function \( f \).
   Which is not a solution of \( f(x) = 0 \)?
   - F 3
   - G 1
   - H 0
   - J -2

5. Which function has no real zeros?
   - A \( y = \frac{1}{x} \)
   - B \( y = x^2 - 3 \)
   - C \( y = |x| \)
   - D \( y = -x^3 \)

6. Between which two integers is the real zero of \( f(x) = x^3 + 12 \) located?
   - F -4 and -3
   - G -3 and -2
   - H 2 and 3
   - J 3 and 4

7. This is the graph of a function \( g \).
   Which appears to be the solution set of \( g(x) = 0 \)?
   - A \{ -4, 0, 4 \}
   - B \{ -4, 4 \}
   - C \{ -2, 2 \}
   - D \( \emptyset \)
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

**OBJECTIVE AII.11** Use matrix multiplication to solve practical problems. Graphing calculators or computer programs with matrix capabilities will be used to find the product.

1. If $M = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, $N = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$, and $P = [0]$, which product is not possible?  
   **A** $M \times M$  
   **B** $M \times N$  
   **C** $N \times M$  
   **D** $N \times P$

2. If $A = \begin{bmatrix} -8 & 3 \\ 5 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 4 \\ -5 & 7 \end{bmatrix}$, which matrix represents $B \times A$?  
   **F** $\begin{bmatrix} -48 & 12 \\ -25 & 14 \end{bmatrix}$  
   **G** $\begin{bmatrix} -63 & 11 \\ 40 & 6 \end{bmatrix}$  
   **H** $\begin{bmatrix} -28 & 75 \\ 10 & -29 \end{bmatrix}$  
   **J** $\begin{bmatrix} -28 & 10 \\ 75 & -29 \end{bmatrix}$

3. Which statement about matrices $T$ and $Q$ is true?  
   $T = \begin{bmatrix} -1 & 4 \\ 6 & 12 \\ 9 & -5 \\ 7 & 5 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$  
   **A** The product $T \times Q$ is a $4 \times 1$ matrix.  
   **B** The product $Q \times T$ is a $1 \times 4$ matrix.  
   **C** The product $Q \times Q$ is a $2 \times 1$ matrix.  
   **D** The products $T \times Q$, $Q \times T$, and $Q \times Q$ do not exist.

4. The Office Store sells two models of desk lamps. Matrix $R$ shows the number of lamps of each model sold at the store during the first four months of 2004, while matrix $S$ gives the price for each model.

   $R = \begin{bmatrix} 55 & 82 \\ 61 & 81 \\ 75 & 70 \\ 80 & 66 \end{bmatrix}$  
   $S = \begin{bmatrix} \$24.50 \\ \$36.00 \end{bmatrix}$

   For which month did the store have the greatest total sales from these desk lamps?  
   **F** January  
   **G** February  
   **H** March  
   **J** April

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Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

OBJECTIVE AII.11 (continued)

5 Matrix $P$ shows the 2004 admission prices to Mount Vernon, George Washington's home. Matrix $A$ gives the number of admissions in each of the four age categories sold during two months last year.

$$P = \begin{bmatrix} \text{Adult} & \text{Senior} & \text{Child (6–11)} & \text{Child (0–5)} \\ \$11.00 & \$10.50 & \$5.00 & \$0.00 \end{bmatrix}$$

$$A = \begin{bmatrix} \text{February} & \text{July} \\ 18,500 & 31,300 \\ 12,200 & 13,800 \\ 6,050 & 9,050 \\ 3,400 & 5,260 \end{bmatrix}$$

What was the total sales from admission tickets for July?

A $896,300
B $534,450
C $361,850
D $354,260

OBJECTIVE AII.12 Represent problem situations with a system of linear equations and solve the system, using the inverse matrix method. Graphing calculators or computer programs with matrix capability will be used to perform computations.

1 Which is the multiplicative inverse of $\begin{bmatrix} 3 & -5 \\ 7 & 2 \end{bmatrix}$?

A $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
B $\begin{bmatrix} -3 & 5 \\ -7 & 2 \end{bmatrix}$
C $\begin{bmatrix} -2 & 5 \\ -7 & 3 \\ 29 & 29 \end{bmatrix}$
D $\begin{bmatrix} -3 & 7 \\ -5 & 2 \\ 29 & 29 \end{bmatrix}$

2 Which matrix is its own multiplicative inverse?

F $\begin{bmatrix} 1 & 0 & -1 & 0 \end{bmatrix}$
G $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$
H $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
J $\begin{bmatrix} 5 & 1 \\ 10 & 10 \end{bmatrix}$
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

**OBJECTIVE AII.12** (continued)

3 Which matrix equation is equivalent to the system of equations below?
\[
\begin{align*}
4x + 5y &= 9 \\
2x - y &= 6
\end{align*}
\]

A \[
\begin{bmatrix}
4 & 5 \\
2 & -1
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix} =
\begin{bmatrix}
9 \\
6
\end{bmatrix}
\]

B \[
\begin{bmatrix}
4 & 5 \\
2 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix} =
\begin{bmatrix}
9 \\
6
\end{bmatrix}
\]

C \[
\begin{bmatrix}
4 & -2 \\
5 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix} =
\begin{bmatrix}
6 \\
9
\end{bmatrix}
\]

D \[
\begin{bmatrix}
4x & 5y \\
2x & y
\end{bmatrix} =
\begin{bmatrix}
9 \\
6
\end{bmatrix}
\]

3 __________

4 At Monticello, the home of Thomas Jefferson, there is one admission price for adults (ages 12 and over) and another price for children aged 6–11. Children under 6 are admitted free. A family of six, 2 parents and 4 children ages 5, 7, 9, and 11, paid $44 for admission. Another family made up of 2 parents, a grandmother, 8-year-old twins, a 10 year-old, and a 14 year-old, paid $70. What is the price of an adult admission?

F $6

G $10

H $13

J $15

4 __________

5 Liam and Nani went shopping at a store that was having a sale where all DVDs were one price and all CDs were another price. Not including sales tax, Liam paid $69 for 2 DVDs and 3 CDs, while Nani paid $73 for 1 DVD and 5 CDs. If \( x \) represents the price of 1 DVD and \( y \) represents the price of 1 CD, which matrix equation can be used to find the price for each DVD and the price for each CD?

A \[
\begin{bmatrix}
x \\
y
\end{bmatrix} =
\begin{bmatrix}
2 & 3 \\
1 & 5
\end{bmatrix}
\begin{bmatrix}
69 \\
73
\end{bmatrix}
\]

B \[
\begin{bmatrix}
x \\
y
\end{bmatrix} =
\begin{bmatrix}
5 & -3 \\
1 & 2
\end{bmatrix}
\begin{bmatrix}
69 \\
73
\end{bmatrix}
\]

C \[
\begin{bmatrix}
x \\
y
\end{bmatrix} =
\begin{bmatrix}
5/7 & -3/7 \\
-1/7 & 2/7
\end{bmatrix}
\begin{bmatrix}
69 \\
73
\end{bmatrix}
\]

D \[
\begin{bmatrix}
x \\
y
\end{bmatrix} =
\begin{bmatrix}
2/7 & 3/7 \\
1/7 & 5/7
\end{bmatrix}
\begin{bmatrix}
69 \\
73
\end{bmatrix}
\]

5 __________

6 Jenna spent a total of $14.32 for a combination of 37-cent and 23-cent postage stamps. The number of 37-cent stamps was 2 more than twice the number of 23-cent stamps. How many 23-cent stamps did she buy?

F 30

G 28

H 15

J 14

6 __________
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

**OBJECTIVE AII.13** Solve practical problems, using systems of linear inequalities and linear programming. A graphing calculator will be used to facilitate solutions to linear programming problems.

1. Which system of inequalities best represents this graph?
   - **A.** \( y \leq -\frac{4}{3}x + 4 \)
     \( x > 1 \)
     \( y < -2 \)
   - **B.** \( y \leq \frac{4}{3}x - 4 \)
     \( x > 1 \)
     \( y > -2 \)
   - **C.** \( 3x + 4y \leq 12 \)
     \( x > 1 \)
     \( y > -2 \)
   - **D.** \( 4x + 3y \leq 12 \)
     \( x > 1 \)
     \( y > -2 \)

2. A company makes Virginia state flags in two sizes, large and jumbo. The company’s profit is $4 on each large flag and $5 on each jumbo flag. The company’s employees can make up to 850 flags each week. To meet the demand for its flags, at least twice as many large flags as jumbo flags must be produced. If \( x \) represents the number of large flags and \( y \) represents the number of jumbo flags manufactured in one week, which set of constraints describes this situation?
   - **F.** \( 4x \geq 5y \)
     \( x \geq 0 \)
     \( y \geq 0 \)
   - **G.** \( 4x + 5y \leq 850 \)
     \( x \geq 2y \)
     \( y \geq 0 \)
   - **H.** \( x + y \leq 850 \)
     \( x \geq 2y \)
     \( x \geq 0 \)
     \( y \geq 0 \)
   - **J.** \( x + y \geq 850 \)
     \( x \geq 2y \)
     \( y \geq 2x \)
     \( y \geq 0 \)

3. What are the vertices of the feasible region determined by the constraints \( x + y \leq 60, x \geq 10, \) and \( y \geq 0 \)?
   - **A.** (0, 10), (0, 60), (50, 10)
   - **B.** (10, 0), (10, 50), (60, 0)
   - **C.** (10, 0), (30, 30), (60, 0)
   - **D.** (10, 50), (30, 30), (60, 0)
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

OBJECTIVE AII.13 (continued)

4 The VBC Company makes two models of office chairs. The company’s profit is $15 on each Model Q chair and $20 on each model R chair. To use linear programming to maximize profit, the company’s finance officer developed this feasible region from the constraints on the company’s resources and the pattern of demand for its products. The number of Model Q chairs to be made each week is represented by $x$, and $y$ represents the number of Model R chairs to be made each week. How many of each model should the company make each week in order to maximize the profit?

- **F** 150 model Q, 50 model R
- **G** 50 model Q, 150 model R
- **H** 100 model Q, 100 model R
- **J** 0 model Q, 200 model R

OBJECTIVE AII.14 Solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions.

1 What is the solution set for this system of equations?

\[
\begin{align*}
\begin{cases}
y = 4 - x^2 \\
x + y = 2
\end{cases}
\end{align*}
\]

- **A** $\{-1, 2\}$
- **B** $\{0, 3\}$
- **C** $\{(0, 2), (3, -1)\}$
- **D** $\{(-1, 3), (2, 0)\}$

2 What is the solution set for this system of equations?

\[
\begin{align*}
\begin{cases}
(x - 1)^2 + y^2 = 4 \\
(x - 3)^2 + y^2 = 4
\end{cases}
\end{align*}
\]

- **F** $\{(2, 3), (2, -3)\}$
- **G** $\{(-2, -\sqrt{3}), (2, \sqrt{3})\}$
- **H** $\{(2, \sqrt{3}), (2, -\sqrt{3})\}$
- **J** $\emptyset$
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

**OBJECTIVE AII.14** (continued)

3 The figure shows the graph of a system of equations. How many real solutions does the system have?
   - A 0
   - B 2
   - C 4
   - D 8

4 Which is the solution set for the system given below?
   \[
   \begin{align*}
   3x^2 + y &= 7 \\
   x - y &= 3
   \end{align*}
   \]
   - F \((-2, -5), \left(\frac{5}{3}, \frac{-4}{3}\right)\)
   - G \((2, 5), \left(\frac{5}{3}, \frac{-4}{3}\right)\)
   - H \((2, -5), \left(-\frac{5}{3}, \frac{4}{3}\right)\)
   - J \emptyset

5 How many real solutions does this system have?
   \[
   \begin{align*}
   (x + 2)^2 + (y - 2)^2 &= 4 \\
   y &= x
   \end{align*}
   \]
   - A 4
   - B 2
   - C 1
   - D 0

6 How many real solutions does this system have?
   \[
   \begin{align*}
   x^2 + y^2 &= 10 \\
   2x^2 - y^2 &= 17
   \end{align*}
   \]
   - F 0
   - G 1
   - H 2
   - J 4

7 The graph of one equation in a system is a circle and the graph of the other equation is a straight line. Which is a list of all the possibilities for the number of real solutions the system could have?
   - A 0 or 1
   - B 1 or 2
   - C 0, 1, or 2
   - D 0, 1, 2, or 4
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

**OBJECTIVE AII.15** Recognize the general shape of polynomial, exponential, and logarithmic functions. The graphing calculator will be used as a tool to investigate the shape and behavior of these functions.

1. Which type of function is shown here?
   - A exponential
   - B logarithmic
   - C quadratic
   - D rational

2. If \(a, b, c, d,\) and \(e\) are real numbers and \(a < 0\), which equation could be represented by this graph?
   - F \(y = ax + b\)
   - G \(y = ax^2 + bx + c\)
   - H \(y = ax^3 + bx^2 + cx + d\)
   - J \(y = ax^4 + bx^3 + cx^2 + dx + e\)

3. Which graph represents a rational function \(f(x) = \frac{p(x)}{q(x)}\), where \(p\) and \(q\) are linear functions?
   - A
   - B
   - C
   - D
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

**OBJECTIVE AII.15** (continued)

4 If \( a \) is a real number, with \( a > 1 \), which equation could be represented by this graph?

- **F** \( y = a^x \)
- **G** \( y = a^{-x} \)
- **H** \( y = \log_a x \)
- **J** \( y = -\log_a x \)

5 Which type of function is shown here?
- **A** absolute value
- **B** linear
- **C** quadratic
- **D** rational

**OBJECTIVE AII.16** Investigate and apply the properties of arithmetic and geometric sequences and series to solve practical problems, including finding the first \( n \) terms, finding the \( n \)th term, and evaluating summation formulas.

1 If \( a_n = \frac{n^2}{n + 2} \), then what is \( a_6 \)?

- **A** \( \frac{2}{9} \)
- **B** \( \frac{25}{7} \)
- **C** \( \frac{9}{2} \)
- **D** \( \frac{32}{5} \)

2 Which sequence is geometric?

- **F** 5, 10, 15, 20, …
- **G** 1, −2, 4, −8, …
- **H** 1, 2, 4, 7, …
- **J** 4, 2, 0, −2, …

3 What is the value of \( \sum_{n=1}^{5} (3n - 6) \)?

- **A** 15
- **B** 18
- **C** 27
- **D** 45
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

**OBJECTIVE AII.16 (continued)**

4. Which represents the series 4.5 + 5.0 + 5.5 + … + 10.0?

- **F** \[ \sum_{n=1}^{12} (0.5n + 4.0) \]
- **G** \[ \sum_{n=1}^{12} (0.5n + 4.5) \]
- **H** \[ \sum_{n=1}^{12} (4.5n + 0.5) \]
- **J** \[ \sum_{n=1}^{10} (0.5n + 4.5) \]

4. ________

5. What are the first 5 terms of the sequence \( a_n = -12\left(-\frac{1}{3}\right)^n \)?

- **A** -12, 4, \(-\frac{4}{3}, \frac{4}{9}, \frac{4}{27}\)
- **B** -4, \(-\frac{4}{3}, \frac{4}{9}, \frac{4}{27}, \frac{4}{81}\)
- **C** 12, -36, 108, -324, 972
- **D** 4, \(-\frac{4}{3}, \frac{4}{9}, \frac{4}{27}, \frac{4}{81}\)

5. ________

6. Which represents the sum of the first 50 odd positive integers?

- **F** \[ \sum_{n=1}^{50} (2n - 1) \]
- **G** \[ \sum_{n=1}^{50} n \]
- **H** \[ \sum_{n=1}^{50} (2n + 1) \]
- **J** \[ \sum_{n=1}^{50} 2n \]

6. ________

7. A high school auditorium has 15 seats in the first row, and each row after the first one has 2 more seats than the row directly in front of it. What is the total number of seats in the 20 rows of this auditorium?

- **A** 627 seats
- **B** 680 seats
- **C** 735 seats
- **D** 1,360 seats

7. ________

8. Christina works part-time as a cashier at a store in Lynchburg. She started her job on October 1, 2003, at a starting wage of $6.50 an hour, and she receives a raise of 25¢ an hour every 3 months. If she continues at this job, what will be her hourly wage on August 15, 2005?

- **F** $7.75
- **G** $8.00
- **H** $8.25
- **J** $8.50

8. ________

9. Marcus rents an apartment in Lexington, near the Washington and Lee University campus. His rent for the first year was $650 a month. At the beginning of each subsequent year, his rent was increased by 5%. Which is closest to his monthly rent during his fifth year there?

- **A** $752
- **B** $780
- **C** $790
- **D** $830

9. ________
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

OBJECTIVE AII.17 Perform operations on complex numbers and express the results in simplest form.

1 What is the sum \((14 - 5i) + (-8 + 12i)\)?
   A 22 - 17i  
   B 6 + 7i  
   C 2 - 13i  
   D 6 - 17i

2 Which represents the product of \(-3i\) and \(9 - 5i\)?
   F 15 - 27i  
   G 15 + 27i  
   H -15 + 27i  
   J -15 - 27i

3 Which is equivalent to \((7 - 6i)(2 + i)\)?
   A 20 - 5i  
   B 8 - 5i  
   C 8 + 5i  
   D 20 + 5i

4 Which product is a real number?
   F \((3 + 4i)^2\)  
   G \((3 - 4i)^2\)  
   H \((3 - 4i)(4 + 3i)\)  
   J \((3 - 4i)(3 + 4i)\)

5 Which is equal to \(i^7\)?
   A 1  
   B -1  
   C i  
   D -i

6 Which is the reciprocal of \(i^{11}\)?
   F 1  
   G -1  
   H i  
   J -i

7 Which represents \(\frac{3 - 2i}{4 - i}\) in the standard form \(a + bi\)?
   A \(\frac{14}{5} - i\)  
   B \(\frac{14}{17} - \frac{5}{17}i\)  
   C \(\frac{14}{15} - 5i\)  
   D \(\frac{10}{17} - \frac{5}{17}i\)

8 Which is equal to \((4 - 12i) - (10 - 8i) + (-4 + 7i)\)?
   F -2 + 3i  
   G -10 + 3i  
   H -10 - 13i  
   J -8 - 13i

9 Which is equivalent to \(\sqrt{4} \cdot \sqrt{-25}\)?
   A 10  
   B 10i  
   C -10  
   D -10i
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

**OBJECTIVE AII.17** (continued)

10 Which is equal to $i^3(4 + 5i)(4 - 5i)$?
- F $-41i$
- G $-9i$
- H 9
- J 41

11 Which is equivalent to $i^{-30}$?
- A 1
- B $-1$
- C $i$
- D $-i$

12 Which is equivalent to $(3 + i\sqrt{7})(3 - i\sqrt{7})$?
- F $-16$
- G $-4$
- H 16
- J 49

**OBJECTIVE AII.18** Identify conic sections (circle, ellipse, parabola, and hyperbola) from equations. Given the equations in $(h, k)$ form, sketch graphs of conic sections, using transformations.

1 Which describes the graph of $\frac{x^2}{16} - \frac{y^2}{9} = 1$?
- A circle
- B ellipse
- C hyperbola
- D parabola

2 When graphed, which equation would produce a parabola?
- F $y = 25 - x^2$
- G $x^2 - y^2 = 25$
- H $x^2 + y^2 = 25$
- J $x - y = 25$

3 Which could be an equation for this graph?
- A $(x - 2)^2 + (y + 1)^2 = 9$
- B $(x + 2)^2 + (y - 1)^2 = 9$
- C $(x - 2)^2 + (y + 1)^2 = 3$
- D $(x + 2)^2 + (y - 1)^2 = 3$

4 Which equation describes a parabola with axis $y = -4$ and vertex $(2, -4)$?
- F $y + 4 = (x - 2)^2$
- G $y - 4 = (x + 2)^2$
- H $x + 2 = (y - 4)^2$
- J $x - 2 = (y + 4)^2$
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

**OBJECTIVE AII.18** (continued)

5 Which could be the graph of \(\frac{(x - 3)^2}{4} + \frac{y^2}{9} = 1\)?

A

B

C

D

6 Which could be the graph of \(y - 4 = -2(x + 3)^2\)?

F

G

H

J

7 Which could be an equation for this graph?

A \(x^2 - y^2 = 4\)

B \(y^2 - x^2 = 4\)

C \(x^2 + y^2 = 4\)

D \(y^2 - x^2 = 2\)
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

OBJECTIVE AII.19 Collect and analyze data to make predictions and solve practical problems. Graphing calculators will be used to investigate scatterplots and to determine the equation for a curve of best fit. Models will include linear, quadratic, exponential, and logarithmic functions.

1 The chart gives the enrollment at Midway High School at the beginning of each school year for several consecutive years. Assuming exponential growth in the enrollment at this school, which is the best estimate for the school enrollment at the beginning of the 2005–2006 school year?

<table>
<thead>
<tr>
<th>Year</th>
<th>Enrollment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998–1999</td>
<td>1,240</td>
</tr>
<tr>
<td>1999–2000</td>
<td>1,303</td>
</tr>
<tr>
<td>2000–2001</td>
<td>1,370</td>
</tr>
<tr>
<td>2001–2002</td>
<td>1,435</td>
</tr>
<tr>
<td>2002–2003</td>
<td>1,510</td>
</tr>
<tr>
<td>2003–2004</td>
<td>1,585</td>
</tr>
</tbody>
</table>

A 1,664 students  
B 1,735 students  
C 1,748 students  
D 1,836 students  

2 Angela works as a cashier in a store at the Manassas Mall. The chart shows the amount of sales tax collected on four recent purchases that were made at her register. If she collects $7.29 in sales tax from her next customer, which of these was the total amount of that customer’s purchase before the tax was added?

<table>
<thead>
<tr>
<th>Amount of Purchase</th>
<th>Sales Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>$40</td>
<td>$1.80</td>
</tr>
<tr>
<td>$134</td>
<td>$6.03</td>
</tr>
<tr>
<td>$90</td>
<td>$4.05</td>
</tr>
<tr>
<td>$66</td>
<td>$2.97</td>
</tr>
</tbody>
</table>

F $182.25  
G $180.00  
H $162.00  
J $145.80  

3 Attendance at an amusement park grew rapidly during the first three years the park was open. Since then attendance has continued to increase each year, but at a slower and slower rate. Which type of function best models this situation?

A exponential  
B linear  
C logarithmic  
D quadratic  

1 C  
2 H  
3 C
Standards Practice

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

OBJECTIVE AII.19 (continued)

4 Which type of function would best fit the data in this scatterplot?
   - F exponential
   - G linear
   - H logarithmic
   - J quadratic

5 Which equation most closely fits the data in this scatterplot?
   - A \( y = -0.32x + 100 \)
   - B \( y = -0.1x^2 + 100 \)
   - C \( y = \frac{100}{x} \)
   - D \( y = -10x^2 + 100 \)

OBJECTIVE AII.20 Identify, create, and solve practical problems involving inverse variation and a combination of direct and inverse variations.

1 Which equation shows that \( z \) is directly proportional to the square root of \( x \) and inversely proportional to the cube of \( y \)?
   - A \( z = \frac{k\sqrt{x}}{y^3} \)
   - B \( z = \frac{ky^2}{\sqrt{y}} \)
   - C \( z = k\sqrt{x} \cdot y^3 \)
   - D \( z = k\sqrt{xy^3} \)

2 The volume of a gas varies directly as its temperature and inversely as its pressure. Alison is working with a gas in a laboratory at Lynchburg College. At a temperature of 375 Kelvin and a pressure of 25 pascals, she measured the volume of this gas as 30 cubic centimeters. What volume should she expect if she decreases the temperature to 320 Kelvin and increases the pressure to 40 pascals?
   - F 4 cm³
   - G 8 cm³
   - H 16 cm³
   - J 46.875 cm³
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

OBJECTIVE AII.20 (continued)

3 The current in a simple electrical circuit varies inversely as the resistance. Ryan is studying electronics technology at Norfolk State College. In one of his labs, he measured a current of 45 amps in a circuit when the resistance was 6 ohms. What will be the current in this circuit if the resistance is 10 ohms?
   A 27 amps  B 41 amps  C 49 amps  D 75 amps

4 The frequency of a vibrating string varies inversely as its length. If a piano string that is 30 inches long vibrates at 200 cycles per second, what would be the frequency of a string that is 48 inches long?
   F 1,500 cycles per second  G 175 cycles per second  H 125 cycles per second  J 75 cycles per second

5 The illumination produced by a light source varies inversely as the square of the distance from the source. If the illumination from a lamp at a distance of 6 meters is 50 candela, what is the illumination at a distance of 10 meters from the lamp?
   A $88\frac{1}{3}$ candelas  B 54 candelas  C 30 candelas  D 18 candelas

6 The time it takes to travel a certain distance is inversely proportional to the average speed for the trip. It took Tran 3 hours to drive from Richmond to Roanoke, a distance of 186 miles. Heavier traffic on his return trip resulted in an average speed that was 8 miles per hour slower than that for his trip to Roanoke. About how much longer did the return trip take than the trip to Roanoke?
   F 12 min  G 27 min  H 44 min  J 62 min

7 The amount of time needed to mow the grass in a city park varies directly as the area to be mowed and inversely as the number of workers. If it takes 2 hours for 3 workers to mow 150,000 square feet, how long will it take for 5 workers to mow 225,000 square feet?
   A 54 min  B 1 h 48 min  C 2 h 30 min  D 3 h
Sample Test

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

For this test you may assume that the value of the denominator of a rational expression is not zero.

1. Which function has a graph with x-intercepts $\frac{-1}{3}$ and $\frac{4}{5}$?
   A. $f(x) = 15x^2 + 7x - 4$
   B. $f(x) = 15x^2 - 7x - 4$
   C. $f(x) = 4x^2 - 17x + 15$
   D. $f(x) = 12x^2 - 11x - 5$

2. The relationship between the times $a$ and $b$ it will take each of two people to complete a job working alone and the time $t$ it will take them to complete the job working together can be described by the equation $\frac{1}{t} = \frac{1}{a} + \frac{1}{b}$. Amir and Hector are students at Shenandoah University in Winchester. They are planning to paint the apartment that they share. If Amir could paint the apartment in 18 hours working alone and Hector could do the job in 15 hours working alone, about how long will it take them to paint the apartment if they work together?
   F. 3 h
   G. 7 h 15 min
   H. 8 h 11 min
   J. 16 h 30 min

3. Which is a factor of $27s^3 + 64t^3$?
   A. $3s - 4t$
   B. $9s^2 + 16t^2$
   C. $9s^2 - 12st + 16t^2$
   D. $9s^2 - 24st + 16t^2$

4. If $b$ is a real number greater than 1, which equation could be represented by this graph?
   F. $y = \log_b x$
   G. $y = \log_{\frac{1}{b}} x$
   H. $y = b^x$
   J. $y = \left(\frac{1}{b}\right)^x$

5. The volume $V$ of a gas varies directly as its temperature $T$ and inversely as its pressure $P$. Which equation describes this relationship?
   A. $V = kTP$
   B. $V = \frac{kP}{T}$
   C. $V = \frac{k}{TP}$
   D. $V = \frac{kT}{P}$
6 Which is a solution of $3x^2 - 9x + 5 = 0$?

F $\frac{9 - \sqrt{21}}{6}$

G $\frac{9 + \sqrt{141}}{6}$

H $\frac{3}{2} + \sqrt{21}$

J $\frac{9 - i\sqrt{51}}{6}$

7 What is the simplest form of $\frac{\frac{2}{x+2} - \frac{2}{x-2}}{x^2 - 4}$?

A $-2$

B $2$

C $\frac{x}{2}$

D $x$

8 Which equation most closely fits the data in this scatterplot?

F $y = 2x - 40$

G $y = (x - 25)^2$

H $y = 0.5(x - 25)^2 + 20$

J $y = 0.1(x - 25)^2 + 20$

9 Which function includes the values in the table?

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-4</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>-4</td>
</tr>
</tbody>
</table>

A $y = (x + 2)^2$

B $y = -(x + 2)^2$

C $y = -(x - 2)^2$

D $y = -|x + 2|$

10 Which could be the graph of $\frac{(x - 2)^2}{4} + \frac{y^2}{9} = 1$?
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

11 A store sells “I Love Virginia” T-shirts in four sizes: small (S), medium (M), large (L), and extra large (XL). Matrix $V$ shows the number of each size sold at the store during the three summer months of 2003, while matrix $P$ gives the store’s profit on a T-shirt of each size.

$$V = \begin{bmatrix}
\text{June} & \text{S} & \text{M} & \text{L} & \text{XL} \\
27 & 39 & 85 & 111 \\
\text{July} & 32 & 45 & 98 & 122 \\
\text{August} & 50 & 43 & 135 & 108 \\
\end{bmatrix}, \quad P = \begin{bmatrix}
\text{Profit} \\
\text{S} & \text{M} & \text{L} & \text{XL} \\
$2.00 & $2.25 & $3.00 & $3.50 \\
\end{bmatrix}$$

What was the store’s profit from selling these T-shirts just for July?

A $2,651.50  
B $979.75  
C $886.25  
D $785.25

12 The chart gives the population of a city at the beginning of four 5-year intervals. Assuming an exponential relationship, which is the best estimate of the population of this city in 1980?

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>45,250</td>
<td>41,600</td>
<td>38,300</td>
<td>35,400</td>
</tr>
</tbody>
</table>

F 49,100  
G 48,600  
H 40,200  
J 32,600

13 What property is illustrated by $75(34 + 56) = (34 + 56)75$?

A Associative Property of Multiplication  
B Commutative Property of Addition  
C Commutative Property of Multiplication  
D Distributive Property

14 Which describes the graph of $x = -(y + 3)^2 + 4$?

F circle  
G ellipse  
H hyperbola  
J parabola

15 What are the solutions of $|4x - 5| = 7$?

A $\frac{1}{2}$ and $-3$  
B 2 and $-3$  
C $-\frac{1}{2}$ and 3  
D 2 and 3

Go on
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

16 Visitors to Colonial Williamsburg may choose between several ticket plans. Two of these are the Colonial Sampler Ticket and the Freedom Pass. Both plans have one price for adults and another price for youths, ages 6–17. One group of teenagers, all under age 18, paid a total of $395 for 12 Colonial Sampler Tickets and 7 Freedom Passes. Another group of teenagers, also all under 18, paid a total of $418 for 8 Colonial Sampler Tickets and 10 Freedom Passes. What is the cost of a youth Freedom Pass?

F $29.00  
G $17.50  
H $16.00  
J $9.50

17 The children in Ms. Steinberg’s kindergarten class are building block walls that follow the pattern shown in the figure. If they want to make their wall 12 levels high, how many blocks will they need?

A 156 blocks  
B 91 blocks  
C 78 blocks  
D 66 blocks

18 The figure shows the graph of a system of equations. What is the solution set for this system?

F \( \left\{-2, -\frac{1}{2}, 2\right\} \)  
G \( \{(−3, 5), (1, −3)\} \)  
H \( \{-3, 1\} \)  
J \( \{(−3, 5), (0, −1), (1, −3)\} \)

19 Which is equivalent to \( \sqrt{9} \cdot \sqrt{36} \cdot i^3 \)?

A \(-18i\)  
B \(18i\)  
C \(-18\)  
D \(18\)

20 Which is the solution set for \( \sqrt{4 – 2x} = x + 1 \)?

F \( \{-2 \pm \sqrt{7}\} \)  
G \( \{-2 + \sqrt{7}\} \)  
H \( \{-3, 1\} \)  
J \( \{2 + \sqrt{7}\} \)
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

21 If \( f(x) = -0.5x(2x + 7)(x - 4) \), what is the solution set of \( f(x) = 0? \)
   \[ f(x) = 0 \]
   \[ \begin{align*}
   21 & \quad \text{__________} \\
   \text{A} & \quad \left\{ \frac{-7}{2}, 4 \right\} \\
   \text{B} & \quad \left\{ \frac{-7}{2}, 0, 4 \right\} \\
   \text{C} & \quad \left\{ \frac{-2}{7}, 4 \right\} \\
   \text{D} & \quad \left\{ -4, 0, \frac{7}{2} \right\}
   \end{align*} \]

22 Which equation has solution set \( \{2 \pm 3i\} ? \)
   \[ \begin{align*}
   22 & \quad \text{__________} \\
   \text{F} & \quad z^2 - 4z - 5 = 0 \\
   \text{G} & \quad z^2 - 4z + 13 = 0 \\
   \text{H} & \quad z^2 - 2z + 7 = 0 \\
   \text{J} & \quad z^2 + 4z + 13 = 0
   \end{align*} \]

23 Which is the graph of a logarithmic function?
   \[ \begin{align*}
   23 & \quad \text{__________} \\
   \text{A} & \quad \text{Graph A} \\
   \text{B} & \quad \text{Graph B} \\
   \text{C} & \quad \text{Graph C} \\
   \text{D} & \quad \text{Graph D}
   \end{align*} \]

24 Which matrix equation is equivalent to the system of equations below?
   \[ \begin{align*}
   24 & \quad \text{__________} \\
   \text{F} & \quad \begin{bmatrix} 5 & 3 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 45 \\ 0 \end{bmatrix} \\
   \text{G} & \quad \begin{bmatrix} 5 & 6 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 45 \\ 0 \end{bmatrix} \\
   \text{H} & \quad \begin{bmatrix} 5 & 6 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 45 \\ 0 \end{bmatrix} \\
   \text{J} & \quad \begin{bmatrix} 5 & 3 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 45 \\ 0 \end{bmatrix}
   \end{align*} \]

25 Which statement about matrices \( M \) and \( N \) is true?
   \[ \begin{align*}
   25 & \quad \text{__________} \\
   \text{M} & \quad \begin{bmatrix} 0.2 & 0.3 \\ 1.4 & 2.5 \\ 0.8 & 1.9 \end{bmatrix} \text{ and } N = \begin{bmatrix} 20 & 40 & 60 \end{bmatrix} \\
   \text{A} & \quad \text{The product } N \times M \text{ is a } 1 \times 2 \text{ matrix.} \\
   \text{B} & \quad \text{The product } M \times N \text{ is a } 3 \times 3 \text{ matrix.} \\
   \text{C} & \quad \text{The product } M \times M \text{ is a } 3 \times 2 \text{ matrix.} \\
   \text{D} & \quad \text{The product } N \times N \text{ is a } 3 \times 3 \text{ matrix.}
   \end{align*} \]
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

26 If an object is thrown upward from a height of \( h_0 \) feet at an initial velocity of \( v_0 \) feet per second, its height \( h \) (in feet) after \( t \) seconds is given by the equation \( h = -16t^2 + v_0t + h_0 \). The Federal Reserve Bank Building in Richmond is 393 feet tall. If a ball is thrown upward from the top of this building at an initial velocity of 50 feet per second, about how long will it take the ball to hit the ground?

F 6.8 s  G 5.6 s
H 5.0 s  J 3.6 s

27 Which equation describes a circle with center at \((-5, 3)\) and radius 6?

A \((x + 5)^2 - (y - 3)^2 = 36\)
B \((x + 5)^2 + (y - 3)^2 = 36\)
C \((x - 5)^2 - (y - 3)^2 = 36\)
D \((x + 5)^2 + (y - 3)^2 = 6\)

28 Let \( A \) and \( B \) be \(2 \times 2\) matrices such that \( A \neq B\), let \( I \) be the \(2 \times 2\) multiplicative identity matrix, and let \( O \) be the \(2 \times 2\) zero matrix. Which statement is not always true?

F \(AI = IA\)  G \(A + O = O + A\)
H \(AB = BA\)  J \(A + B = B + A\)

29 Which are the zeros of \(f(x) = 3x^3 - x^2 - 2x\)?

A \(-\frac{3}{2}, \ 0, \text{ and } 1\)
B \(-1 \text{ and } \frac{2}{3}\)
C \(-\frac{2}{3}, \ 0, \text{ and } 1\)
D \(-\frac{2}{3} \text{ and } 1\)

30 Kitchen canisters are cylindrical and come in a variety of sizes. For a given volume, the height of a cylinder varies inversely as the square of the radius. Tricia has two canisters in her kitchen that have the same volume. The one she uses to store spaghetti is 12 inches tall and has a radius of \(2\frac{1}{4}\) inches. The one she uses to store flour has a radius of 3 inches. How tall is the flour canister?

F 9 in.  G \(6\frac{3}{4}\) in.
H \(5\frac{1}{3}\) in.  J 3 in.
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

31 The VBU Company makes beach chairs and beach umbrellas. The company’s profit is $5 on each chair and $8 on each umbrella. To use linear programming to maximize profit, the company’s financial officer developed this feasible region from the constraints on the company’s equipment and the number of hours of employee time available. The number of chairs to be made each day is represented by $x$. The number of umbrellas to be made each day is represented by $y$. How many chairs and how many umbrellas should the company make each day in order to maximize the profit?

A 10 chairs, 20 umbrellas 
B 20 chairs, 20 umbrellas 
C 10 chairs, 0 umbrellas 
D 40 chairs, 0 umbrellas

32 Which graph shows the solution set for $|3 - 2x| \geq 5$?

F 
G 
H 
J

33 Which could be an equation for this graph?

A $\frac{x^2}{9} - \frac{y^2}{4} = 1$ 
B $\frac{x^2}{9} + \frac{y^2}{4} = 1$ 
C $\frac{y^2}{9} - \frac{x^2}{4} = 1$ 
D $\frac{y^2}{4} - \frac{x^2}{9} = 1$

34 Which equation has exactly one real solution?

F $9x^2 - 30x + 25 = 0$ 
G $9x^2 + 25 = 0$ 
H $9x^2 - 25 = 0$ 
J $x^2 - 5x + 6 = 0$
Sample Test (continued)

Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

35 Which expression is equal to \( \frac{4r}{9x} + \frac{5s}{3x^2} - \frac{7t}{12x^3} \)?

A \( \frac{-140rst}{36x^3} \)
B \( \frac{4r + 5s - 7t}{36x^3} \)
C \( 36rx + 15sx^2 - 84tx^3 \)
D \( \frac{16rx^2 + 60sx - 21t}{36x^3} \)

36 What are the vertices of the feasible region determined by the constraints \( 2x + y \leq 80, 10 \leq x \leq 30, y \geq 0 \)?

F (0, 0), (0, 80), (40, 0)
G (10, 0), (10, 40), (30, 40), (30, 0)
H (10, 0), (10, 60), (30, 20), (30, 0)
J (10, 0), (10, 60), (40, 0)

37 Which is the solution set for the system given below?
\[
\begin{cases} 
2x^2 - 4y^2 = 8 \\
x^2 + 2y^2 = 8 
\end{cases}
\]

A \( \left\{ (-i, -\sqrt{6}), (-i, \sqrt{6}), (i, -\sqrt{6}), (i, \sqrt{6}) \right\} \)
B \( \left\{ (-\sqrt{6}, -i), (-\sqrt{6}, i), (\sqrt{6}, -i), (\sqrt{6}, i) \right\} \)
C \( \left\{ (-1, -\sqrt{6}), (-1, \sqrt{6}), (1, -\sqrt{6}), (1, \sqrt{6}) \right\} \)
D \( \left\{ (-\sqrt{6}, -1), (-\sqrt{6}, 1), (\sqrt{6}, -1), (\sqrt{6}, 1) \right\} \)

38 Which function is represented by the graph?
F \( y = 2|x - 2| - 3 \)
G \( y = 2(x + 2)^2 - 3 \)
H \( y = 2|x + 2| - 3 \)
J \( y = |x - 2| - 3 \)

39 Which is equivalent to \( \frac{7}{\sqrt{11} - 2} \)?

A \( \sqrt{11} + 2 \)
B \( \frac{7\sqrt{2}}{\sqrt{22} - 2} \)
C \( \frac{7(\sqrt{11} + 2)}{9} \)
D \( \sqrt{11} - 2 \)

40 What is the value of \( \sum_{n=1}^{5} \frac{n}{n + 1} \)?

F \( \frac{71}{20} \)
G \( \frac{163}{60} \)
H \( \frac{29}{20} \)
J \( \frac{3}{4} \)

Go on
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

41 Which has the set of all real numbers as its solution set?
A \( |4x + 3| = 0 \)  
B \( |4x + 3| < 0 \)  
C \( |4x + 3| > 0 \)  
D \( |4x + 3| \geq 0 \)  

42 This is the graph of a polynomial function \( g \).
Which is not a solution of \( g(x) = 0 \)?
F \(-2\)  
G \(0\)  
H \(2\)  
J \(4\)  

43 If \( g(x) = x^3 + 3 \) and \( h(x) = -(x + 2)^2 \), which is the value of \( h[g(-2)] \)?
A \(-169\)  
B \(-9\)  
C \(3\)  
D \(9\)  

44 Which is the completely factored form of \( 2x^2 + 4x - 70 \)?
F \((2x - 10)(x + 7)\)  
G \((2x + 14)(x - 5)\)  
H \(2(x + 7)(x - 5)\)  
J \(2(x - 7)(x + 5)\)  

45 Which type of function would best fit the data in this scatterplot?
A exponential  
B linear  
C logarithmic  
D quadratic  

46 How many real solutions does \( \frac{2}{x + 1} + \frac{3}{x - 1} = 5 \) have?
F \(0\)  
G \(1\)  
H \(2\)  
J \(3\)
Read each question and choose the best answer. Then write the letter for the answer you have chosen in the blank at the right of each question.

47 Which is another way to write $\sqrt[3]{a^3b^4c^6}$?

A $(abc)^{\frac{13}{6}}$  
B $a^{\frac{2}{3}}b^{\frac{1}{2}}c$  
C $a^2b^2c$  
D $a^3b^2c^0$

48 Which is equivalent to $(4 - i)(6 + 2i)$?

F $26 + 2i$  
G $22 + 2i$  
H $26$  
J $22$

49 Brett likes to purchase concert tickets online for himself and his friends. The ticket service he uses charges a service fee that is a fixed percentage of the price of the tickets purchased. The chart shows the purchase prices and service fees for Brett’s last three transactions. If the service fee on Brett’s next purchase is $10.14, what is the purchase price for this transaction?

<table>
<thead>
<tr>
<th>Purchase Price</th>
<th>$48</th>
<th>$124</th>
<th>$86</th>
</tr>
</thead>
<tbody>
<tr>
<td>Service Fee</td>
<td>$3.12</td>
<td>$8.06</td>
<td>$5.59</td>
</tr>
</tbody>
</table>

A $169$  
B $156$  
C $150$  
D $145$

50 The table lists the seven Presidents of the United States who are buried in Virginia, in the order in which they served, and their ages at death. If this information is considered as a function made up of ordered pairs (President, Age at Death), how should the range of the function be written?

<table>
<thead>
<tr>
<th>President</th>
<th>Age at Death</th>
</tr>
</thead>
<tbody>
<tr>
<td>George Washington</td>
<td>67</td>
</tr>
<tr>
<td>Thomas Jefferson</td>
<td>83</td>
</tr>
<tr>
<td>James Madison</td>
<td>85</td>
</tr>
<tr>
<td>James Monroe</td>
<td>73</td>
</tr>
<tr>
<td>John Tyler</td>
<td>71</td>
</tr>
<tr>
<td>William Howard Taft</td>
<td>72</td>
</tr>
<tr>
<td>John Kennedy</td>
<td>46</td>
</tr>
</tbody>
</table>

F 7  
G 39  
H \{46, 67, 71, 72, 73, 83, 85\}  
J \{x\mid 46 \leq x \leq 85\}
1-1  

**Expressions and Formulas**

Find the value of each expression.

1. $3(4 - 7) - 11$
2. $4(12 - 4^2)$
3. $1 + 2 - 3(4) + 2$
4. $12 - [20 - 2(6^2 + 3 	imes 2^2)]$
5. $20 \div (5 - 3) + 5^2(3)$
6. $(-2)^3 - (3)(8) + (5)(10)$
7. $18 - [5 - [34 - (17 - 11)]]$
8. $[4(5 - 3) - 2(4 - 8)] \div 16$
9. $\frac{1}{2}[6 - 4^2]$
10. $\frac{1}{4}[-5 + 5(-3)]$
11. $\frac{-8(13 - 37)}{6}$
12. $\frac{(-8)^2}{5 - 9} - (-1)^2 + 4(-9)$

Evaluate each expression if $a = \frac{3}{4}$, $b = -8$, $c = -2$, $d = 3$, and $e = \frac{1}{3}$.

13. $ab^2 - d$
14. $(c + d)b$
15. $\frac{ab}{c} + d^2$
16. $\frac{d(b - c)}{ac}$
17. $(b - de)e^2$
18. $ac^3 - b^2de$
19. $-b[a + (c - d)^2]$
20. $\frac{ac^4}{d} - \frac{c}{e^2}$
21. $9bc - \frac{1}{e}$
22. $2ab^2 - (d^3 - c)$

23. **TEMPERATURE** The formula $F = \frac{9}{5}C + 32$ gives the temperature in degrees Fahrenheit for a given temperature in degrees Celsius. What is the temperature in degrees Fahrenheit when the temperature is $-40$ degrees Celsius?

24. **PHYSICS** The formula $h = 120t - 16t^2$ gives the height $h$ in feet of an object $t$ seconds after it is shot upward from Earth’s surface with an initial velocity of 120 feet per second. What will the height of the object be after 6 seconds?

25. **AGRICULTURE** Faith owns an organic apple orchard. From her experience the last few seasons, she has developed the formula $P = 20x - 0.01x^2 - 240$ to predict her profit $P$ in dollars this season if her trees produce $x$ bushels of apples. What is Faith’s predicted profit this season if her orchard produces 300 bushels of apples?
1. \(6425\)
2. \(\sqrt{7}\)
3. \(2\pi\)
4. 0
5. \(\sqrt{\frac{25}{36}}\)
6. \(-\sqrt{16}\)
7. -35
8. -31.8

Name the property illustrated by each equation.
9. \(5x \cdot (4y + 3x) = 5x \cdot (3x + 4y)\)
10. \(7x + (9x + 8) = (7x + 9x) + 8\)
11. \(5(3x + y) = 5(3x + 1y)\)
12. \(7n + 2n = (7 + 2)n\)
13. \(3(2x)y = (3 \cdot 2)(xy)\)
14. \(3x \cdot 2y = 3 \cdot 2 \cdot x \cdot y\)
15. \((6 + -6)y = 0y\)
16. \(\frac{1}{4} \cdot 4y = 1y\)
17. \(5(x + y) = 5x + 5y\)
18. \(4n + 0 = 4n\)

Name the additive inverse and multiplicative inverse for each number.
19. 0.4
20. -1.6
21. \(-\frac{11}{16}\)
22. \(5\frac{5}{6}\)

Simplify each expression.
23. \(5x - 3y - 2x + 3y\)
24. \(-11a - 13b + 7a - 3b\)
25. \(8x - 7y - (3 - 6y)\)
26. \(4c - 2c - (4c + 2c)\)
27. \(3(r - 10s) - 4(7s + 2r)\)
28. \(\frac{1}{5}(10a - 15) + \frac{1}{2}(8 + 4a)\)
29. \(2(4 - 2x + y) - 4(5 + x - y)\)
30. \(\frac{5}{6}\left(\frac{3}{2}x + 12y\right) - \frac{1}{4}(2x - 12y)\)

31. **TRAVEL** Olivia drives her car at 60 miles per hour for \(t\) hours. Ian drives his car at 50 miles per hour for \((t + 2)\) hours. Write a simplified expression for the sum of the distances traveled by the two cars.

32. **NUMBER THEORY** Use the properties of real numbers to tell whether the following statement is true or false: If \(a > b\), it follows that \(a\left(\frac{1}{a}\right) > b\left(\frac{1}{b}\right)\). Explain your reasoning.
1-3 Practice
Solving Equations

Write an algebraic expression to represent each verbal expression.
1. 2 more than the quotient of a number and 5  2. the sum of two consecutive integers

3. 5 times the sum of a number and 1  4. 1 less than twice the square of a number

Write a verbal expression to represent each equation.
5. 5 – 2x = 4  6. 3y = 4y^3

7. 3c = 2(c – 1)  8. \( \frac{m}{5} = 3(2m + 1) \)

Name the property illustrated by each statement.
9. If \( t – 13 = 52 \), then \( 52 = t – 13 \).  10. If \( 8(2q + 1) = 4 \), then \( 2(2q + 1) = 1 \).
11. If \( h + 12 = 22 \), then \( h = 10 \).  12. If \( 4m = -15 \), then \( -12m = 45 \).

Solve each equation. Check your solution.
13. 14 = 8 – 6r  14. 9 + 4n = -59
15. \( \frac{3}{4} - \frac{1}{2}n = \frac{5}{8} \)  16. \( \frac{5}{6}s + \frac{3}{4} = \frac{11}{12} \)
17. \( -1.6r + 5 = -7.8 \)  18. \( 6x - 5 = 7 - 9x \)
19. \( 5(6 - 4v) = v + 21 \)  20. \( 6y - 5 = -3(2y + 1) \)

Solve each equation or formula for the specified variable.
21. \( E = mc^2 \), for \( m \)  22. \( c = \frac{2d + 1}{3} \), for \( d \)
23. \( h = vt - gt^2 \), for \( v \)  24. \( E = \frac{1}{2}lw^2 + U \), for \( I \)

Define a variable, write an equation, and solve the problem.
25. GEOMETRY The length of a rectangle is twice the width. Find the width if the perimeter is 60 centimeters.
26. GOLF Luis and three friends went golfing. Two of the friends rented clubs for $6 each. The total cost of the rented clubs and the green fees for each person was $76. What was the cost of the green fees for each person?
Evaluate each expression if $a = -1$, $b = -8$, $c = 5$, and $d = -1.4$.

1. $|6a|$  
2. $|2b + 4|$  
3. $-|10d + a|$  
4. $|17c| + |3b - 5|$  
5. $-6|10a - 12|$  
6. $|2b - 1| - |-8b + 5|$  
7. $|5a - 7| + |3c - 4|$  
8. $|1 - 7c| - |a|$  
9. $-3|0.5c + 2| - |-0.5b|$  
10. $|4d| + |5 - 2a|$  
11. $|a - b| + |b - a|$  
12. $|2 - 2d| - 3|b|$  

Solve each equation. Check your solutions.

13. $|n - 4| = 13$  
14. $|x - 13| = 2$  
15. $|2y - 3| = 29$  
16. $7|x + 3| = 42$  
17. $|3u - 6| = 42$  
18. $|5x - 4| = -6$  
19. $-3|4x - 9| = 24$  
20. $-6|5 - 2y| = -9$  
21. $|8 + p| = 2p - 3$  
22. $|4w - 1| = 5w + 37$  
23. $4|2y - 7| + 5 = 9$  
24. $-2|7 - 3y| - 6 = -14$  
25. $2|4 - s| = -3s$  
26. $5 - 3|2 + 2w| = -7$  
27. $5|2r + 3| - 5 = 0$  
28. $3 - 5|2d - 3| = 4$  

29. **WEATHER** A thermometer comes with a guarantee that the stated temperature differs from the actual temperature by no more than 1.5 degrees Fahrenheit. Write and solve an equation to find the minimum and maximum actual temperatures when the thermometer states that the temperature is 87.4 degrees Fahrenheit.

30. **OPINION POLLS** Public opinion polls reported in newspapers are usually given with a margin of error. For example, a poll with a margin of error of $\pm 5\%$ is considered accurate to within plus or minus $5\%$ of the actual value. A poll with a stated margin of error of $\pm 3\%$ predicts that candidate Tonwe will receive 51% of an upcoming vote. Write and solve an equation describing the minimum and maximum percent of the vote that candidate Tonwe is expected to receive.
Practice

Solving Inequalities

Solve each inequality. Describe the solution set using set-builder or interval notation. Then, graph the solution set on a number line.

1. \(8x - 6 \geq 10\)

2. \(23 - 4u < 11\)

3. \(-16 - 8r \geq 0\)

4. \(14s < 9s + 5\)

5. \(9x - 11 > 6x - 9\)

6. \(-3(4w - 1) > 18\)

7. \(1 - 8u \leq 3u - 10\)

8. \(17.5 < 19 - 2.5x\)

9. \(9(2r - 5) - 3 < 7r - 4\)

10. \(1 + 5(x - 8) \leq 2 - (x + 5)\)

11. \(\frac{4x - 3}{2} \geq -3.5\)

12. \(q - 2(2 - q) \leq 0\)

13. \(-36 - 2(w + 77) > -4(2w + 52)\)

14. \(4n - 5(n - 3) > 3(n + 1) - 4\)

Define a variable and write an inequality for each problem. Then solve.

15. Twenty less than a number is more than twice the same number.

16. Four times the sum of twice a number and \(-3\) is less than 5.5 times that same number.

17. **HOTELS** The Lincoln’s hotel room costs $90 a night. An additional 10% tax is added. Hotel parking is $12 per day. The Lincoln’s expect to spend $30 in tips during their stay. Solve the inequality \(90x + 90(0.1)x + 12x + 30 \leq 600\) to find how many nights the Lincoln’s can stay at the hotel without exceeding total hotel costs of $600.

18. **BANKING** Jan’s account balance is $3800. Of this, $750 is for rent. Jan wants to keep a balance of at least $500. Write and solve an inequality describing how much she can withdraw and still meet these conditions.
1. all numbers greater than 4 or less than −4

2. all numbers between −1.5 and 1.5, including −1.5 and 1.5

Write an absolute value inequality for each graph.

3. \( |n| \leq 10 \)

4. \( |n| < 1.5 \)

Solve each inequality. Graph the solution set on a number line.

5. \(-8 \leq 3y - 20 < 52\)

6. \(3(5x - 2) < 24 \) or \(6x - 4 > 4 + 5x\)

7. \(2x - 3 > 15 \) or \(3 - 7x < 17\)

8. \(15 - 5x \leq 0 \) and \(5x + 6 \geq -14\)

9. \(|2w| \geq 5\)

10. \(|y + 5| < 2\)

11. \(|x - 8| \geq 3\)

12. \(|2z - 2| \leq 3\)

13. \(|2x + 2| - 7 \leq -5\)

14. \(|x| > x - 1\)

15. \(|3b + 5| \leq -2\)

16. \(|3n - 2| - 2 < 1\)

17. **RAINFALL** In 90% of the last 30 years, the rainfall at Shell Beach has varied no more than 6.5 inches from its mean value of 24 inches. Write and solve an absolute value inequality to describe the rainfall in the other 10% of the last 30 years.

18. **MANUFACTURING** A company’s guidelines call for each can of soup produced not to vary from its stated volume of 14.5 fluid ounces by more than 0.08 ounces. Write and solve an absolute value inequality to describe acceptable can volumes.
Practice

Relations and Functions

Determine whether each relation is a function. Write yes or no.

1. No

2. Yes

3. Yes

4. No

Graph each relation or equation and find the domain and range. Then determine whether the relation or equation is a function.

5. D: {(-4, -1), (4, 0), (0, 3), (2, 0)}
   R: all reals, R; yes

6. y = 2x - 1

Find each value if \( f(x) = \frac{5}{x + 2} \) and \( g(x) = -2x + 3 \).

7. \( f(3) \)

8. \( f(-4) \)

9. \( g\left(\frac{1}{2}\right) \)

10. \( f(-2) \)

11. \( g(-6) \)

12. \( f(m - 2) \)

13. MUSIC The ordered pairs (1, 16), (2, 16), (3, 32), (4, 32), and (5, 48) represent the cost of buying various numbers of CDs through a music club. Identify the domain and range of the relation. Is the relation a function?

14. COMPUTING If a computer can do one calculation in 0.0000000015 second, then the function \( T(n) = 0.0000000015n \) gives the time required for the computer to do \( n \) calculations. How long would it take the computer to do 5 billion calculations?
2-2 Practice

Linear Equations

State whether each equation or function is linear. Write yes or no. If no, explain your reasoning.

1. \(h(x) = 23\)  
2. \(y = \frac{2}{3}x\)  
3. \(y = \frac{5}{x}\)  
4. \(9 - 5xy = 2\)

Write each equation in standard form. Identify \(A\), \(B\), and \(C\).

5. \(y = 7x - 5\)  
6. \(y = \frac{3}{8}x + 5\)  
7. \(3y - 5 = 0\)  
8. \(x = -\frac{2}{7}y + \frac{3}{4}\)

Find the \(x\)-intercept and the \(y\)-intercept of the graph of each equation. Then graph the equation.

9. \(y = 2x + 4\)  
10. \(2x + 7y = 14\)

11. \(y = -2x - 4\)  
12. \(6x + 2y = 6\)

13. MEASURE The equation \(y = 2.54x\) gives the length in centimeters corresponding to a length \(x\) in inches. What is the length in centimeters of a 1-foot ruler?

14. What is the total cost of talking 8 hours? of talking 20 hours?

15. What is the effective cost per minute (the total cost divided by the number of minutes talked) of talking 8 hours? of talking 20 hours?
2-3 Practice

Slope

Find the slope of the line that passes through each pair of points.

1. \((3, -8), (-5, 2)\) 
2. \((-10, -3), (7, 2)\) 
3. \((-7, -6), (3, -6)\)

4. \((8, 2), (8, -1)\) 
5. \((4, 3), (7, -2)\) 
6. \((-6, -3), (-8, 4)\)

Graph the line passing through the given point with the given slope.

7. \((0, -3), m = 3\) 
8. \((2, 1), m = -\frac{3}{4}\)

9. \((0, 2), m = 0\) 
10. \((2, -3), m = \frac{4}{5}\)

Graph the line that satisfies each set of conditions.

11. passes through \((3, 0)\), perpendicular to a line whose slope is \(\frac{3}{2}\) 
12. passes through \((-3, -1)\), parallel to a line whose slope is \(-1\)

DEPRECIATION For Exercises 13–15, use the following information.

A machine that originally cost $15,600 has a value of $7500 at the end of 3 years. The same machine has a value of $2800 at the end of 8 years.

13. Find the average rate of change in value (depreciation) of the machine between its purchase and the end of 3 years.

14. Find the average rate of change in value of the machine between the end of 3 years and the end of 8 years.

15. Interpret the sign of your answers.
State the slope and y-intercept of the graph of each equation.

1. \( y = 8x + 12 \)  
2. \( y = 0.25x - 1 \)  
3. \( y = -\frac{3}{5}x \)

4. \( 3y = 7 \)  
5. \( 3x = -15 + 5y \)  
6. \( 2x - 3y = 10 \)

Write an equation in slope-intercept form for each graph.

7. 

8. 

9. 

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

10. slope \(-5\), passes through \((-3, -8)\)  
11. slope \(\frac{4}{5}\), passes through \((10, -3)\)

12. slope \(0\), passes through \((0, -10)\)  
13. slope \(-\frac{2}{3}\), passes through \((6, -8)\)

14. passes through \((3, 11)\) and \((-6, 5)\)  
15. passes through \((7, -2)\) and \((3, -1)\)

16. \(x\)-intercept 3, \(y\)-intercept 2  
17. \(x\)-intercept \(-5\), \(y\)-intercept 7

18. passes through \((-8, -7)\), perpendicular to the graph of \(y = 4x - 3\)

19. RESERVOIRS  The surface of Grand Lake is at an elevation of 648 feet. During the current drought, the water level is dropping at a rate of 3 inches per day. If this trend continues, write an equation that gives the elevation in feet of the surface of Grand Lake after \(x\) days.

20. BUSINESS  Tony Marconi’s company manufactures CD-ROM drives. The company will make $150,000 profit if it manufactures 100,000 drives, and $1,750,000 profit if it manufactures 500,000 drives. The relationship between the number of drives manufactured and the profit is linear. Write an equation that gives the profit \(P\) when \(n\) drives are manufactured.
Practice

Modeling Real-World Data: Using Scatter Plots

For Exercises 1–3, complete parts a–c for each set of data.

a. Draw a scatter plot.
b. Use two ordered pairs to write a prediction equation.
c. Use your prediction equation to predict the missing value.

1. FUEL ECONOMY The table gives the approximate weights in tons and estimates for overall fuel economy in miles per gallon for several cars.

<table>
<thead>
<tr>
<th>Weight (tons)</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
<th>1.8</th>
<th>2</th>
<th>2.1</th>
<th>2.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles per Gallon</td>
<td>29</td>
<td>24</td>
<td>23</td>
<td>21</td>
<td>?</td>
<td>17</td>
<td>15</td>
</tr>
</tbody>
</table>

2. ALTITUDE In most cases, temperature decreases with increasing altitude. As Anchara drives into the mountains, her car thermometer registers the temperatures (°F) shown in the table at the given altitudes (feet).

<table>
<thead>
<tr>
<th>Altitude (ft)</th>
<th>7500</th>
<th>8200</th>
<th>8600</th>
<th>9200</th>
<th>9700</th>
<th>10,400</th>
<th>12,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°F)</td>
<td>61</td>
<td>58</td>
<td>56</td>
<td>53</td>
<td>50</td>
<td>46</td>
<td>?</td>
</tr>
</tbody>
</table>

3. HEALTH Alton has a treadmill that uses the time on the treadmill and the speed of walking or running to estimate the number of Calories he burns during a workout. The table gives workout times and Calories burned for several workouts.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>18</th>
<th>24</th>
<th>30</th>
<th>40</th>
<th>42</th>
<th>48</th>
<th>52</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calories Burned</td>
<td>260</td>
<td>280</td>
<td>320</td>
<td>380</td>
<td>400</td>
<td>440</td>
<td>475</td>
<td>?</td>
</tr>
</tbody>
</table>
Graph each function. Identify the domain and range.

1. \( f(x) = \lfloor 0.5x \rfloor \)

2. \( f(x) = \lfloor x \rfloor - 2 \)

3. \( g(x) = -2|x| \)

4. \( f(x) = |x + 1| \)

5. \( f(x) = \begin{cases} x + 2 & \text{if } x \leq -2 \\ 3x & \text{if } x > -2 \end{cases} \)

6. \( h(x) = \begin{cases} 4 - x & \text{if } x > 0 \\ -2x - 2 & \text{if } x < 0 \end{cases} \)

7. **BUSINESS** A Stitch in Time charges $40 per hour or any fraction thereof for labor. Draw a graph of the step function that represents this situation.

8. **BUSINESS** A wholesaler charges a store $3.00 per pound for less than 20 pounds of candy and $2.50 per pound for 20 or more pounds. Draw a graph of the function that represents this situation.
Graph each inequality.

1. \( y \leq -3 \)  
2. \( x > 2 \)  
3. \( x + y \leq -4 \)

4. \( y < -3x + 5 \)  
5. \( y < \frac{1}{2}x + 3 \)  
6. \( y - 1 \geq -x \)

7. \( x - 3y \leq 6 \)  
8. \( y > |x| - 1 \)  
9. \( y > -3|x + 1| - 2 \)

**COMPUTERS** For Exercises 10–12, use the following information.

A school system is buying new computers. They will buy desktop computers costing $1000 per unit, and notebook computers costing $1200 per unit. The total cost of the computers cannot exceed $80,000.

10. Write an inequality that describes this situation.

11. Graph the inequality.

12. If the school wants to buy 50 of the desktop computers and 25 of the notebook computers, will they have enough money?
3-1 Practice

Solving Systems of Equations By Graphing

Solve each system of equations by graphing.

1. \( x - 2y = 0 \)
   \[ y = 2x - 3 \]

2. \( x + 2y = 4 \)
   \[ 2x - 3y = 1 \]

3. \( 2x + y = 3 \)
   \[ y = \frac{1}{2}x - \frac{9}{2} \]

4. \( y - x = 3 \)
   \[ y = 1 \]

5. \( 2x - y = 6 \)
   \[ x + 2y = -2 \]

6. \( 5x - y = 4 \)
   \[ -2x + 6y = 4 \]

Graph each system of equations and describe it as consistent and independent, consistent and dependent, or inconsistent.

7. \( 2x - y = 4 \)
   \[ x - y = 2 \]

8. \( y = -x - 2 \)
   \[ x + y = -4 \]

9. \( 2y - 8 = x \)
   \[ y = \frac{1}{2}x + 4 \]

SOFTWARE For Exercises 10–12, use the following information.

Location Mapping needs new software. Software A costs $13,000 plus $500 per additional site license. Software B costs $2500 plus $1200 per additional site license.

10. Write two equations that represent the cost of each software.

11. Graph the equations. Estimate the break-even point of the software costs.

12. If Location Mapping plans to buy 10 additional site licenses, which software will cost less?

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3-2 Practice

Solving Systems of Equations Algebraically

Solve each system of equations by using substitution.

1. \(2x + y = 4\)  
   \(3x + 2y = 1\)

2. \(x - 3y = 9\)  
   \(x + 2y = -1\)

3. \(g + 3h = 8\)  
   \(\frac{1}{3}g + h = 9\)

4. \(2a - 4b = 6\)  
   \(-a + 2b = -3\)

5. \(2m + n = 6\)  
   \(5m + 6n = 1\)

6. \(4x - 3y = -6\)  
   \(-x - 2y = 7\)

7. \(u - 2v = \frac{1}{2}\)  
   \(-u + 2v = 5\)

8. \(x - 3y = 16\)  
   \(4x - y = 9\)

9. \(w + 3z = 1\)  
   \(3w - 5z = -4\)

Solve each system of equations by using elimination.

10. \(2r + s = 5\)  
    \(3r - s = 20\)

11. \(2m - n = -1\)  
    \(3m + 2n = 30\)

12. \(6x + 3y = 6\)  
    \(8x + 5y = 12\)

13. \(3j - k = 10\)  
    \(4j - k = 16\)

14. \(2x - y = -4\)  
    \(-4x + 2y = 6\)

15. \(2g + h = 6\)  
    \(3g - 2h = 16\)

16. \(2t + 4v = 6\)  
    \(-t - 2v = -3\)

17. \(3x - 2y = 12\)  
    \(2x + \frac{2}{3}y = 14\)

18. \(\frac{1}{2}x + 3y = 11\)  
    \(8x - 5y = 17\)

Solve each system of equations by using either substitution or elimination.

19. \(8x + 3y = -5\)  
    \(10x + 6y = -13\)

20. \(8q - 15r = -40\)  
    \(4q + 2r = 56\)

21. \(3x - 4y = 12\)  
    \(\frac{1}{3}x - \frac{4}{9}y = \frac{4}{3}\)

22. \(4b - 2d = 5\)  
    \(-2b + d = 1\)

23. \(s + 3y = 4\)  
    \(s = 1\)

24. \(4m - 2p = 0\)  
    \(-3m + 9p = 5\)

25. \(5g + 4k = 10\)  
    \(-3g - 5k = 7\)

26. \(0.5x + 2y = 5\)  
    \(x - 2y = -8\)

27. \(h - z = 3\)  
    \(-3h + 3z = 6\)

SPORTS For Exercises 28 and 29, use the following information.

Last year the volleyball team paid $5 per pair for socks and $17 per pair for shorts on a total purchase of $315. This year they spent $342 to buy the same number of pairs of socks and shorts because the socks now cost $6 a pair and the shorts cost $18.

28. Write a system of two equations that represents the number of pairs of socks and shorts bought each year.

29. How many pairs of socks and shorts did the team buy each year?
3-3 Practice

Solving Systems of Inequalities by Graphing

Solve each system of inequalities by graphing.

1. \( y + 1 < -x \)
   \( y \geq 1 \)
2. \( x > -2 \)
3. \( y \leq 2x - 3 \)
   \( 2y \geq 3x + 6 \)
   \( y \leq -\frac{1}{2}x + 2 \)
4. \( x + y > -2 \)
   \( 3x - y \geq -2 \)
5. \( |y| \leq 1 \)
   \( y < x - 1 \)
6. \( 3y > 4x \)
   \( 2x - 3y > -6 \)

Find the coordinates of the vertices of the figure formed by each system of inequalities.

7. \( y \geq 1 - x \)
   \( y \leq x - 1 \)
   \( x \leq 3 \)
8. \( x - y \leq 2 \)
   \( x + y \leq 2 \)
   \( x \geq -2 \)
9. \( y \geq 2x - 2 \)
   \( 2x + 3y \geq 6 \)
   \( y < 4 \)

DRAMA For Exercises 10 and 11, use the following information.

The drama club is selling tickets to its play. An adult ticket costs $15 and a student ticket costs $11. The auditorium will seat 300 ticket-holders. The drama club wants to collect at least $3630 from ticket sales.

10. Write and graph a system of four inequalities that describe how many of each type of ticket the club must sell to meets its goal.

11. List three different combinations of tickets sold that satisfy the inequalities.
Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

1. \(2x - 4 \leq y\)  
   \(-2x - 4 \leq y\)  
   \(y \leq 2\)  
   \(f(x, y) = -2x + y\)

2. \(3x - y \leq 7\)  
   \(2x - y \geq 3\)  
   \(y \geq x - 3\)  
   \(f(x, y) = x - 4y\)

3. \(x \geq 0\)  
   \(y \geq 0\)  
   \(y \leq 6\)  
   \(y \leq -3x + 15\)  
   \(f(x, y) = 3x + y\)

4. \(x \leq 0\)  
   \(y \leq 0\)  
   \(4x + y \geq -7\)  
   \(f(x, y) = -x - 4y\)

5. \(y \leq 3x + 6\)  
   \(4y + 3x \leq 3\)  
   \(x \geq -2\)  
   \(f(x, y) = -x + 3y\)

6. \(2x + 3y \geq 6\)  
   \(2x - y \leq 2\)  
   \(x \geq 0\)  
   \(y \geq 0\)  
   \(f(x, y) = x + 4y + 3\)

**PRODUCTION** For Exercises 7–9, use the following information.

A glass blower can form 8 simple vases or 2 elaborate vases in an hour. In a work shift of no more than 8 hours, the worker must form at least 40 vases.

7. Let \(s\) represent the hours forming simple vases and \(e\) the hours forming elaborate vases. Write a system of inequalities involving the time spent on each type of vase.

8. If the glass blower makes a profit of $30 per hour worked on the simple vases and $35 per hour worked on the elaborate vases, write a function for the total profit on the vases.

9. Find the number of hours the worker should spend on each type of vase to maximize profit. What is that profit?
3-5 Practice

Solving Systems of Equations in Three Variables

Solve each system of equations.

1. \(2x - y + 2z = 15\)
   \(-x + y + z = 3\)
   \(3x - y + 2z = 18\)
2. \(x - 4y + 3z = -27\)
   \(2x + 2y - 3z = 22\)
   \(4z = -16\)
3. \(a + b = 3\)
   \(-b + c = 3\)
   \(a + 2c = 10\)

4. \(3m - 2n + 4p = 15\)
   \(m - n + p = 3\)
   \(m + 4n - 5p = 0\)
5. \(2g + 3h - 8j = 10\)
   \(g - 4h = 1\)
   \(-2g - 3h + 8j = 5\)
6. \(2x + y - z = -8\)
   \(4x - y + 2z = -3\)
   \(-3x + y + 2z = 5\)

7. \(2x - 5y + z = 5\)
   \(3x + 2y - z = 17\)
   \(4x - 3y + 2z = 17\)
8. \(2x + 3y + 4z = 2\)
   \(5x - 2y + 3z = 0\)
   \(x - 5y - 2z = -4\)
9. \(p + 4r = -7\)
   \(p - 3q = -8\)
   \(q + r = 1\)

10. \(4x + 4y - 2z = 8\)
    \(3x - 5y + 3z = 0\)
    \(2x + 2y - z = 4\)
11. \(d + 3e + f = 0\)
    \(-d + 2e + f = -1\)
    \(4d + e - f = 1\)
12. \(4x + y + 5z = -9\)
    \(x - 4y - 2z = -2\)
    \(2x + 3y - 2z = 21\)

13. \(5x + 9y + z = 20\)
    \(2x - y - z = -21\)
    \(5x + 2y + 2z = -21\)
14. \(2x + y - 3z = -3\)
    \(3x + 2y + 4z = 5\)
    \(-6x - 3y + 9z = 9\)
15. \(3x + 3y + z = 10\)
    \(5x + 2y + 2z = 7\)
    \(3x - 2y + 3z = -9\)

16. \(2u + v + w = 2\)
    \(-3u + 2v + 3w = 7\)
    \(-u - v + 2w = 7\)
17. \(x + 5y - 3z = -18\)
    \(3x - 2y + 5z = 22\)
    \(-2x - 3y + 8z = 28\)
18. \(x - 2y + z = -1\)
    \(-x + 2y - z = 6\)
    \(-4y + 2z = 1\)

19. \(2x - 2y - 4z = -2\)
    \(3x - 3y - 6z = -3\)
    \(-2x + 3y + z = 7\)
20. \(x - y + 9z = -27\)
    \(2x - 4y - z = -1\)
    \(3x + 6y - 3z = 27\)
21. \(2x - 5y - 3z = 7\)
    \(-4x + 10y + 2z = 6\)
    \(6x - 15y - z = -19\)

22. The sum of three numbers is 6. The third number is the sum of the first and second numbers. The first number is one more than the third number. Find the numbers.

23. The sum of three numbers is \(-4\). The second number decreased by the third is equal to the first. The sum of the first and second numbers is \(-5\). Find the numbers.

24. SPORTS Alexandria High School scored 37 points in a football game. Six points are awarded for each touchdown. After each touchdown, the team can earn one point for the extra kick or two points for a 2-point conversion. The team scored one fewer 2-point conversions than extra kicks. The team scored 10 times during the game. How many touchdowns were made during the game?
State the dimensions of each matrix.

1. \([ -3 \ -3 \ 7 ]\)
2. \([ \begin{array}{ccc} 5 & 8 & -1 \\ -2 & 1 & 8 \end{array} ]\)
3. \([ \begin{array}{ccc} -2 & 2 & 3 \\ 5 & 16 & 0 \\ 4 & 7 & -1 \end{array} ]\)

Solve each equation.

4. \([4x \ 42] = [24 \ 6y]\)
5. \([ -2x \ 22 \ -3z] = [6x \ -2y \ 45]\)
6. \([ \begin{array}{c} 6x \\ 2y + 3 \end{array} ] = [ \begin{array}{c} -36 \\ 17 \end{array} ]\)
7. \([ \begin{array}{c} 7x - 8 \\ 8y - 3 \end{array} ] = [ \begin{array}{c} 20 \\ 2y + 3 \end{array} ]\)
8. \([ \begin{array}{c} -4x - 3 \\ 6y \end{array} ] = [ \begin{array}{c} -3x \\ -2y + 16 \end{array} ]\)
9. \([ \begin{array}{c} 6x - 12 \\ -3y + 6 \end{array} ] = [ \begin{array}{c} -3x - 21 \\ 8y - 5 \end{array} ]\)
10. \([ \begin{array}{c} -5 \\ 2y - 1 \end{array} ] = [ \begin{array}{c} -5 \\ 3y \end{array} ]\)
11. \([ \begin{array}{c} 3x \\ y + 4 \end{array} ] = [ \begin{array}{c} y + 8 \\ 17 \end{array} ]\)
12. \([ \begin{array}{c} 5x + 8y \\ 3x - 11 \end{array} ] = [ \begin{array}{c} -1 \\ y \end{array} ]\)
13. \([ \begin{array}{c} 2x + y \\ 3x + 2y \end{array} ] = [ \begin{array}{c} 0 \\ -2 \end{array} ]\)

14. **TICKET PRICES** The table at the right gives ticket prices for a concert. Write a \(2 \times 3\) matrix that represents the cost of a ticket.

<table>
<thead>
<tr>
<th>Child</th>
<th>Student</th>
<th>Adult</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6</td>
<td>$12</td>
<td>$18</td>
</tr>
<tr>
<td>$8</td>
<td>$15</td>
<td>$22</td>
</tr>
</tbody>
</table>

15. Write a matrix for the amount of gravel in each load.

<table>
<thead>
<tr>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load 1 40 tons</td>
<td>Load 1 40 tons</td>
<td>Load 1 32 tons</td>
</tr>
<tr>
<td>Load 2 32 tons</td>
<td>Load 2 40 tons</td>
<td>Load 2 24 tons</td>
</tr>
<tr>
<td>Load 3 24 tons</td>
<td>Load 3 32 tons</td>
<td>Load 3 24 tons</td>
</tr>
</tbody>
</table>

16. What are the dimensions of the matrix?
Perform the indicated matrix operations. If the matrix does not exist, write impossible.

1. \[
\begin{pmatrix}
2 & -1 \\
3 & 7 \\
\end{pmatrix}
+ 
\begin{pmatrix}
-6 & 9 \\
14 & -9 \\
\end{pmatrix}
\]

2. \[
\begin{pmatrix}
4 \\
-71
\end{pmatrix}
- 
\begin{pmatrix}
-67 \\
18
\end{pmatrix}
\]

3. \[
-3\begin{pmatrix}
-1 \\
17
\end{pmatrix}
+ 4\begin{pmatrix}
-3 \\
-21
\end{pmatrix}
\]

4. \[
7\begin{pmatrix}
2 & -1 & 8 \\
4 & 7 & 9
\end{pmatrix}
- 2\begin{pmatrix}
-1 & 4 & -3 \\
7 & 2 & -6
\end{pmatrix}
\]

5. \[
-2\begin{pmatrix}
1 \\
2
\end{pmatrix}
+ 4\begin{pmatrix}
0 \\
5
\end{pmatrix}
- \begin{pmatrix}
10 \\
18
\end{pmatrix}
\]

6. \[
\frac{3}{4}\begin{pmatrix}
8 \\
-16
\end{pmatrix}
+ \frac{2}{3}\begin{pmatrix}
27 \\
54
\end{pmatrix}
\]

Use \(A = \begin{pmatrix}
-4 & -1 \\
6 & 2
\end{pmatrix}\), \(B = \begin{pmatrix}
-2 & 4 & 5 \\
1 & 0 & -9
\end{pmatrix}\), and \(C = \begin{pmatrix}
10 & -8 & 6 \\
-6 & -4 & 20
\end{pmatrix}\) to find the following.

7. \(A - B\)

8. \(A - C\)

9. \(-3B\)

10. \(4B - A\)

11. \(-2B - 3C\)

12. \(A + 0.5C\)

ECONOMICS For Exercises 13 and 14, use the table that shows loans by an economic development board to women and men starting new businesses.

13. Write two matrices that represent the number of new businesses and loan amounts, one for women and one for men.

14. Find the sum of the numbers of new businesses and loan amounts for both men and women over the three-year period expressed as a matrix.

15. PET NUTRITION Use the table that gives nutritional information for two types of dog food. Find the difference in the percent of protein, fat, and fiber between Mix B and Mix A expressed as a matrix.
4-3 Practice

Multiplying Matrices

Determine whether each matrix product is defined. If so, state the dimensions of the product.

1. \(A_7 \times 4 \cdot B_4 \times 3\)
2. \(A_3 \times 5 \cdot M_5 \times 8\)
3. \(M_2 \times 1 \cdot A_1 \times 6\)
4. \(M_3 \times 2 \cdot A_3 \times 2\)
5. \(P_1 \times 9 \cdot Q_9 \times 1\)
6. \(P_9 \times 1 \cdot Q_1 \times 9\)

Find each product, if possible.

7. \[
\begin{bmatrix}
2 & 4 \\
3 & -1
\end{bmatrix}
\begin{bmatrix}
3 & -2 & 7 \\
6 & 0 & -5
\end{bmatrix}
\]

8. \[
\begin{bmatrix}
2 & 4 \\
7 & -1
\end{bmatrix}
\begin{bmatrix}
-3 & 0 \\
2 & 5
\end{bmatrix}
\]

9. \[
\begin{bmatrix}
-3 & 0 \\
2 & 5
\end{bmatrix}
\begin{bmatrix}
2 & 4 \\
7 & -1
\end{bmatrix}
\]

10. \[
\begin{bmatrix}
3 & -2 & 7 \\
6 & 0 & -5
\end{bmatrix}
\begin{bmatrix}
3 & -2 & 7 \\
6 & 0 & -5
\end{bmatrix}
\]

11. \[
\begin{bmatrix}
4 & 0 & 2
\end{bmatrix}
\begin{bmatrix}
1 & 3 \\
3 & -1
\end{bmatrix}
\]

12. \[
\begin{bmatrix}
1 & 3 \\
-1
\end{bmatrix}
\begin{bmatrix}
4 & 0 & 2
\end{bmatrix}
\]

13. \[
\begin{bmatrix}
-6 & 2 \\
3 & -1
\end{bmatrix}
\begin{bmatrix}
5 & 0 \\
0 & 5
\end{bmatrix}
\]

14. \[
\begin{bmatrix}
-15 & -9
\end{bmatrix}
\begin{bmatrix}
6 & 11 \\
23 & -10
\end{bmatrix}
\]

Use \(A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}\), \(B = \begin{bmatrix} -4 & 0 \\ -2 & -1 \end{bmatrix}\), \(C = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\), and scalar \(c = 3\) to determine whether the following equations are true for the given matrices.

15. \(AC = CA\)

16. \(A(B + C) = BA + CA\)

17. \((AB)c = c(AB)\)

18. \((A + C)B = B(A + C)\)

RENTALS For Exercises 19–21, use the following information.

For their one-week vacation, the Montoyas can rent a 2-bedroom condominium for $1796, a 3-bedroom condominium for $2165, or a 4-bedroom condominium for $2538. The table shows the number of units in each of three complexes.

<table>
<thead>
<tr>
<th>Complex</th>
<th>2-Bedroom</th>
<th>3-Bedroom</th>
<th>4-Bedroom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun Haven</td>
<td>36</td>
<td>24</td>
<td>22</td>
</tr>
<tr>
<td>Surfside</td>
<td>29</td>
<td>32</td>
<td>42</td>
</tr>
<tr>
<td>Seabreeze</td>
<td>18</td>
<td>22</td>
<td>18</td>
</tr>
</tbody>
</table>

19. Write a matrix that represents the number of each type of unit available at each complex and a matrix that represents the weekly charge for each type of unit.

20. If all of the units in the three complexes are rented for the week at the rates given the Montoyas, express the income of each of the three complexes as a matrix.

21. What is the total income of all three complexes for the week?
Practice

Transformations with Matrices

For Exercises 1–3, use the following information.
Quadrilateral $WXYZ$ with vertices $W(−3, 2)$, $X(−2, 4)$, $Y(4, 1)$, and $Z(3, 0)$ is translated 1 unit left and 3 units down.

1. Write the translation matrix.

2. Find the coordinates of quadrilateral $W′X′Y′Z′$.

3. Graph the preimage and the image.

For Exercises 4–6, use the following information.
The vertices of $\triangle RST$ are $R(6, 2)$, $S(3, −3)$, and $T(−2, 5)$. The triangle is dilated so that its perimeter is one half the original perimeter.

4. Write the coordinates of $\triangle RST$ in a vertex matrix.

5. Find the coordinates of the image $\triangle R′S′T′$.

6. Graph $\triangle RST$ and $\triangle R′S′T′$.

For Exercises 7–10, use the following information.
The vertices of quadrilateral $ABCD$ are $A(−3, 2)$, $B(0, 3)$, $C(4, −4)$, and $D(−2, −2)$. The quadrilateral is reflected over the $y$-axis.

7. Write the coordinates of $ABCD$ in a vertex matrix.

8. Write the reflection matrix for this situation.

9. Find the coordinates of $A′B′C′D′$.

10. Graph $ABCD$ and $A′B′C′D′$.

11. ARCHITECTURE Using architectural design software, the Bradleys plot their kitchen plans on a grid with each unit representing 1 foot. They place the corners of an island at $(2, 8)$, $(8, 11)$, $(3, 5)$, and $(9, 8)$. If the Bradleys wish to move the island 1.5 feet to the right and 2 feet down, what will the new coordinates of its corners be?

12. BUSINESS The design of a business logo calls for locating the vertices of a triangle at $(1.5, 5)$, $(4, 1)$, and $(1, 0)$ on a grid. If design changes require rotating the triangle $90^\circ$ counter-clockwise, what will the new coordinates of the vertices be?
Find the value of each determinant.

1. \[ \begin{vmatrix} 1 & 6 \\ 2 & 7 \end{vmatrix} \]
2. \[ \begin{vmatrix} 9 & 6 \\ 3 & 2 \end{vmatrix} \]
3. \[ \begin{vmatrix} 4 & 1 \\ -2 & -5 \end{vmatrix} \]
4. \[ \begin{vmatrix} -14 & -3 \\ 2 & -2 \end{vmatrix} \]
5. \[ \begin{vmatrix} 4 & -3 \\ -12 & 4 \end{vmatrix} \]
6. \[ \begin{vmatrix} 2 & -5 \\ 5 & -11 \end{vmatrix} \]
7. \[ \begin{vmatrix} 4 & 0 \\ -2 & 9 \end{vmatrix} \]
8. \[ \begin{vmatrix} 3 & -4 \\ 7 & 9 \end{vmatrix} \]
9. \[ \begin{vmatrix} -1 & -11 \\ 10 & -2 \end{vmatrix} \]
10. \[ \begin{vmatrix} 3 & -4 \\ 3.75 & 5 \end{vmatrix} \]
11. \[ \begin{vmatrix} 2 & -1 \\ 3 & -9.5 \end{vmatrix} \]
12. \[ \begin{vmatrix} 0.5 & -0.7 \\ 0.4 & -0.3 \end{vmatrix} \]

Evaluate each determinant using expansion by minors.

13. \[ \begin{vmatrix} -2 & 3 & 1 \\ 0 & 4 & -3 \\ 2 & 5 & -1 \end{vmatrix} \]
14. \[ \begin{vmatrix} 2 & -4 & 1 \\ 3 & 0 & 9 \\ -1 & 5 & 7 \end{vmatrix} \]
15. \[ \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -2 \\ 1 & 1 & -1 \end{vmatrix} \]
16. \[ \begin{vmatrix} 0 & -4 & 0 \\ 2 & -1 & 1 \\ 3 & -2 & 5 \end{vmatrix} \]
17. \[ \begin{vmatrix} 2 & 7 & -6 \\ 8 & 4 & 0 \\ 1 & -1 & 3 \end{vmatrix} \]
18. \[ \begin{vmatrix} -12 & 0 & 3 \\ 7 & 5 & -1 \\ 4 & 2 & -6 \end{vmatrix} \]

Evaluate each determinant using diagonals.

19. \[ \begin{vmatrix} -4 & 3 & -1 \\ 2 & 1 & -2 \\ 4 & 1 & -4 \end{vmatrix} \]
20. \[ \begin{vmatrix} 2 & 2 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & 1 \end{vmatrix} \]
21. \[ \begin{vmatrix} 1 & -4 & -1 \\ 1 & -6 & -2 \\ 2 & 3 & 1 \end{vmatrix} \]
22. \[ \begin{vmatrix} 1 & 2 & -4 \\ 1 & 4 & -6 \\ 2 & 3 & 3 \end{vmatrix} \]
23. \[ \begin{vmatrix} 2 & -1 & -2 \\ 4 & 0 & -2 \\ 0 & 3 & 2 \end{vmatrix} \]
24. \[ \begin{vmatrix} 2 & 1 & 3 \\ 1 & 8 & 0 \\ 0 & 5 & -1 \end{vmatrix} \]

25. GEOMETRY Find the area of a triangle whose vertices have coordinates \((3, 5), (6, -5),\) and \((-4, 10).\)

26. LAND MANAGEMENT A fish and wildlife management organization uses a GIS (geographic information system) to store and analyze data for the parcels of land it manages. All of the parcels are mapped on a grid in which 1 unit represents 1 acre. If the coordinates of the corners of a parcel are \((-8, 10), (6, 17),\) and \((2, -4),\) how many acres is the parcel?
4-6 Practice

Cramer's Rule

Use Cramer's Rule to solve each system of equations.

1. \(2x + y = 0\)
   \(3x + 2y = -2\)
2. \(5c + 9d = 19\)
   \(2c - d = -20\)
3. \(2x + 3y = 5\)
   \(3x - 2y = 1\)
4. \(20m - 3n = 28\)
   \(2m + 3n = 16\)
5. \(x - 3y = 6\)
   \(3x + y = -22\)
6. \(5x - 6y = -45\)
   \(9x + 8y = 13\)
7. \(-2e + f = 4\)
   \(-3e + 5f = -15\)
8. \(2x - y = -1\)
   \(2x - 4y = 8\)
9. \(8a + 3b = 24\)
   \(2a + b = 4\)
10. \(-3x + 15y = 45\)
    \(-2x + 7y = 18\)
11. \(3u - 5v = 11\)
    \(6u + 7v = -12\)
12. \(-6g + h = -10\)
    \(-3g - 4h = 4\)
13. \(x - 3y = 8\)
    \(x - 0.5y = 3\)
14. \(0.2x - 0.5y = -1\)
    \(0.6x - 3y = -9\)
15. \(0.3d - 0.6g = 1.8\)
    \(0.2d + 0.3g = 0.5\)

16. GEOMETRY The two sides of an angle are contained in the lines whose equations are
    \(x - \frac{4}{3}y = 6\) and \(2x + y = 1\). Find the coordinates of the vertex of the angle.

17. GEOMETRY Two sides of a parallelogram are contained in the lines whose equations are
    \(0.2x - 0.5y = 1\) and \(0.02x - 0.3y = -0.9\). Find the coordinates of a vertex of the
    parallelogram.

Use Cramer's Rule to solve each system of equations.

18. \(x + 3y + 3z = 4\)
    \(-x + 2y + z = -1\)
    \(4x + y - 2z = -1\)
19. \(-5a + b - 4c = 7\)
    \(-3a + 2b - c = 0\)
    \(2a + 3b - c = 17\)
20. \(2x + y - 3z = -5\)
    \(5x + 2y - 2z = 8\)
    \(3x - 3y + 5z = 17\)
21. \(2c + 3d - e = 17\)
    \(4c + d + 5e = -9\)
    \(c + 2d - e = 12\)
22. \(2j + k - 3m = -3\)
    \(3j + 2k + 4m = 5\)
    \(-4j - k + 2m = 4\)
23. \(3x - 2y + 5z = 3\)
    \(2x + 2y - 4z = 3\)
    \(-5x + 10y + 7z = -3\)

24. LANDSCAPING A memorial garden being planted in front of a municipal library will contain three circular beds that are tangent to each other. A landscape architect has prepared a sketch of the design for the garden using CAD (computer-aided drafting) software, as shown at the right. The centers of the three circular beds are represented by points \(A\), \(B\), and \(C\). The distance from \(A\) to \(B\) is 15 feet, the distance from \(B\) to \(C\) is 13 feet, and the distance from \(A\) to \(C\) is 16 feet. What is the radius of each of the circular beds?
Determine whether each pair of matrices are inverses.

1. \( M = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}, \quad N = \begin{bmatrix} -2 & 1 \\ 3 & -2 \end{bmatrix} \)  
   yes

2. \( X = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}, \quad Y = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} \)  
   yes

3. \( A = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 \\ 5 & 10 \end{bmatrix} \)  
   no

4. \( P = \begin{bmatrix} 6 & -2 \\ 3 & -3 \end{bmatrix}, \quad Q = \begin{bmatrix} 3 & 1 \\ 14 & 7 \end{bmatrix} \)  
   yes

Determine whether each statement is true or false.

5. All square matrices have multiplicative inverses.  
   false

6. All square matrices have multiplicative identities.  
   true

Find the inverse of each matrix, if it exists.

7. \( \begin{bmatrix} 4 & 5 \\ -4 & -3 \end{bmatrix} \)  
   \( \begin{bmatrix} 5 & -4 \\ 3 & 4 \end{bmatrix} \)

8. \( \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix} \)  
   \( \begin{bmatrix} 5 & -2 \\ -3 & 2 \end{bmatrix} \)

9. \( \begin{bmatrix} -1 & 3 \\ 4 & -7 \end{bmatrix} \)  
   \( \begin{bmatrix} -3 & 1 \\ 4 & -1 \end{bmatrix} \)

10. \( \begin{bmatrix} 2 & 5 \\ -1 & 3 \end{bmatrix} \)  
    \( \begin{bmatrix} 3 & -2 \\ -5 & 2 \end{bmatrix} \)

11. \( \begin{bmatrix} 2 & -5 \\ 3 & 1 \end{bmatrix} \)  
    \( \begin{bmatrix} 1 & 5 \\ -2 & 6 \end{bmatrix} \)

12. \( \begin{bmatrix} 4 & 6 \\ 6 & 9 \end{bmatrix} \)  
    \( \begin{bmatrix} 3 & -2 \\ -1 & 5 \end{bmatrix} \)

GEOMETRY For Exercises 13–16, use the figure at the right.

13. Write the vertex matrix \( A \) for the rectangle.

14. Use matrix multiplication to find \( BA \) if \( B = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix} \).

15. Graph the vertices of the transformed rectangle on the previous graph.  
    Describe the transformation.

16. Make a conjecture about what transformation \( B^{-1} \) describes on a coordinate plane.

17. CODES Use the alphabet table below and the inverse of coding matrix \( C = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \) to decode this message:
   19 14 11 13 11 22 55 65 57 60 2 1 52 47 33 51 56 55.

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>A 1</td>
</tr>
<tr>
<td>B 2</td>
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<tr>
<td>C 3</td>
</tr>
<tr>
<td>D 4</td>
</tr>
<tr>
<td>E 5</td>
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<td>F 6</td>
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<tr>
<td>G 7</td>
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<td>H 8</td>
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<td>Y 25</td>
</tr>
<tr>
<td>Z 26</td>
</tr>
<tr>
<td>- 0</td>
</tr>
</tbody>
</table>
4-8 Practice

Using Matrices to Solve Systems of Equations

Write a matrix equation for each system of equations.

1. \(-3x + 2y = 9\)
   \[5x - 3y = -13\]

2. \(6x - 2y = -2\)
   \[3x + 3y = 10\]

3. \(2a + b = 0\)
   \[3a + 2b = -2\]

4. \(r + 5s = 10\)
   \[2r - 3s = 7\]

5. \(3x - 2y + 5z = 3\)
   \[x + y - 4z = 2\]
   \[-2x + 2y + 7z = -5\]

6. \(2m + n - 3p = -5\)
   \[5m + 2n - 2p = 8\]
   \[3m - 3n + 5p = 17\]

Solve each matrix equation or system of equations by using inverse matrices.

7. \[
\begin{bmatrix}
2 & 1 \\
3 & 2
\end{bmatrix}
\cdot
\begin{bmatrix}
g \\
h
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
-2
\end{bmatrix}
\]

8. \[
\begin{bmatrix}
-2 & 3 \\
1 & 5
\end{bmatrix}
\cdot
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
-7 \\
10
\end{bmatrix}
\]

9. \[
\begin{bmatrix}
-1 & -3 \\
3 & 4
\end{bmatrix}
\cdot
\begin{bmatrix}
a \\
b
\end{bmatrix}
= 
\begin{bmatrix}
12 \\
-11
\end{bmatrix}
\]

10. \[
\begin{bmatrix}
-5 & 3 \\
6 & 4
\end{bmatrix}
\cdot
\begin{bmatrix}
c \\
d
\end{bmatrix}
= 
\begin{bmatrix}
16 \\
34
\end{bmatrix}
\]

11. \[
\begin{bmatrix}
-4 & 7 \\
2 & 4
\end{bmatrix}
\cdot
\begin{bmatrix}
r \\
s
\end{bmatrix}
= 
\begin{bmatrix}
17 \\
-26
\end{bmatrix}
\]

12. \[
\begin{bmatrix}
8 & 3 \\
12 & 6
\end{bmatrix}
\cdot
\begin{bmatrix}
y \\
z
\end{bmatrix}
= 
\begin{bmatrix}
-1 \\
-1
\end{bmatrix}
\]

13. \(2x + 3y = 5\)
   \(3x - 2y = 1\)

14. \(8d + 9f = 13\)
   \(-6d + 5f = -45\)

15. \(5m + 9n = 19\)
   \(2m - n = -20\)

16. \(-4j + 9k = -8\)
   \(6j + 12k = -5\)

17. AIRLINE TICKETS Last Monday at 7:30 A.M., an airline flew 89 passengers on a commuter flight from Boston to New York. Some of the passengers paid $120 for their tickets and the rest paid $230 for their tickets. The total cost of all of the tickets was $14,200. How many passengers bought $120 tickets? How many bought $230 tickets?

18. NUTRITION A single dose of a dietary supplement contains 0.2 gram of calcium and 0.2 gram of vitamin C. A single dose of a second dietary supplement contains 0.1 gram of calcium and 0.4 gram of vitamin C. If a person wants to take 0.6 gram of calcium and 1.2 grams of vitamin C, how many doses of each supplement should she take?
Deciduous and coniferous trees are hard to distinguish in a black-and-white photo. But because deciduous trees reflect infrared energy better than coniferous trees, the two types of trees are more distinguishable in an infrared photo. If an infrared wavelength measures about \(8 \times 10^{-7}\) meters and a blue wavelength measures about \(4.5 \times 10^{-7}\) meters, about how many times longer is the infrared wavelength than the blue wavelength?
5-2 Practice
Polynomials

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

1. \(5x^3 + 2xy^4 + 6xy\)  
2. \(\frac{4}{3}ac - a^5d^3\)  
3. \(\frac{12m^8n^9}{(m-n)^2}\)
4. \(25x^2z - x\sqrt{78}\)  
5. \(6e^{-2} + c - 1\)  
6. \(\frac{5}{r} + \frac{6}{s}\)

Simplify.

7. \((3n^2 + 1) + (8n^2 - 8)\)  
8. \((6w - 11w^2) - (4 + 7w^2)\)
9. \((-6n - 13n^2) + (-3n + 9n^2)\)  
10. \((8x^2 - 3x) - (4x^2 + 5x - 3)\)
11. \((5m^2 - 2mp - 6p^2) - (-3m^2 + 5mp + p^2)\)  
12. \((2x^2 - xy + y^2) + (-3x^2 + 4xy + 3y^2)\)
13. \((5t - 7) + (2t^2 + 3t + 12)\)  
14. \((u - 4) - (6 + 3u^2 - 4u)\)
15. \(-9(y^2 - 7w)\)  
16. \(-9r^4y^2(-3ry^7 + 2r^3y^4 - 8r^{10})\)
17. \(-6a^2w(a^3w - aw^4)\)  
18. \(5a^2w^3(a^2w^6 - 3a^4w^2 + 9aw^6)\)
19. \(2x^2(x^2 + xy - 2y^2)\)  
20. \(-\frac{3}{5}ab^3d^2(-5ab^2d^5 - 5ab)\)
21. \((v^2 - 6)(v^2 + 4)\)  
22. \((7a + 9y)(2a - y)\)
23. \((y - 8)^2\)  
24. \((x^2 + 5y)^2\)
25. \((5x + 4w)(5x - 4w)\)  
26. \((2n^4 - 3)(2n^4 + 3)\)
27. \((w + 2s)(w^2 - 2ws + 4s^2)\)  
28. \((x + y)(x^2 - 3xy + 2y^2)\)

29. BANKING Terry invests $1500 in two mutual funds. The first year, one fund grows 3.8% and the other grows 6%. Write a polynomial to represent the amount Terry’s $1500 grows to in that year if \(x\) represents the amount he invested in the fund with the lesser growth rate.

30. GEOMETRY The area of the base of a rectangular box measures \(2x^2 + 4x - 3\) square units. The height of the box measures \(x\) units. Find a polynomial expression for the volume of the box.
Practice

Dividing Polynomials

Simplify.

1. \[ \frac{15r^{10} - 5r^8 + 40r^2}{5r^4} \]

2. \[ \frac{6k^2m - 12k^3m^2 + 9m^3}{2km^2} \]

3. \[ (-30x^3y + 12x^2y^2 - 18x^2y) \div (-6x^2y) \]

4. \[ (-6w^3z^4 - 3w^2z^5 + 4w + 5z) \div (2w^2z) \]

5. \[ (4a^3 - 8a^2 + a^2)(4a)^{-1} \]

6. \[ (28d^3h^2 + d^2h^2 - 4dk^2)(4dk^2)^{-1} \]

7. \[ \frac{f^2 + 7f + 10}{f + 2} \]

8. \[ \frac{2x^2 + 3x - 14}{x - 2} \]

9. \[ (a^3 - 64) \div (a - 4) \]

10. \[ (b^3 + 27) \div (b + 3) \]

11. \[ \frac{2x^3 + 6x + 152}{x + 4} \]

12. \[ \frac{2x^3 + 4x - 6}{x + 3} \]

13. \[ (3w^3 + 7w^2 - 4w + 3) \div (w + 3) \]

14. \[ (6y^4 + 15y^3 - 28y - 6) \div (y + 2) \]

15. \[ (x^4 - 3x^3 - 11x^2 + 3x + 10) \div (x - 5) \]

16. \[ (3m^5 + m - 1) \div (m + 1) \]

17. \[ (x^4 - 3x^3 + 5x - 6)(x + 2)^{-1} \]

18. \[ (6y^2 - 5y - 15)(2y + 3)^{-1} \]

19. \[ \frac{4x^2 - 2x + 6}{2x - 3} \]

20. \[ \frac{6x^2 - x - 7}{3x + 1} \]

21. \[ (2r^3 + 5r^2 - 2r - 15) \div (2r - 3) \]

22. \[ (6t^3 + 5t^2 - 2t + 1) \div (3t + 1) \]

23. \[ \frac{4p^4 - 17p^2 + 14p^3 - 3}{2p - 3} \]

24. \[ \frac{2h^4 - h^3 + h^2 + h - 3}{h^2 - 1} \]

25. GEOMETRY The area of a rectangle is \(2x^2 - 11x + 15\) square feet. The length of the rectangle is \(2x - 5\) feet. What is the width of the rectangle?

26. GEOMETRY The area of a triangle is \(15x^4 + 3x^3 + 4x^2 - x - 3\) square meters. The length of the base of the triangle is \(6x^2 - 2\) meters. What is the height of the triangle?
5-4 Practice
Factoring Polynomials

Factor completely. If the polynomial is not factorable, write prime.

1. \(15a^2b - 10ab^2\)  
2. \(3st^2 - 9s^3t + 6s^2t^2\)  
3. \(3x^3y^2 - 2x^2y + 5xy\)

4. \(2x^3y - x^2y + 5xy^2 + xy^3\)  
5. \(21 - 7t + 3r - rt\)  
6. \(x^2 - xy + 2x - 2y\)

7. \(y^2 + 20y + 96\)  
8. \(4ab + 2a + 6b + 3\)  
9. \(6n^2 - 11n - 2\)

10. \(6x^2 + 7x - 3\)  
11. \(x^2 - 8x - 8\)  
12. \(6p^2 - 17p - 45\)

13. \(r^3 + 3r^2 - 54r\)  
14. \(8a^2 + 2a - 6\)  
15. \(c^2 - 49\)

16. \(x^3 + 8\)  
17. \(16r^2 - 169\)  
18. \(b^4 - 81\)

19. \(8m^3 - 25\)  
20. \(2t^3 + 32t^2 + 128t\)

21. \(5y^5 + 135y^2\)  
22. \(81x^4 - 16\)

Simplify. Assume that no denominator is equal to 0.

23. \(\frac{x^2 - 16}{x^2 + x - 20}\)  
24. \(\frac{x^2 - 16x + 64}{x^2 + x - 72}\)  
25. \(\frac{3x^2 - 27}{x^3 - 27}\)

26. DESIGN Bobbi Jo is using a software package to create a drawing of a cross section of a brace as shown at the right. Write a simplified, factored expression that represents the area of the cross section of the brace.

27. COMBUSTION ENGINES In an internal combustion engine, the up and down motion of the pistons is converted into the rotary motion of the crankshaft, which drives the flywheel. Let \(r_1\) represent the radius of the flywheel at the right and let \(r_2\) represent the radius of the crankshaft passing through it. If the formula for the area of a circle is \(A = \pi r^2\), write a simplified, factored expression for the area of the cross section of the flywheel outside the crankshaft.
5-5 **Practice**  
*Roots of Real Numbers*

Use a calculator to approximate each value to three decimal places.

1. $\sqrt{7.8}$
2. $-\sqrt{89}$
3. $3\sqrt{25}$
4. $3\sqrt{-4}$

5. $\sqrt[4]{1.1}$
6. $\sqrt[5]{-0.1}$
7. $\sqrt[6]{5555}$
8. $\sqrt[8]{(0.94)^2}$

Simplify.

9. $\sqrt{0.81}$
10. $-\sqrt{324}$
11. $-\sqrt{256}$
12. $\sqrt{64}$

13. $\sqrt[3]{-64}$
14. $\sqrt[3]{0.512}$
15. $\sqrt[3]{-243}$
16. $-\sqrt[3]{1296}$

17. $\sqrt[5]{\frac{-1024}{243}}$
18. $\sqrt[5]{243x^{10}}$
19. $\sqrt{(14a)^2}$
20. $\sqrt{-(14a)^2}$

21. $\sqrt{49m^2r^8}$
22. $\sqrt{16m^2}{\frac{25}{25}}$
23. $\sqrt[3]{-64r^6w^{15}}$
24. $\sqrt{(2x)^8}$

25. $-\sqrt[4]{625s^8}$
26. $\sqrt[3]{216p^3q^9}$
27. $\sqrt{676x^4y^6}$
28. $\sqrt[3]{-27x^9y^{12}}$

29. $-\sqrt{144m^8n^6}$
30. $\sqrt[5]{-32x^5y^{10}}$
31. $\sqrt[6]{(m + 4)^6}$
32. $\sqrt[3]{(2x + 1)^3}$

33. $-\sqrt{49a^{10}b^{16}}$
34. $\sqrt[4]{(x - 5)^8}$
35. $\sqrt[3]{343d^6}$
36. $\sqrt{x^2 + 10x + 25}$

**37. RADIANT TEMPERATURE**  Thermal sensors measure an object’s *radiant* temperature, which is the amount of energy radiated by the object. The *internal* temperature of an object is called its *kinetic* temperature. The formula $T_r = T_k \sqrt{e}$ relates an object’s radiant temperature $T_r$ to its kinetic temperature $T_k$. The variable $e$ in the formula is a measure of how well the object radiates energy. If an object’s kinetic temperature is $30^\circ C$ and $e = 0.94$, what is the object’s radiant temperature to the nearest tenth of a degree?

**38. HERO’S FORMULA** Salvatore is buying fertilizer for his triangular garden. He knows the lengths of all three sides, so he is using Hero’s formula to find the area. Hero’s formula states that the area of a triangle is $\sqrt{s(s - a)(s - b)(s - c)}$, where $a$, $b$, and $c$ are the lengths of the sides of the triangle and $s$ is half the perimeter of the triangle. If the lengths of the sides of Salvatore’s garden are 15 feet, 17 feet, and 20 feet, what is the area of the garden? Round your answer to the nearest whole number.
5-6 Practice

Radical Expressions

Simplify.

1. \( \sqrt{540} \)  
2. \( \sqrt[3]{-432} \)  
3. \( \sqrt[3]{128} \)

4. \( -\sqrt[4]{405} \)  
5. \( \sqrt[3]{-5000} \)  
6. \( \sqrt[5]{-1215} \)

7. \( \sqrt[3]{125t^6w^2} \)  
8. \( \sqrt[3]{48v^8z^{13}} \)  
9. \( \sqrt[3]{8g^3k^8} \)

10. \( \sqrt{45x^3y^8} \)  
11. \( \sqrt{\frac{11}{9}} \)  
12. \( \sqrt[3]{\frac{216}{24}} \)

13. \( \sqrt[3]{\frac{1}{128}cd^7} \)  
14. \( \sqrt[3]{\frac{9a^5}{64b^4}} \)  
15. \( \sqrt[4]{\frac{8}{9a^3}} \)

16. \( (3\sqrt{15})(-4\sqrt{45}) \)  
17. \( (2\sqrt{24})(7\sqrt{18}) \)  
18. \( \sqrt{810} + \sqrt{240} - \sqrt{250} \)

19. \( 6\sqrt{20} + 8\sqrt{5} - 5\sqrt{45} \)  
20. \( 8\sqrt{48} - 6\sqrt{75} + 7\sqrt{80} \)  
21. \( (3\sqrt{2} + 2\sqrt{3})^2 \)

22. \( (3 - \sqrt{7})^2 \)  
23. \( (\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2}) \)  
24. \( (\sqrt{2} + \sqrt{10})(\sqrt{2} - \sqrt{10}) \)

25. \( (1 + \sqrt{6})(5 - \sqrt{7}) \)  
26. \( (\sqrt{3} + 4\sqrt{7})^2 \)  
27. \( (\sqrt{108} - 6\sqrt{3})^2 \)

28. \( \frac{\sqrt{3}}{\sqrt{5} - 2} \)  
29. \( \frac{6}{\sqrt{2} - 1} \)  
30. \( \frac{5 + \sqrt{3}}{4 + \sqrt{3}} \)

31. \( \frac{3 + \sqrt{2}}{2 - \sqrt{2}} \)  
32. \( \frac{3 + \sqrt{6}}{5 - \sqrt{24}} \)  
33. \( \frac{3 + \sqrt{x}}{2 - \sqrt{x}} \)

34. BRAKING The formula \( s = 2\sqrt{5\ell} \) estimates the speed \( s \) in miles per hour of a car when it leaves skid marks \( \ell \) feet long. Use the formula to write a simplified expression for \( s \) if \( \ell = 85 \). Then evaluate \( s \) to the nearest mile per hour.

35. PYTHAGOREAN THEOREM The measures of the legs of a right triangle can be represented by the expressions \( 6x^2y \) and \( 9x^2y \). Use the Pythagorean Theorem to find a simplified expression for the measure of the hypotenuse.
5-7

Practice

Rational Exponents

Write each expression in radical form.

1. \(5^{\frac{1}{3}}\)  
2. \(6^{\frac{2}{5}}\)  
3. \(m^{\frac{4}{7}}\)  
4. \((n^3)^{\frac{2}{5}}\)

Write each radical using rational exponents.

5. \(\sqrt[3]{79}\)  
6. \(\sqrt[5]{153}\)  
7. \(\sqrt[4]{27m^6n^4}\)  
8. \(5\sqrt{2a^{10}b}\)

Evaluate each expression.

9. \(81^{\frac{1}{4}}\)  
10. \(1024^{-\frac{1}{5}}\)  
11. \(8^{-\frac{5}{3}}\)

12. \(-256^{-\frac{3}{4}}\)  
13. \((-64)^{-\frac{2}{3}}\)  
14. \(27^{\frac{1}{3}} \cdot 27^{\frac{4}{3}}\)

15. \(\left(\frac{125}{216}\right)^{\frac{2}{3}}\)  
16. \(\frac{64^{\frac{2}{3}}}{343^{\frac{2}{3}}}\)  
17. \(\left(25^{\frac{1}{2}}\right)\left(-64^{-\frac{1}{3}}\right)\)

Simplify each expression.

18. \(g^{\frac{4}{7}} \cdot g^{\frac{3}{7}}\)  
19. \(s^{\frac{3}{4}} \cdot s^{\frac{13}{4}}\)  
20. \(u^{-\frac{1}{3}}^{\frac{4}{9}}\)  
21. \(y^{\frac{1}{2}}\)

22. \(b^{\frac{3}{5}}\)  
23. \(\frac{q^{\frac{3}{2}}}{q^{\frac{5}{2}}}\)  
24. \(\frac{t^{\frac{2}{3}}}{5t^{\frac{1}{3}} \cdot t^{-\frac{3}{4}}}\)  
25. \(\frac{2z^2}{z^{\frac{1}{2}} - 1}\)

26. \(\sqrt[10]{85}\)  
27. \(\sqrt{12} \cdot \sqrt[3]{12^3}\)  
28. \(\sqrt{6} \cdot 3\sqrt{6}\)  
29. \(\frac{a}{\sqrt{3b}}\)

30. **ELECTRICITY** The amount of current in amperes \(I\) that an appliance uses can be calculated using the formula \(I = \left(\frac{P}{R}\right)^{\frac{1}{2}}\), where \(P\) is the power in watts and \(R\) is the resistance in ohms. How much current does an appliance use if \(P = 500\) watts and \(R = 10\) ohms? Round your answer to the nearest tenth.

31. **BUSINESS** A company that produces DVDs uses the formula \(C = 88n^{\frac{1}{3}} + 330\) to calculate the cost \(C\) in dollars of producing \(n\) DVDs per day. What is the company’s cost to produce 150 DVDs per day? Round your answer to the nearest dollar.
5-8 Practice

Radical Equations and Inequalities

Solve each equation or inequality.

1. \( \sqrt{x} = 8 \)

2. \( 4 - \sqrt{x} = 3 \)

3. \( \sqrt{2p} + 3 = 10 \)

4. \( 4\sqrt{3h} - 2 = 0 \)

5. \( \frac{1}{2} + 6 = 9 \)

6. \( 18 + 7h^\frac{1}{2} = 12 \)

7. \( \sqrt[3]{d} + 2 = 7 \)

8. \( \sqrt[5]{w} - 7 = 1 \)

9. \( 6 + \sqrt[3]{q} - 4 = 9 \)

10. \( \sqrt[4]{y} - 9 + 4 = 0 \)

11. \( \sqrt{2m} - 6 - 16 = 0 \)

12. \( \sqrt[3]{4m} + 1 - 2 = 2 \)

13. \( \sqrt{8n} - 5 - 1 = 2 \)

14. \( \sqrt{1 - 4t} - 8 = -6 \)

15. \( \sqrt{2t} - 5 - 3 = 3 \)

16. \( (7v - 2)^\frac{1}{4} + 12 = 7 \)

17. \( (3g + 1)^\frac{1}{2} - 6 = 4 \)

18. \( (6u - 5)^\frac{1}{3} + 2 = -3 \)

19. \( \sqrt{2d} - 5 = \sqrt{d} - 1 \)

20. \( \sqrt[4]{4r} - 6 = \sqrt{r} \)

21. \( \sqrt{6x} - 4 = \sqrt{2x} + 10 \)

22. \( \sqrt{2x} + 5 = \sqrt{2x} + 1 \)

23. \( 3\sqrt{a} \geq 12 \)

24. \( \sqrt{z} + 5 + 4 \leq 13 \)

25. \( 8 + \sqrt{2q} \leq 5 \)

26. \( \sqrt{2a} - 3 < 5 \)

27. \( 9 - \sqrt{c} + 4 \leq 6 \)

28. \( \frac{1}{3}x - 1 < -2 \)

29. **STATISTICS** Statisticians use the formula \( \sigma = \sqrt{v} \) to calculate a standard deviation \( \sigma \), where \( v \) is the variance of a data set. Find the variance when the standard deviation is 15.

30. **GRAVITATION** Helena drops a ball from 25 feet above a lake. The formula

\[ t = \frac{1}{4} \sqrt{25 - h} \]

describes the time \( t \) in seconds that the ball is \( h \) feet above the water.

How many feet above the water will the ball be after 1 second?
5-9 Practice
Complex Numbers

Simplify.

1. \( \sqrt{-49} \)  
2. \( 6\sqrt{-12} \)  
3. \( \sqrt{-121s^8} \)

4. \( \sqrt{-36a^3b^4} \)  
5. \( \sqrt{-8 \cdot \sqrt{-32}} \)

6. \( \sqrt{-15 \cdot \sqrt{-25}} \)

7. \( (-3i)(4i)(-5i) \)  
8. \( (7i)^2(6i) \)  
9. \( i^{42} \)

10. \( i^{55} \)  
11. \( i^{89} \)  
12. \( (5 - 2i) + (-13 - 8i) \)

13. \( (7 - 6i) + (9 + 11i) \)  
14. \( (-12 + 48i) + (15 + 21i) \)

15. \( (10 + 15i) - (48 - 30i) \)

16. \( (28 - 4i) - (10 - 30i) \)  
17. \( (6 - 4i)(6 + 4i) \)  
18. \( (8 - 11i)(8 - 11i) \)

19. \( (4 + 3i)(2 - 5i) \)  
20. \( (7 + 2i)(9 - 6i) \)

21. \( \frac{6 + 5i}{2i} \)

22. \( \frac{2}{7 - 8i} \)  
23. \( \frac{3 - i}{2 - i} \)  
24. \( \frac{2 - 4i}{1 + 3i} \)

Solve each equation.

25. \( 5n^2 + 35 = 0 \)  
26. \( 2m^2 + 10 = 0 \)

27. \( 4m^2 + 76 = 0 \)  
28. \( -2m^2 - 6 = 0 \)

29. \( -5m^2 - 65 = 0 \)  
30. \( \frac{3}{4}x^2 + 12 = 0 \)

Find the values of \( m \) and \( n \) that make each equation true.

31. \( 15 - 28i = 3m + 4ni \)  
32. \( (6 - m) + 3ni = -12 + 27i \)

33. \( (3m + 4) + (3 - n)i = 16 - 3i \)  
34. \( (7 + n) + (4m - 10)i = 3 - 6i \)

35. **ELECTRICITY** The impedance in one part of a series circuit is \( 1 + 3j \) ohms and the impedance in another part of the circuit is \( 7 - 5j \) ohms. Add these complex numbers to find the total impedance in the circuit.

36. **ELECTRICITY** Using the formula \( E = IZ \), find the voltage \( E \) in a circuit when the current \( I \) is \( 3 - j \) amps and the impedance \( Z \) is \( 3 + 2j \) ohms.

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6-1 Practice

Graphing Quadratic Functions

Complete parts a–c for each quadratic function.

a. Find the \( y \)-intercept, the equation of the axis of symmetry, and the \( x \)-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

1. \( f(x) = x^2 - 8x + 15 \)
2. \( f(x) = -x^2 - 4x + 12 \)
3. \( f(x) = 2x^2 - 2x + 1 \)

Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function.

4. \( f(x) = x^2 + 2x - 8 \)
5. \( f(x) = x^2 - 6x + 14 \)
6. \( v(x) = -x^2 + 14x - 57 \)

7. \( f(x) = 2x^2 + 4x - 6 \)
8. \( f(x) = -x^2 + 4x - 1 \)
9. \( f(x) = \frac{2}{3}x^2 + 8x - 24 \)

10. GRAVITATION From 4 feet above a swimming pool, Susan throws a ball upward with a velocity of 32 feet per second. The height \( h(t) \) of the ball \( t \) seconds after Susan throws it is given by \( h(t) = -16t^2 + 32t + 4 \). Find the maximum height reached by the ball and the time that this height is reached.

11. HEALTH CLUBS Last year, the SportsTime Athletic Club charged $20 to participate in an aerobics class. Seventy people attended the classes. The club wants to increase the class price this year. They expect to lose one customer for each $1 increase in the price.

   a. What price should the club charge to maximize the income from the aerobics classes?

   b. What is the maximum income the SportsTime Athletic Club can expect to make?
Practice

Solving Quadratic Equations By Graphing

Use the related graph of each equation to determine its solutions.

1. \(-3x^2 + 3 = 0\)  
2. \(3x^2 + x + 3 = 0\)  
3. \(x^2 - 3x + 2 = 0\)

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. \(-2x^2 - 6x + 5 = 0\)  
5. \(x^2 + 10x + 24 = 0\)  
6. \(2x^2 - x - 6 = 0\)

Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.

7. Their sum is 1, and their product is \(-6\).  
8. Their sum is 5, and their product is 8.

For Exercises 9 and 10, use the formula \(h(t) = v_0t - 16t^2\), where \(h(t)\) is the height of an object in feet, \(v_0\) is the object’s initial velocity in feet per second, and \(t\) is the time in seconds.

9. BASEBALL Marta throws a baseball with an initial upward velocity of 60 feet per second. Ignoring Marta’s height, how long after she releases the ball will it hit the ground?

10. VOLCANOES A volcanic eruption blasts a boulder upward with an initial velocity of 240 feet per second. How long will it take the boulder to hit the ground if it lands at the same elevation from which it was ejected?
6-3 Practice
Solving Quadratic Equations by Factoring

Solve each equation by factoring.

1. \(x^2 - 4x - 12 = 0\)  
2. \(x^2 - 16x + 64 = 0\)  
3. \(x^2 - 20x + 100 = 0\)

4. \(x^2 - 6x + 8 = 0\)  
5. \(x^2 + 3x + 2 = 0\)  
6. \(x^2 - 9x + 14 = 0\)

7. \(x^2 - 4x = 0\)  
8. \(7x^2 = 4x\)  
9. \(x^2 + 25 = 10x\)

10. \(10x^2 = 9x\)  
11. \(x^2 = 2x + 99\)

12. \(x^2 + 12x = -36\)  
13. \(5x^2 - 35x + 60 = 0\)

14. \(36x^2 = 25\)  
15. \(2x^2 - 8x - 90 = 0\)

16. \(3x^2 + 2x - 1 = 0\)  
17. \(6x^2 = 9x\)

18. \(3x^2 + 24x + 45 = 0\)  
19. \(15x^2 + 19x + 6 = 0\)

20. \(3x^2 - 8x = -4\)  
21. \(6x^2 = 5x + 6\)

Write a quadratic equation with the given roots. Write the equation in the form \(ax^2 + bx + c = 0\), where \(a\), \(b\), and \(c\) are integers.

22. \(7, 2\)  
23. \(0, 3\)  
24. \(-5, 8\)

25. \(-7, -8\)  
26. \(-6, -3\)  
27. \(3, -4\)

28. \(1, \frac{1}{2}\)  
29. \(\frac{1}{3}, 2\)  
30. \(0, -\frac{7}{2}\)

31. \(\frac{1}{3}, -3\)  
32. \(4, \frac{1}{3}\)  
33. \(-\frac{2}{3}, -\frac{4}{5}\)

34. NUMBER THEORY Find two consecutive even positive integers whose product is 624.

35. NUMBER THEORY Find two consecutive odd positive integers whose product is 323.

36. GEOMETRY The length of a rectangle is 2 feet more than its width. Find the dimensions of the rectangle if its area is 63 square feet.

37. PHOTOGRAPHY The length and width of a 6-inch by 8-inch photograph are reduced by the same amount to make a new photograph whose area is half that of the original. By how many inches will the dimensions of the photograph have to be reduced?
6-4 Practice

Completing the Square

Solve each equation by using the Square Root Property.

1. $x^2 + 8x + 16 = 1$
2. $x^2 + 6x + 9 = 1$
3. $x^2 + 10x + 25 = 16$

4. $x^2 - 14x + 49 = 9$
5. $4x^2 + 12x + 9 = 4$
6. $x^2 - 8x + 16 = 8$

7. $x^2 - 6x + 9 = 5$
8. $x^2 - 2x + 1 = 2$
9. $9x^2 - 6x + 1 = 2$

Find the value of $c$ that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

10. $x^2 + 12x + c$
11. $x^2 - 20x + c$
12. $x^2 + 11x + c$

13. $x^2 + 0.8x + c$
14. $x^2 - 2.2x + c$
15. $x^2 - 0.36x + c$

16. $x^2 + \frac{5}{6}x + c$
17. $x^2 - \frac{1}{4}x + c$
18. $x^2 - \frac{5}{3}x + c$

Solve each equation by completing the square.

19. $x^2 + 6x + 8 = 0$
20. $3x^2 + x - 2 = 0$
21. $3x^2 - 5x + 2 = 0$

22. $x^2 + 18 = 9x$
23. $x^2 - 14x + 19 = 0$
24. $x^2 + 16x - 7 = 0$

25. $2x^2 + 8x - 3 = 0$
26. $x^2 + x - 5 = 0$
27. $2x^2 - 10x + 5 = 0$

28. $x^2 + 3x + 6 = 0$
29. $2x^2 + 5x + 6 = 0$
30. $7x^2 + 6x + 2 = 0$

31. GEOMETRY When the dimensions of a cube are reduced by 4 inches on each side, the surface area of the new cube is 864 square inches. What were the dimensions of the original cube?

32. INVESTMENTS The amount of money $A$ in an account in which $P$ dollars is invested for 2 years is given by the formula $A = P(1 + r)^2$, where $r$ is the interest rate compounded annually. If an investment of $800 in the account grows to $882 in two years, at what interest rate was it invested?
6-5 Practice

The Quadratic Formula and the Discriminant

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.
b. Describe the number and type of roots.
c. Find the exact solutions by using the Quadratic Formula.

1. \(x^2 - 16x + 64 = 0\)  
2. \(x^2 = 3x\)  
3. \(9x^2 - 24x + 16 = 0\)

4. \(x^2 - 3x = 40\)  
5. \(3x^2 + 9x - 2 = 0\)  
6. \(2x^2 + 7x = 0\)

7. \(5x^2 - 2x + 4 = 0\)  
8. \(12x^2 - x - 6 = 0\)  
9. \(7x^2 + 6x + 2 = 0\)

10. \(12x^2 + 2x - 4 = 0\)  
11. \(6x^2 - 2x - 1 = 0\)  
12. \(x^2 + 3x + 6 = 0\)

13. \(4x^2 - 3x - 6 = 0\)  
14. \(16x^2 - 8x + 1 = 0\)  
15. \(2x^2 - 5x - 6 = 0\)

Solve each equation by using the method of your choice. Find exact solutions.

16. \(7x^2 - 5x = 0\)  
17. \(4x^2 - 9 = 0\)

18. \(3x^2 + 8x = 3\)  
19. \(x^2 - 21 = 4x\)

20. \(3x^2 - 13x + 4 = 0\)  
21. \(15x^2 + 22x = -8\)

22. \(x^2 - 6x + 3 = 0\)  
23. \(x^2 - 14x + 53 = 0\)

24. \(3x^2 = -54\)  
25. \(25x^2 - 20x - 6 = 0\)

26. \(4x^2 - 4x + 17 = 0\)  
27. \(8x - 1 = 4x^2\)

28. \(x^2 = 4x - 15\)  
29. \(4x^2 - 12x + 7 = 0\)

30. **GRAVITATION** The height \(h(t)\) in feet of an object \(t\) seconds after it is propelled straight up from the ground with an initial velocity of 60 feet per second is modeled by the equation \(h(t) = -16t^2 + 60t\). At what times will the object be at a height of 56 feet?

31. **STOPPING DISTANCE** The formula \(d = 0.05s^2 + 1.1s\) estimates the minimum stopping distance \(d\) in feet for a car traveling \(s\) miles per hour. If a car stops in 200 feet, what is the fastest it could have been traveling when the driver applied the brakes?
Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening.

1. \( y = -6(x + 2)^2 - 1 \)  
2. \( y = 2x^2 + 2 \)  
3. \( y = -4x^2 + 8x \)

4. \( y = x^2 + 10x + 20 \)  
5. \( y = 2x^2 + 12x + 18 \)  
6. \( y = 3x^2 - 6x + 5 \)

7. \( y = -2x^2 - 16x - 32 \)  
8. \( y = -3x^2 + 18x - 21 \)  
9. \( y = 2x^2 + 16x + 29 \)

Graph each function.

10. \( y = (x + 3)^2 - 1 \)  
11. \( y = -x^2 + 6x - 5 \)  
12. \( y = 2x^2 - 2x + 1 \)

Write an equation for the parabola with the given vertex that passes through the given point.

13. vertex: \((1, 3)\)  
   point: \((-2, -15)\)

14. vertex: \((-3, 0)\)  
   point: \((3, 18)\)

15. vertex: \((10, -4)\)  
   point: \((5, 6)\)

16. Write an equation for a parabola with vertex at \((4, 4)\) and \(x\)-intercept 6.

17. Write an equation for a parabola with vertex at \((-3, -1)\) and \(y\)-intercept 2.

18. **BASEBALL** The height \( h \) of a baseball \( t \) seconds after being hit is given by \( h(t) = -16t^2 + 80t + 3 \). What is the maximum height that the baseball reaches, and when does this occur?

19. **SCULPTURE** A modern sculpture in a park contains a parabolic arc that starts at the ground and reaches a maximum height of 10 feet after a horizontal distance of 4 feet. Write a quadratic function in vertex form that describes the shape of the outside of the arc, where \( y \) is the height of a point on the arc and \( x \) is its horizontal distance from the left-hand starting point of the arc.

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Graph each inequality.
1. \( y \leq x^2 + 4 \)
2. \( y > x^2 + 6x + 6 \)
3. \( y < 2x^2 - 4x - 2 \)

Use the graph of its related function to write the solutions of each inequality.
4. \( x^2 - 8x > 0 \)
5. \( -x^2 - 2x + 3 \geq 0 \)
6. \( x^2 - 9x + 14 \leq 0 \)

Solve each inequality algebraically.
7. \( x^2 - x - 20 > 0 \)
8. \( x^2 - 10x + 16 < 0 \)
9. \( x^2 + 4x + 5 \leq 0 \)
10. \( x^2 + 14x + 49 \geq 0 \)
11. \( x^2 - 5x > 14 \)
12. \( -x^2 - 15 \geq 8x \)
13. \( -x^2 + 5x - 7 \leq 0 \)
14. \( 9x^2 + 36x + 36 \leq 0 \)
15. \( 9x \leq 12x^2 \)
16. \( 4x^2 + 4x + 1 > 0 \)
17. \( 5x^2 + 10 \geq 27x \)
18. \( 9x^2 + 31x + 12 \leq 0 \)

19. FENCING Vanessa has 180 feet of fencing that she intends to use to build a rectangular play area for her dog. She wants the play area to enclose at least 1800 square feet. What are the possible widths of the play area?

20. BUSINESS A bicycle maker sold 300 bicycles last year at a profit of $300 each. The maker wants to increase the profit margin this year, but predicts that each $20 increase in profit will reduce the number of bicycles sold by 10. How many $20 increases in profit can the maker add in and expect to make a total profit of at least $100,000?
7-1 Practice

Polynomial Functions

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

1. \((3x^2 + 1)(2x^2 - 9)\)  
2. \(\frac{1}{5}a^3 - \frac{3}{5}a^2 + \frac{4}{5}a\)  
3. \(\frac{2}{m^3} + 3m - 12\)  
4. \(27 + 3xy^3 - 12x^2y^2 - 10y\)

Find \(p(-2)\) and \(p(3)\) for each function.

5. \(p(x) = x^3 - x^5\)  
6. \(p(x) = -7x^2 + 5x + 9\)  
7. \(p(x) = -x^5 + 4x^3\)

8. \(p(x) = 3x^3 - x^2 + 2x - 5\)  
9. \(p(x) = x^4 + \frac{1}{2}x^3 - \frac{1}{2}x\)  
10. \(p(x) = \frac{1}{3}x^3 + \frac{2}{3}x^2 + 3x\)

If \(p(x) = 3x^2 - 4\) and \(r(x) = 2x^2 - 5x + 1\), find each value.

11. \(p(8a)\)  
12. \(r(a^2)\)  
13. \(-5r(2a)\)

14. \(r(x + 2)\)  
15. \(p(x^2 - 1)\)  
16. \(5[p(x + 2)]\)

For each graph,

a. describe the end behavior,

b. determine whether it represents an odd-degree or an even-degree polynomial function, and

c. state the number of real zeroes.

17.  
18.  
19.  

20. WIND CHILL The function \(C(s) = 0.013s^2 - s - 7\) estimates the wind chill temperature \(C(s)\) at 0°F for wind speeds \(s\) from 5 to 30 miles per hour. Estimate the wind chill temperature at 0°F if the wind speed is 20 miles per hour.
Practice

Graphing Polynomial Functions

Complete each of the following.

a. Graph each function by making a table of values.

b. Determine consecutive values of \( x \) between which each real zero is located.

c. Estimate the \( x \)-coordinates at which the relative and relative minima occur.

1. \( f(x) = -x^3 + 3x^2 - 3 \)

2. \( f(x) = x^3 - 1.5x^2 - 6x + 1 \)

3. \( f(x) = 0.75x^4 + x^3 - 3x^2 + 4 \)

4. \( f(x) = x^4 + 4x^3 + 6x^2 + 4x - 3 \)

PRICES For Exercises 5 and 6, use the following information.

The Consumer Price Index (CPI) gives the relative price for a fixed set of goods and services. The CPI from September, 2000 to July, 2001 is shown in the graph.

Source: U. S. Bureau of Labor Statistics

5. Describe the turning points of the graph.

6. If the graph were modeled by a polynomial equation, what is the least degree the equation could have?

7. LABOR A town’s jobless rate can be modeled by \((1, 3.3), (2, 4.9), (3, 5.3), (4, 6.4), (5, 4.5), (6, 5.6), (7, 2.5), (8, 2.7)\). How many turning points would the graph of a polynomial function through these points have? Describe them.
### 7-3 Practice

#### Solving Equations Using Quadratic Techniques

Write each expression in quadratic form, if possible.

1. \(10b^4 + 3b^2 - 11\)  
2. \(-5x^8 + x^2 + 6\)  
3. \(28d^6 + 25d^3\)

4. \(4s^8 + 4s^4 + 7\)  
5. \(500x^4 - x^2\)  
6. \(8b^5 - 8b^3 - 1\)

7. \(32w^5 - 56w^3 + 8w\)  
8. \(e^{\frac{2}{3}} + 7e^{\frac{1}{3}} - 10\)  
9. \(x^5 + 29x^{\frac{1}{10}} + 2\)

Solve each equation.

10. \(y^4 - 7y^3 - 18y^2 = 0\)  
11. \(s^5 + 4s^4 - 32s^3 = 0\)

12. \(m^4 - 625 = 0\)  
13. \(n^4 - 49n^2 = 0\)

14. \(x^4 - 50x^2 + 49 = 0\)  
15. \(r^4 - 21r^2 + 80 = 0\)

16. \(4r^6 - 9r^4 = 0\)  
17. \(x^4 - 24 = -2x^2\)

18. \(d^4 = 16d^2 - 48\)  
19. \(t^3 - 343 = 0\)

20. \(x^\frac{1}{2} - 5x^\frac{1}{4} + 6 = 0\)  
21. \(x^\frac{4}{3} - 29x^{\frac{2}{3}} + 100 = 0\)

22. \(y^3 - 28y^\frac{3}{2} + 27 = 0\)  
23. \(n - 10\sqrt{n} + 25 = 0\)

24. \(w - 12\sqrt{w} + 27 = 0\)  
25. \(x - 2\sqrt{x} - 80 = 0\)

26. **PHYSICS** A proton in a magnetic field follows a path on a coordinate grid modeled by the function \(f(x) = x^4 - 2x^2 - 15\). What are the \(x\)-coordinates of the points on the grid where the proton crosses the \(x\)-axis?

27. **SURVEYING** Vista county is setting aside a large parcel of land to preserve it as open space. The county has hired Meghan's surveying firm to survey the parcel, which is in the shape of a right triangle. The longer leg of the triangle measures 5 miles less than the square of the shorter leg, and the hypotenuse of the triangle measures 13 miles less than twice the square of the shorter leg. The length of each boundary is a whole number. Find the length of each boundary.
Practice

7-4

The Remainder and Factor Theorems

Use synthetic substitution to find \( f(-3) \) and \( f(4) \) for each function.

1. \( f(x) = x^2 + 2x + 3 \)  
2. \( f(x) = x^2 - 5x + 10 \)
3. \( f(x) = x^2 - 5x - 4 \)  
4. \( f(x) = x^3 - x^2 - 2x + 3 \)
5. \( f(x) = x^3 + 2x^2 + 5 \)  
6. \( f(x) = x^3 - 6x^2 + 2x \)
7. \( f(x) = x^3 - 2x^2 - 2x + 8 \)  
8. \( f(x) = x^3 - x^2 + 4x - 4 \)
9. \( f(x) = x^3 + 3x^2 + 2x - 50 \)  
10. \( f(x) = x^4 + x^3 - 3x^2 - x + 12 \)
11. \( f(x) = x^4 - 2x^2 - x + 7 \)  
12. \( f(x) = 2x^4 - 3x^3 + 4x^2 - 2x + 1 \)
13. \( f(x) = 2x^4 - x^3 + 2x^2 - 26 \)  
14. \( f(x) = 3x^4 - 4x^3 + 3x^2 - 5x - 3 \)
15. \( f(x) = x^5 + 7x^3 - 4x - 10 \)  
16. \( f(x) = x^6 + 2x^5 - x^4 + x^3 - 9x^2 + 20 \)

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

17. \( x^3 + 3x^2 - 6x - 8; x - 2 \)  
18. \( x^3 + 7x^2 + 7x - 15; x - 1 \)
19. \( x^3 - 9x^2 + 27x - 27; x - 3 \)  
20. \( x^3 - x^2 - 8x + 12; x + 3 \)
21. \( x^3 + 5x^2 - 2x - 24; x - 2 \)  
22. \( x^3 - x^2 - 14x + 24; x + 4 \)
23. \( 3x^3 - 4x^2 - 17x + 6; x + 2 \)  
24. \( 4x^3 - 12x^2 - x + 3; x - 3 \)
25. \( 18x^3 + 9x^2 - 2x - 1; 2x + 1 \)  
26. \( 6x^3 + 5x^2 - 3x - 2; 3x - 2 \)
27. \( x^5 + x^4 - 5x^3 - 5x^2 + 4x + 4; x + 1 \)  
28. \( x^5 - 2x^4 + 4x^3 - 8x^2 - 5x + 10; x - 2 \)

29. **Population**  
The projected population in thousands for a city over the next several years can be estimated by the function \( P(x) = x^3 + 2x^2 - 8x + 520 \), where \( x \) is the number of years since 2000. Use synthetic substitution to estimate the population for 2005.

30. **Volume**  
The volume of water in a rectangular swimming pool can be modeled by the polynomial \( 2x^3 - 9x^2 + 7x + 6 \). If the depth of the pool is given by the polynomial \( 2x + 1 \), what polynomials express the length and width of the pool?
Solve each equation. State the number and type of roots.

1. \(-9x - 15 = 0\)

2. \(x^4 - 5x^2 + 4 = 0\)

3. \(x^5 = 81x\)

4. \(x^3 + x^2 - 3x - 3 = 0\)

5. \(x^3 + 6x + 20 = 0\)

6. \(x^4 - x^3 - x^2 - x - 2 = 0\)

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

7. \(f(x) = 4x^3 - 2x^2 + x + 3\)

8. \(p(x) = 2x^4 - 2x^3 + 2x^2 - x - 1\)

9. \(q(x) = 3x^4 + x^3 - 3x^2 + 7x + 5\)

10. \(h(x) = 7x^4 + 3x^3 - 2x^2 - x + 1\)

Find all the zeros of each function.

11. \(h(x) = 2x^3 + 3x^2 - 65x + 84\)

12. \(p(x) = x^3 - 3x^2 + 9x - 7\)

13. \(h(x) = x^3 - 7x^2 + 17x - 15\)

14. \(q(x) = x^4 + 50x^2 + 49\)

15. \(g(x) = x^4 + 4x^3 - 3x^2 - 14x - 8\)

16. \(f(x) = x^4 - 6x^3 + 6x^2 + 24x - 40\)

Write a polynomial function of least degree with integral coefficients that has the given zeros.

17. \(-5, 3i\)

18. \(-2, 3 + i\)

19. \(-1, 4, 3i\)

20. \(2, 5, 1 + i\)

21. **CRAFTS** Stephan has a set of plans to build a wooden box. He wants to reduce the volume of the box to 105 cubic inches. He would like to reduce the length of each dimension in the plan by the same amount. The plans call for the box to be 10 inches by 8 inches by 6 inches. Write and solve a polynomial equation to find out how much Stephen should take from each dimension.
7-6 Practice

Rational Zero Theorem

List all of the possible rational zeros of each function.

1. \( h(x) = x^3 - 5x^2 + 2x + 12 \)
2. \( s(x) = x^4 - 8x^3 + 7x - 14 \)

3. \( f(x) = 3x^5 - 5x^2 + x + 6 \)
4. \( p(x) = 3x^2 + x + 7 \)

5. \( g(x) = 5x^3 + x^2 - x + 8 \)
6. \( q(x) = 6x^5 + x^3 - 3 \)

Find all of the rational zeros of each function.

7. \( q(x) = x^3 + 3x^2 - 6x - 8 = 0 \)
8. \( v(x) = x^3 - 9x^2 + 27x - 27 \)

9. \( c(x) = x^3 - x^2 - 8x + 12 \)
10. \( f(x) = x^4 - 49x^2 \)

11. \( h(x) = x^3 - 7x^2 + 17x - 15 \)
12. \( b(x) = x^3 + 6x + 20 \)

13. \( f(x) = x^3 - 6x^2 + 4x - 24 \)
14. \( g(x) = 2x^3 + 3x^2 - 4x - 4 \)

15. \( h(x) = 2x^3 - 7x^2 - 21x + 54 = 0 \)
16. \( z(x) = x^4 - 3x^3 + 5x^2 - 27x - 36 \)

17. \( d(x) = x^4 + x^3 + 16 \)
18. \( n(x) = x^4 - 2x^3 - 3 \)

19. \( p(x) = 2x^4 - 7x^3 + 4x^2 + 7x - 6 \)
20. \( q(x) = 6x^4 + 29x^3 + 40x^2 + 7x - 12 \)

Find all of the zeros of each function.

21. \( f(x) = 2x^4 + 7x^3 - 2x^2 - 19x - 12 \)
22. \( q(x) = x^4 - 4x^3 + x^2 + 16x - 20 \)

23. \( h(x) = x^6 - 8x^3 \)
24. \( g(x) = x^6 - 1 \)

25. **TRAVEL** The height of a box that Joan is shipping is 3 inches less than the width of the box. The length is 2 inches more than twice the width. The volume of the box is 1540 in\(^3\). What are the dimensions of the box?

26. **GEOMETRY** The height of a square pyramid is 3 meters shorter than the side of its base. If the volume of the pyramid is 432 m\(^3\), how tall is it? Use the formula \( V = \frac{1}{3}Bh \).
Practice

Operations on Functions

Find \((f + g)(x), (f - g)(x), (f \cdot g)(x),\) and \((\frac{f}{g})(x)\) for each \(f(x)\) and \(g(x)\).

1. \(f(x) = 2x + 1\)
   \(g(x) = x - 3\)

2. \(f(x) = 8x^2\)
   \(g(x) = \frac{1}{x^2}\)

3. \(f(x) = x^2 + 7x + 12\)
   \(g(x) = x^2 - 9\)

For each set of ordered pairs, find \(f \circ g\) and \(g \circ f\) if they exist.

4. \(f = \{(−9, −1), (−1, 0), (3, 4)\}\)
   \(g = \{(0, −9), (−1, 3), (4, −1)\}\)

5. \(f = \{(-4, 3), (0, −2), (1, −2)\}\)
   \(g = \{(-2, 0), (3, 1)\}\)

6. \(f = \{(-4, −5), (0, 3), (1, 6)\}\)
   \(g = \{(6, 1), (−5, 0), (3, −4)\}\)

7. \(f = \{(0, −3), (1, −3), (6, 8)\}\)
   \(g = \{(8, 2), (−3, 0), (−3, 1)\}\)

Find \([g \circ h](x)\) and \([h \circ g](x)\).

8. \(g(x) = 3x\)
   \(h(x) = x - 4\)

9. \(g(x) = −8x\)
   \(h(x) = 2x + 3\)

10. \(g(x) = x + 6\)
    \(h(x) = 3x^2\)

11. \(g(x) = x + 3\)
    \(h(x) = 2x^2\)

12. \(g(x) = −2x\)
    \(h(x) = x^2 + 3x + 2\)

13. \(g(x) = x - 2\)
    \(h(x) = 3x^2 + 1\)

If \(f(x) = x^2, g(x) = 5x, \) and \(h(x) = x + 4\), find each value.

14. \(f[g(1)]\)

15. \(g[h(−2)]\)

16. \(h[f(4)]\)

17. \(f[h(−9)]\)

18. \(h[g(−3)]\)

19. \(g[f(8)]\)

20. \(h[f(20)]\)

21. \([f \circ (h \circ g)](−1)\)

22. \([f \circ (g \circ h)](4)\)

23. BUSINESS The function \(f(x) = 1000 - 0.01x^2\) models the manufacturing cost per item when \(x\) items are produced, and \(g(x) = 150 - 0.001x^2\) models the service cost per item.

Write a function \(C(x)\) for the total manufacturing and service cost per item.

24. MEASUREMENT The formula \(f = \frac{n}{12}\) converts inches \(n\) to feet \(f\), and \(m = \frac{f}{5280}\) converts feet to miles \(m\). Write a composition of functions that converts inches to miles.
7-8 Practice

Inverse Functions and Relations

Find the inverse of each relation.

1. \{ (0, 3), (4, 2), (5, -6) \}  
2. \{ (-5, 1), (-5, -1), (-5, 8) \}

3. \{ (-3, -7), (0, -1), (5, 9), (7, 13) \}  
4. \{ (8, -2), (10, 5), (12, 6), (14, 7) \}

5. \{ (-5, -4), (1, 2), (3, 4), (7, 8) \}  
6. \{ (-3, 9), (-2, 4), (0, 0), (1, 1) \}

Find the inverse of each function. Then graph the function and its inverse.

7. \( f(x) = \frac{3}{4}x \)  
8. \( g(x) = 3 + x \)  
9. \( y = 3x - 2 \)

Find the inverse of each function. Then graph the function and its inverse.

7. \( f(x) = \frac{3}{4}x \)  
8. \( g(x) = 3 + x \)  
9. \( y = 3x - 2 \)

Determine whether each pair of functions are inverse functions.

10. \( f(x) = x + 6 \)  
    \( g(x) = x - 6 \)

11. \( f(x) = -4x + 1 \)  
    \( g(x) = \frac{1}{4}(1 - x) \)

12. \( g(x) = 13x - 13 \)  
    \( h(x) = \frac{1}{13}x - 1 \)

13. \( f(x) = 2x \)  
    \( g(x) = -2x \)

14. \( f(x) = \frac{6}{7}x \)  
    \( g(x) = \frac{7}{6}x \)

15. \( g(x) = 2x - 8 \)  
    \( h(x) = \frac{1}{2}x + 4 \)

16. MEASUREMENT  The points (63, 121), (71, 180), (67, 140), (65, 108), and (72, 165) give the weight in pounds as a function of height in inches for 5 students in a class. Give the points for these students that represent height as a function of weight.

17. What will the new flooring cost the Cleary’s?

REMODELING  For Exercises 17 and 18, use the following information.

The Clearys are replacing the flooring in their 15 foot by 18 foot kitchen. The new flooring costs $17.99 per square yard. The formula \( f(x) = 9x \) converts square yards to square feet.

17. Find the inverse \( f^{-1}(x) \). What is the significance of \( f^{-1}(x) \) for the Clearys?

18. What will the new flooring cost the Cleary’s?
Graph each function. State the domain and range of each function.

1. \[ y = \sqrt{5x} \]
2. \[ y = -\sqrt{x - 1} \]
3. \[ y = 2\sqrt{x} + 2 \]
4. \[ y = \sqrt{3x - 4} \]
5. \[ y = \sqrt{x + 7} - 4 \]
6. \[ y = 1 - \sqrt{2x + 3} \]

Graph each inequality.

7. \[ y \geq -\sqrt{6x} \]
8. \[ y \leq \sqrt{x} - 5 + 3 \]
9. \[ y > -2\sqrt{3x + 2} \]

10. ROLLER COASTERS The velocity of a roller coaster as it moves down a hill is \[ v = \sqrt{v_0^2 + 64h} \], where \( v_0 \) is the initial velocity and \( h \) is the vertical drop in feet. If \( v = 70 \) feet per second and \( v_0 = 8 \) feet per second, find \( h \).

11. WEIGHT Use the formula \( d = \sqrt{\frac{3960^2 \cdot W_E}{W_s}} - 3960 \), which relates distance from Earth \( d \) in miles to weight. If an astronaut’s weight on Earth \( W_E \) is 148 pounds and in space \( W_s \) is 115 pounds, how far from Earth is the astronaut?
Practice

Midpoint and Distance Formulas

Find the midpoint of each line segment with endpoints at the given coordinates.

1. \((8, -3), (-6, -11)\)  
2. \((-14, 5), (10, 6)\)  
3. \((-7, -6), (1, -2)\)  
4. \((8, -2), (8, -8)\)  
5. \((9, -4), (1, -1)\)  
6. \((3, 3), (4, 9)\)  
7. \((4, -2), (3, -7)\)  
8. \((6, 7), (4, 4)\)  
9. \((-4, -2), (-8, 2)\)  
10. \((5, -2), (3, 7)\)  
11. \((-6, 3), (-5, -7)\)  
12. \((-9, -8), (8, 3)\)  
13. \((2.6, -4.7), (8.4, 2.5)\)  
14. \((\frac{-1}{3}, 6), (\frac{2}{3}, 4)\)  
15. \((-2.5, -4.2), (8.1, 4.2)\)  
16. \((\frac{1}{8}, \frac{1}{2}), (-\frac{5}{8}, -\frac{1}{2})\)

Find the distance between each pair of points with the given coordinates.

17. \((5, 2), (2, -2)\)  
18. \((-2, -4), (4, 4)\)  
19. \((-3, 8), (-1, -5)\)  
20. \((0, 1), (9, -6)\)  
21. \((-5, 6), (-6, 6)\)  
22. \((-3, 5), (12, -3)\)  
23. \((-2, -3), (9, 3)\)  
24. \((-9, -8), (-7, 8)\)  
25. \((9, 3), (9, -2)\)  
26. \((-1, -7), (0, 6)\)  
27. \((10, -3), (-2, -8)\)  
28. \((-0.5, -6), (1.5, 0)\)  
29. \(\left(\frac{2}{5}, \frac{3}{5}\right), \left(\frac{1}{5}, \frac{7}{5}\right)\)  
30. \((-4\sqrt{2}, -\sqrt{5}), (-5\sqrt{2}, 4\sqrt{5})\)

31. GEOMETRY Circle \(O\) has a diameter \(AB\). If \(A\) is at \((-6, -2)\) and \(B\) is at \((-3, 4)\), find the center of the circle and the length of its diameter.

32. GEOMETRY Find the perimeter of a triangle with vertices at \((1, -3), (-4, 9),\) and \((-2, 1)\).
8-2 Practice
Parabolas

Write each equation in standard form.

1. \( y = 2x^2 - 12x + 19 \)  
2. \( y = \frac{1}{2}x^2 + 3x + \frac{1}{2} \)  
3. \( y = -3x^2 - 12x - 7 \)

Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola.

4. \( y = (x - 4)^2 + 3 \)
5. \( x = -\frac{1}{3}y^2 + 1 \)
6. \( x = 3(y + 1)^2 - 3 \)

Write an equation for each parabola described below. Then draw the graph.

7. vertex \((0, -4)\),  
focus \((0, -3\frac{7}{8})\)
8. vertex \((-2, 1)\),  
directrix \(x = -3\)
9. vertex \((1, 3)\),  
axis of symmetry \(x = 1\),  
latus rectum: 2 units, \(a < 0\)

10. TELEVISION Write the equation in the form \( y = ax^2 \) for a satellite dish. Assume that the bottom of the upward-facing dish passes through \((0, 0)\) and that the distance from the bottom to the focus point is 8 inches.
8-3 Practice

Circles

Write an equation for the circle that satisfies each set of conditions.

1. center (−4, 2), radius 8 units
2. center (0, 0), radius 4 units

3. center \( \left( -\frac{1}{4}, -\sqrt{3} \right) \), radius \( 5\sqrt{2} \) units
4. center (2.5, 4.2), radius 0.9 unit

5. endpoints of a diameter at (−2, −9) and (0, −5)
6. center at (−9, −12), passes through (−4, −5)
7. center at (−6, 5), tangent to x-axis

Find the center and radius of the circle with the given equation. Then graph the circle.

8. \( (x + 3)^2 + y^2 = 16 \)
9. \( 3x^2 + 3y^2 = 12 \)
10. \( x^2 + y^2 + 2x + 6y = 26 \)

WEATHER For Exercises 14 and 15, use the following information.

On average, the circular eye of a hurricane is about 15 miles in diameter. Gale winds can affect an area up to 300 miles from the storm’s center. In 1992, Hurricane Andrew devastated southern Florida. A satellite photo of Andrew’s landfall showed the center of its eye on one coordinate system could be approximated by the point (80, 26).

14. Write an equation to represent a possible boundary of Andrew’s eye.

15. Write an equation to represent a possible boundary of the area affected by gale winds.
8-4 Practice

Ellipses

Write an equation for each ellipse.

1. \[
\begin{aligned}
&y^2 + \frac{(x-4)^2}{4} = 1 \\
&y = \frac{\sqrt{3}}{2}x - 3
\end{aligned}
\]

2. \[
\begin{aligned}
&y^2 + \frac{(x-1)^2}{9} = 1 \\
&y = \frac{\sqrt{5}}{3}x - 2
\end{aligned}
\]

3. \[
\begin{aligned}
&y^2 + \frac{(x+2)^2}{4} = 1 \\
&y = \frac{\sqrt{5}}{2}x + 1
\end{aligned}
\]

Write an equation for the ellipse that satisfies each set of conditions.

4. endpoints of major axis at \((-9, 0)\) and \((9, 0)\), endpoints of minor axis at \((0, 3)\) and \((0, -3)\)

5. endpoints of major axis at \((4, 2)\) and \((4, -8)\), endpoints of minor axis at \((1, -3)\) and \((7, -3)\)

6. major axis 20 units long and parallel to \(x\)-axis, minor axis 10 units long, center at \((2, 1)\)

7. major axis 10 units long, minor axis 6 units long and parallel to \(x\)-axis, center at \((2, -4)\)

8. major axis 16 units long, center at \((0, 0)\), foci at \((0, 2\sqrt{15})\) and \((0, -2\sqrt{15})\)

9. endpoints of minor axis at \((0, 2)\) and \((0, -2)\), foci at \((-4, 0)\) and \((4, 0)\)

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

10. \[
\frac{y^2}{16} + \frac{x^2}{9} = 1
\]

11. \[
\frac{(y - 1)^2}{36} + \frac{(x - 3)^2}{1} = 1
\]

12. \[
\frac{(x + 4)^2}{49} + \frac{(y + 3)^2}{25} = 1
\]

13. **SPORTS** An ice skater traces two congruent ellipses to form a figure eight. Assume that the center of the first loop is at the origin, with the second loop to its right. Write an equation to model the first loop if its major axis (along the \(x\)-axis) is 12 feet long and its minor axis is 6 feet long. Write another equation to model the second loop.
Write an equation for each hyperbola.

1. 

Write an equation for the hyperbola that satisfies each set of conditions.

4. vertices (0, 7) and (0, −7), conjugate axis of length 18 units

5. vertices (1, −1) and (1, −9), conjugate axis of length 6 units

6. vertices (−5, 0) and (5, 0), foci ($±\sqrt{26}$, 0)

7. vertices (1, 1) and (1, −3), foci (1, $\pm 1 ± \sqrt{5}$)

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

8. $\frac{y^2}{16} - \frac{x^2}{4} = 1$

9. $\frac{(y - 2)^2}{1} - \frac{(x - 1)^2}{4} = 1$

10. $\frac{(y + 2)^2}{4} - \frac{(x - 3)^2}{4} = 1$

11. **ASTRONOMY** Astronomers use special X-ray telescopes to observe the sources of celestial X rays. Some X-ray telescopes are fitted with a metal mirror in the shape of a hyperbola, which reflects the X rays to a focus. Suppose the vertices of such a mirror are located at (−3, 0) and (3, 0), and one focus is located at (5, 0). Write an equation that models the hyperbola formed by the mirror.
Practice

Conic Sections

Write each equation in standard form. State whether the graph of the equation is a **parabola**, **circle**, **ellipse**, or **hyperbola**. Then graph the equation.

1. \(y^2 = -3x\)  
2. \(x^2 + y^2 + 6x = 7\)  
3. \(5x^2 - 6y^2 - 30x - 12y = -9\)

4. \(196y^2 = 1225 - 100x^2\)  
5. \(3x^2 = 9 - 3y^2 - 6y\)  
6. \(9x^2 + y^2 + 54x - 6y = -81\)

Without writing the equation in standard form, state whether the graph of each equation is a **parabola**, **circle**, **ellipse**, or **hyperbola**.

7. \(6x^2 + 6y^2 = 36\)  
8. \(4x^2 - y^2 = 16\)  
9. \(9x^2 + 16y^2 - 64y - 80 = 0\)

10. \(5x^2 + 5y^2 - 45 = 0\)  
11. \(x^2 + 2x = y\)  
12. \(4y^2 - 36x^2 + 4x - 144 = 0\)

13. **ASTRONOMY** A satellite travels in an hyperbolic orbit. It reaches the vertex of its orbit at \((5, 0)\) and then travels along a path that gets closer and closer to the line \(y = \frac{2}{5}x\).

Write an equation that describes the path of the satellite if the center of its hyperbolic orbit is at \((0, 0)\).
8-7 Practice
Solving Quadratic Systems

Find the exact solution(s) of each system of equations.

1. \((x - 2)^2 + y^2 = 5\) \(x - y = 1\)  
2. \(x = 2(y + 1)^2 - 6\) \(x + y = 3\)  
3. \(y^2 - 3x^2 = 6\) \(y = 2x - 1\)  
4. \(x^2 + 2y^2 = 1\) \(y = -x + 1\)  

5. \(4y^2 - 9x^2 = 36\) \(4x^2 - 9y^2 = 36\)  
6. \(y = x^2 - 3\) \(x^2 + y^2 = 9\)  
7. \(x^2 + y^2 = 25\) \(4y = 3x\)  
8. \(y^2 = 10 - 6x^2\) \(4y^2 = 40 - 2x^2\)  

9. \(x^2 + y^2 = 25\) \(x = 3y - 5\)  
10. \(4x^2 + 9y^2 = 36\) \(2x^2 - 9y^2 = 18\)  
11. \(x = -(y - 3)^2 + 2\) \(x = (y - 3)^2 + 3\)  
12. \(\frac{x^2}{9} - \frac{y^2}{16} = 1\) \(x^2 + y^2 = 9\)  

13. \(25x^2 + 4y^2 = 100\) \(x = -\frac{5}{2}\)  
14. \(x^2 + y^2 = 4\) \(\frac{x^2}{4} + \frac{y^2}{8} = 1\)  
15. \(x^2 - y^2 = 3\) \(y^2 - x^2 = 3\)  

16. \(\frac{x^2}{7} + \frac{y^2}{7} = 1\) \(3x^2 - y^2 = 9\)  
17. \(x + 2y = 3\) \(x^2 + y^2 = 9\)  
18. \(x^2 + y^2 = 64\) \(x^2 - y^2 = 8\)  

Solve each system of inequalities by graphing.

19. \(y \geq x^2\) \(y > -x + 2\)  
20. \(x^2 + y^2 < 36\) \(x^2 + y^2 \geq 16\)  
21. \(\frac{(y - 3)^2}{16} + \frac{(x + 2)^2}{4} \leq 1\) \((x + 1)^2 + (y - 2)^2 \leq 4\)  

22. **GEOMETRY** The top of an iron gate is shaped like half an ellipse with two congruent segments from the center of the ellipse to the ellipse as shown. Assume that the center of the ellipse is at \((0, 0)\). If the ellipse can be modeled by the equation \(x^2 + 4y^2 = 4\) for \(y \geq 0\) and the two congruent segments can be modeled by \(y = \frac{\sqrt{3}}{2}x\) and \(y = -\frac{\sqrt{3}}{2}x\), what are the coordinates of points \(A\) and \(B\)?
9-1 Practice

Multiplying and Dividing Rational Expressions

Simplify each expression.

1. \( \frac{9a^2b^3}{27a^4b^4c} \)

2. \( \frac{(2m^3n^2)^3}{-18m^5n^4} \)

3. \( \frac{10y^2 + 15y}{35y^2 - 5y} \)

4. \( \frac{2k^2 - k - 15}{k^2 - 9} \)

5. \( \frac{25 - v^2}{3v^2 - 13v - 10} \)

6. \( \frac{x^4 + x^3 - 2x^2}{x^4 - x^3} \)

7. \( \frac{-2u^3v}{15x^2} \cdot \frac{25x^3}{14u^2y^2} \)

8. \( \frac{a + y}{6} \cdot \frac{4}{y + a} \)

9. \( \frac{n^5}{n - 6} \cdot \frac{n^2 - 6n}{n^8} \)

10. \( \frac{a - y}{w + n} \cdot \frac{w^2 - n^2}{y - a} \)

11. \( \frac{x^2 - 5x - 24}{6x + 2x^2} \cdot \frac{5x^2}{8 - x} \)

12. \( \frac{x - 5}{10x - 2} \cdot \frac{25x^2 - 1}{x^2 - 10x + 25} \)

13. \( \frac{a^5y^3}{wy^7} \div \frac{a^3w^2}{w^5y^2} \)

14. \( \frac{(2xy)^3}{w^2} \cdot \frac{24x^2}{w^5} \)

15. \( \frac{x + y}{6} \div \frac{x^2 - y^2}{3} \)

16. \( \frac{3x + 6}{x^2 - 9} \div \frac{6x^2 + 12x}{4x + 12} \)

17. \( \frac{2s^2 - 7s - 15}{(s + 4)^2} \div \frac{s^2 - 10s + 25}{s + 4} \)

18. \( \frac{9 - a^2}{a^2 + 5a + 6} \div \frac{2a - 6}{5a + 10} \)

19. \( \frac{2x + 1}{x} \div \frac{4 - x}{x} \)

20. \( \frac{x^2 - 9}{4} \div \frac{3 - x}{8} \)

21. \( \frac{x^3 + 2^3}{x^2 - 2x} \div \frac{(x + 2)^3}{x^2 + 4x + 4} \)

22. GEOMETRY A right triangle with an area of \( x^2 - 4 \) square units has a leg that measures \( 2x + 4 \) units. Determine the length of the other leg of the triangle.

23. GEOMETRY A rectangular pyramid has a base area of \( \frac{x^2 + 3x - 10}{2x} \) square centimeters and a height of \( \frac{x^2 - 3x}{x^2 - 5x + 6} \) centimeters. Write a rational expression to describe the volume of the rectangular pyramid.
9-2

**Practice**

**Adding and Subtracting Rational Expressions**

Find the LCM of each set of polynomials.

1. $x^2y, xy^3$
2. $a^2b^3c, abc^4$
3. $x + 1, x + 3$
4. $g - 1, g^2 + 3g - 4$
5. $2r + 2, r^2 + r, r + 1$
6. $3, 4w + 2, 4w^2 - 1$
7. $x^2 + 2x - 8, x + 4$
8. $x^2 - x - 6, x^2 + 6x + 8$
9. $d^2 + 6d + 9, 2(d^2 - 9)$

Simplify each expression.

10. $\frac{5}{6ab} - \frac{7}{8a}$
11. $\frac{5}{12x^4y} - \frac{1}{5x^2y^3}$
12. $\frac{1}{6c^2d} + \frac{3}{4cd^3}$
13. $\frac{4m}{3mn} + 2$
14. $2x - 5 - \frac{x - 8}{x + 4}$
15. $\frac{4}{a - 3} + \frac{9}{a - 5}$
16. $\frac{16}{x^2 - 16} + \frac{2}{x + 4}$
17. $\frac{2 - 5m}{m - 9} + \frac{4m - 5}{9 - m}$
18. $\frac{y - 5}{y^2 - 3y - 10} + \frac{y}{y^2 + y - 2}$
19. $\frac{5}{2x - 12} - \frac{20}{x^2 - 4x - 12}$
20. $\frac{2p - 3}{p^2 - 5p + 6} - \frac{5}{p^2 - 9}$
21. $\frac{1}{5n} - \frac{3}{4} + \frac{7}{10n}$
22. $\frac{2a}{a - 3} - \frac{2a}{a + 3} + \frac{36}{a^2 - 9}$
23. $\frac{2}{x - y} + \frac{1}{x + y}$
24. $\frac{1}{r} - \frac{1}{r + 2}$

25. **GEOMETRY** The expressions $\frac{5x}{2}, \frac{20}{x + 4}$, and $\frac{10}{x - 4}$ represent the lengths of the sides of a triangle. Write a simplified expression for the perimeter of the triangle.

26. **KAYAKING** Mai is kayaking on a river that has a current of 2 miles per hour. If $r$ represents her rate in calm water, then $r + 2$ represents her rate with the current, and $r - 2$ represents her rate against the current. Mai kayaks 2 miles downstream and then back to her starting point. Use the formula for time, $t = \frac{d}{r}$, where $d$ is the distance, to write a simplified expression for the total time it takes Mai to complete the trip.
Determine the equations of any vertical asymptotes and the values of $x$ for any holes in the graph of each rational function.

1. $f(x) = \frac{6}{x^2 + 3x - 10}$

2. $f(x) = \frac{x - 7}{x^2 - 10x + 21}$

3. $f(x) = \frac{x - 2}{x^2 + 4x + 4}$

4. $f(x) = \frac{x^2 - 100}{x + 10}$

5. $f(x) = \frac{x^2 - 2x - 24}{x - 6}$

6. $f(x) = \frac{x^2 + 9x + 20}{x + 5}$

Graph each rational function.

7. $f(x) = \frac{-4}{x - 2}$

8. $f(x) = \frac{x - 3}{x - 2}$

9. $f(x) = \frac{3x}{(x + 3)^2}$

10. **PAINTING** Working alone, Tawa can give the shed a coat of paint in 6 hours. It takes her father $x$ hours working alone to give the shed a coat of paint. The equation $f(x) = \frac{6 + x}{6x}$ describes the portion of the job Tawa and her father working together can complete in 1 hour. Graph $f(x) = \frac{6 + x}{6x}$ for $x \geq 0, y \geq 0$. If Tawa's father can complete the job in 4 hours alone, what portion of the job can they complete together in 1 hour?

11. **LIGHT** The relationship between the illumination an object receives from a light source of $I$ foot-candles and the square of the distance $d$ in feet of the object from the source can be modeled by $I(d) = \frac{4500}{d^2}$. Graph the function $I(d) = \frac{4500}{d^2}$ for $0 \leq I \leq 80$ and $0 \leq d \leq 80$. What is the illumination in foot-candles that the object receives at a distance of 20 feet from the light source?
State whether each equation represents a direct, joint, or inverse variation. Then name the constant of variation.

1. \( u = 8wz \)  
2. \( p = 4s \)  
3. \( L = \frac{5}{k} \)  
4. \( xy = 4.5 \)

5. \( \frac{C}{d} = \pi \)  
6. \( 2d = mn \)  
7. \( \frac{1.25}{g} = h \)  
8. \( y = \frac{3}{4x} \)

Find each value.

9. If \( y \) varies directly as \( x \) and \( y = 8 \) when \( x = 2 \), find \( y \) when \( x = 6 \).

10. If \( y \) varies directly as \( x \) and \( y = -16 \) when \( x = 6 \), find \( x \) when \( y = -4 \).

11. If \( y \) varies directly as \( x \) and \( y = 132 \) when \( x = 11 \), find \( y \) when \( x = 33 \).

12. If \( y \) varies directly as \( x \) and \( y = 7 \) when \( x = 1.5 \), find \( y \) when \( x = 4 \).

13. If \( y \) varies jointly as \( x \) and \( z \) and \( y = 24 \) when \( x = 2 \) and \( z = 1 \), find \( y \) when \( x = 12 \) and \( z = 2 \).

14. If \( y \) varies jointly as \( x \) and \( z \) and \( y = 60 \) when \( x = 3 \) and \( z = 4 \), find \( y \) when \( x = 6 \) and \( z = 8 \).

15. If \( y \) varies jointly as \( x \) and \( z \) and \( y = 12 \) when \( x = -2 \) and \( z = 3 \), find \( y \) when \( x = 4 \) and \( z = -1 \).

16. If \( y \) varies inversely as \( x \) and \( y = 16 \) when \( x = 4 \), find \( y \) when \( x = 3 \).

17. If \( y \) varies inversely as \( x \) and \( y = 3 \) when \( x = 5 \), find \( x \) when \( y = 2.5 \).

18. If \( y \) varies inversely as \( x \) and \( y = -18 \) when \( x = 6 \), find \( y \) when \( x = 5 \).

19. If \( y \) varies directly as \( x \) and \( y = 5 \) when \( x = 0.4 \), find \( x \) when \( y = 37.5 \).

20. **GASES** The volume \( V \) of a gas varies inversely as its pressure \( P \). If \( V = 80 \) cubic centimeters when \( P = 2000 \) millimeters of mercury, find \( V \) when \( P = 320 \) millimeters of mercury.

21. **SPRINGS** The length \( S \) that a spring will stretch varies directly with the weight \( F \) that is attached to the spring. If a spring stretches 20 inches with 25 pounds attached, how far will it stretch with 15 pounds attached?

22. **GEOMETRY** The area \( A \) of a trapezoid varies jointly as its height and the sum of its bases. If the area is 480 square meters when the height is 20 meters and the bases are 28 meters and 20 meters, what is the area of a trapezoid when its height is 8 meters and its bases are 10 meters and 15 meters?
9-5 Practice

Classes of Functions

Identify the type of function represented by each graph.

1. [Graph 1]
2. [Graph 2]
3. [Graph 3]

Match each graph with an equation below.
A. \( y = |2x + 1| \)  
B. \( y = \lfloor 2x + 1 \rfloor \)  
C. \( y = \frac{x - 3}{2} \)  
D. \( y = \sqrt{-x} \)

Identify the type of function represented by each equation. Then graph the equation.
7. \( y = -3 \)
8. \( y = 2x^2 + 1 \)
9. \( y = \frac{x^2 + 5x + 6}{x + 2} \)

10. BUSINESS A startup company uses the function \( P = 1.3x^2 + 3x - 7 \) to predict its profit or loss during its first 7 years of operation. Describe the shape of the graph of the function.

11. PARKING A parking lot charges $10 to park for the first day or part of a day. After that, it charges an additional $8 per day or part of a day. Describe the graph and find the cost of parking for \( 6\frac{1}{2} \) days.
9-6 Practice

Solving Rational Equations and Inequalities

Solve each equation or inequality. Check your solutions.

1. \( \frac{12}{x} + \frac{3}{4} = \frac{3}{2} \)

2. \( \frac{x}{x - 1} - 1 = \frac{x}{2} \)

3. \( \frac{p + 10}{p^2 - 2} = \frac{4}{p} \)

4. \( \frac{s}{s + 2} + s = \frac{5s + 8}{s + 2} \)

5. \( \frac{5}{y - 5} = \frac{y}{y - 5} - 1 \)

6. \( \frac{1}{3x - 2} + \frac{5}{x} = 0 \)

7. \( \frac{5}{t} < \frac{9}{2t + 1} \)

8. \( \frac{1}{2h} + \frac{5}{h} = \frac{3}{h - 1} \)

9. \( \frac{4}{w - 2} = \frac{-1}{w + 3} \)

10. \( 5 - \frac{3}{a} < \frac{7}{a} \)

11. \( \frac{4}{5x} + \frac{1}{10} < \frac{3}{2x} \)

12. \( 8 + \frac{3}{y} > \frac{19}{y} \)

13. \( \frac{4}{p} + \frac{1}{3p} < \frac{1}{5} \)

14. \( \frac{6}{x - 1} = \frac{4}{x - 2} + \frac{2}{x + 1} \)

15. \( g + \frac{g}{g - 2} = \frac{2}{g - 2} \)

16. \( b + \frac{2b}{b - 1} = 1 - \frac{b - 3}{b - 1} \)

17. \( 2 = \frac{x + 2}{x - 3} + \frac{x - 2}{x - 6} \)

18. \( 5 - \frac{3d + 2}{d - 1} = \frac{2d - 4}{d + 2} \)

19. \( \frac{1}{n + 2} + \frac{1}{n - 2} = \frac{3}{n^2 - 4} \)

20. \( \frac{c + 1}{c - 3} = 4 - \frac{12}{c^2 - 2c - 3} \)

21. \( \frac{3}{k - 3} + \frac{4}{k - 4} = \frac{25}{k^2 - 7k + 12} \)

22. \( \frac{4v}{v - 1} - \frac{5v}{v - 2} = \frac{2}{v^2 - 3v + 2} \)

23. \( \frac{y}{y + 2} + \frac{7}{y - 5} = \frac{14}{y^2 - 3y - 10} \)

24. \( \frac{x^2 + 4}{x^2 - 4} + \frac{x}{2 - x} = \frac{2}{x + 2} \)

25. \( \frac{r}{r + 4} + \frac{4}{r - 4} = \frac{r^2 + 16}{r^2 - 16} \)

26. \( 3 = \frac{6a - 1}{2a + 7} + \frac{22}{a + 5} \)

27. BASKETBALL Kiana has made 9 of 19 free throws so far this season. Her goal is to make 60% of her free throws. If Kiana makes her next \( x \) free throws in a row, the function \( f(x) = \frac{9 + x}{19 + x} \) represents Kiana’s new ratio of free throws made. How many successful free throws in a row will raise Kiana’s percent made to 60%?

28. OPTICS The lens equation \( \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \) relates the distance \( p \) of an object from a lens, the distance \( q \) of the image of the object from the lens, and the focal length \( f \) of the lens. What is the distance of an object from a lens if the image of the object is 5 centimeters from the lens and the focal length of the lens is 4 centimeters?
Practice

Exponential Functions

Sketch the graph of each function. Then state the function’s domain and range.

1. \( y = 1.5(2)^x \)
2. \( y = 4(3)^x \)
3. \( y = 3(0.5)^x \)

Determine whether each function represents exponential growth or decay.

4. \( y = 5(0.6)^x \)
5. \( y = 0.1(2)^x \)
6. \( y = 5 \cdot 4^{-x} \)

Write an exponential function whose graph passes through the given points.

7. \((0, 1)\) and \((-1, 4)\)
8. \((0, 2)\) and \((1, 10)\)
9. \((0, -3)\) and \((1, -1.5)\)

10. \((0, 0.8)\) and \((1, 1.6)\)
11. \((0, -0.4)\) and \((2, -10)\)
12. \((0, \pi)\) and \((3, 8\pi)\)

Simplify each expression.

13. \((2\sqrt{2})\sqrt{8}\)
14. \((n\sqrt{3})\sqrt{75}\)
15. \(y\sqrt{6} \cdot y^5\sqrt{6}\)
16. \(13\sqrt{6} \cdot 13\sqrt{24}\)
17. \(n^3 \div n^\pi\)
18. \(125\sqrt{14} \div 5\sqrt{11}\)

Solve each equation or inequality. Check your solution.

19. \(3^{6x} - 5 > 81\)
20. \(7^{6x} = 7^{2x - 20}\)
21. \(3^{6n} - 5 < 9^{4n} - 3\)
22. \(9^{2x} - 1 = 27^x + 4\)
23. \(2^{3n} - 1 \geq \left(\frac{1}{8}\right)^n\)
24. \(16^{4n} - 1 = 128^{2n} + 1\)

BIOLOGY For Exercises 25 and 26, use the following information.

The initial number of bacteria in a culture is 12,000. The number after 3 days is 96,000.

25. Write an exponential function to model the population \(y\) of bacteria after \(x\) days.
26. How many bacteria are there after 6 days?

EDUCATION A college with a graduating class of 4000 students in the year 2002 predicts that it will have a graduating class of 4862 in 4 years. Write an exponential function to model the number of students \(y\) in the graduating class \(t\) years after 2002.
10-2 Practice

Logarithms and Logarithmic Functions

Write each equation in logarithmic form.

1. \(5^3 = 125\)  
2. \(7^0 = 1\)  
3. \(3^4 = 81\)

4. \(3^{-4} = \frac{1}{81}\)  
5. \(\left(\frac{1}{4}\right)^3 = \frac{1}{64}\)  
6. \(7776^{\frac{1}{6}} = 6\)

Write each equation in exponential form.

7. \(\log_6 216 = 3\)  
8. \(\log_2 64 = 6\)  
9. \(\log_3 \frac{1}{81} = -4\)

10. \(\log_{10} 0.00001 = -5\)  
11. \(\log_{25} 5 = \frac{1}{2}\)  
12. \(\log_{32} 8 = \frac{3}{5}\)

Evaluate each expression.

13. \(\log_3 81\)  
14. \(\log_{10} 0.0001\)  
15. \(\log_2 \frac{1}{16}\)  
16. \(\log_3 27\)

17. \(\log_9 1\)  
18. \(\log_8 4\)  
19. \(\log_7 \frac{1}{49}\)  
20. \(\log_6 6^4\)

21. \(\log_3 \frac{1}{3}\)  
22. \(\log_4 \frac{1}{256}\)  
23. \(\log_9 9^{(n + 1)}\)  
24. \(2^{\log_2 32}\)

Solve each equation or inequality. Check your solutions.

25. \(\log_{10} n = -3\)  
26. \(\log_4 x > 3\)  
27. \(\log_4 x = \frac{3}{2}\)

28. \(\log_5 x = -3\)  
29. \(\log_7 q < 0\)  
30. \(\log_6 (2y + 8) \geq 2\)

31. \(\log_y 16 = -4\)  
32. \(\log_n \frac{1}{8} = -3\)  
33. \(\log_b 1024 = 5\)

34. \(\log_8 (3x + 7) < \log_8 (7x + 4)\)  
35. \(\log_7 (8x + 20) = \log_7 (x + 6)\)  
36. \(\log_3 (x^2 - 2) = \log_3 x\)

37. SOUND Sounds that reach levels of 130 decibels or more are painful to humans. What is the relative intensity of 130 decibels?

38. INVESTING Maria invests $1000 in a savings account that pays 8% interest compounded annually. The value of the account \(A\) at the end of five years can be determined from the equation \(\log A = \log[1000(1 + 0.08)^5]\). Find the value of \(A\) to the nearest dollar.
Use \( \log_{10} 5 = 0.6990 \) and \( \log_{10} 7 = 0.8451 \) to approximate the value of each expression.

1. \( \log_{10} 35 \)
2. \( \log_{10} 25 \)
3. \( \log_{10} \frac{7}{5} \)
4. \( \log_{10} \frac{5}{7} \)
5. \( \log_{10} 245 \)
6. \( \log_{10} 175 \)
7. \( \log_{10} 0.2 \)
8. \( \log_{10} \frac{25}{7} \)

Solve each equation. Check your solutions.

9. \( \log_{7} n = \frac{2}{3} \log_{7} 8 \)
10. \( \log_{10} u = \frac{3}{2} \log_{10} 4 \)
11. \( \log_{6} x + \log_{6} 9 = \log_{6} 54 \)
12. \( \log_{8} 48 - \log_{8} w = \log_{8} 4 \)
13. \( \log_{9} (3u + 14) - \log_{9} 5 = \log_{9} 2u \)
14. \( 4 \log_{2} x + \log_{2} 5 = \log_{2} 405 \)
15. \( \log_{3} y = -\log_{3} 16 + \frac{1}{3} \log_{3} 64 \)
16. \( \log_{2} d = 5 \log_{2} 2 - \log_{2} 8 \)
17. \( \log_{10} (3m - 5) + \log_{10} m = \log_{10} 2 \)
18. \( \log_{10} (b + 3) + \log_{10} b = \log_{10} 4 \)
19. \( \log_{8} (t + 10) - \log_{8} (t - 1) = \log_{8} 12 \)
20. \( \log_{3} (a + 3) + \log_{3} (a + 2) = \log_{3} 6 \)
21. \( \log_{10} (r + 4) - \log_{10} r = \log_{10} (r + 1) \)
22. \( \log_{4} (x^2 - 4) - \log_{4} (x + 2) = \log_{4} 1 \)
23. \( \log_{10} 4 + \log_{10} w = 2 \)
24. \( \log_{8} (n - 3) + \log_{8} (n + 4) = 1 \)
25. \( 3 \log_{5} (x^2 + 9) - 6 = 0 \)
26. \( \log_{16} (9x + 5) - \log_{16} (x^2 - 1) = \frac{1}{2} \)
27. \( \log_{6} (2x - 5) + 1 = \log_{6} (7x + 10) \)
28. \( \log_{2} (5y + 2) - 1 = \log_{2} (1 - 2y) \)
29. \( \log_{10} (c^2 - 1) - 2 = \log_{10} (c + 1) \)
30. \( \log_{7} x + 2 \log_{7} x - \log_{7} 3 = \log_{7} 72 \)

31. **SOUND** The loudness \( L \) of a sound in decibels is given by \( L = 10 \log_{10} R \), where \( R \) is the sound's relative intensity. If the intensity of a certain sound is tripled, by how many decibels does the sound increase?

32. **EARTHQUAKES** An earthquake rated at 3.5 on the Richter scale is felt by many people, and an earthquake rated at 4.5 may cause local damage. The Richter scale magnitude reading \( m \) is given by \( m = \log_{10} x \), where \( x \) represents the amplitude of the seismic wave causing ground motion. How many times greater is the amplitude of an earthquake that measures 4.5 on the Richter scale than one that measures 3.5?
Practice

Common Logarithms

Use a calculator to evaluate each expression to four decimal places.

1. \( \log 101 \)
2. \( \log 2.2 \)
3. \( \log 0.05 \)

Use the formula \( \text{pH} = -\log[H^+] \) to find the pH of each substance given its concentration of hydrogen ions.

4. milk: \( [H^+] = 2.51 \times 10^{-7} \) mole per liter
5. acid rain: \( [H^+] = 2.51 \times 10^{-6} \) mole per liter
6. black coffee: \( [H^+] = 1.0 \times 10^{-5} \) mole per liter
7. milk of magnesia: \( [H^+] = 3.16 \times 10^{-11} \) mole per liter

Solve each equation or inequality. Round to four decimal places.

8. \( 2^x < 25 \)
9. \( 5^a = 120 \)
10. \( 6^z = 45.6 \)
11. \( 9^m \geq 100 \)
12. \( 3.5^x = 47.9 \)
13. \( 8.2^y = 64.5 \)
14. \( 2^b + 1 \leq 7.31 \)
15. \( 4^2x = 27 \)
16. \( 2^a - 4 = 82.1 \)
17. \( 9^{x - 2} > 38 \)
18. \( 5^w + 3 = 17 \)
19. \( 30^{x^3} = 50 \)
20. \( 5^{x^2 - 3} = 72 \)
21. \( 4^{2x} = 9^x + 1 \)
22. \( 2^n + 1 = 5^{2n - 1} \)

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.

23. \( \log_5 12 \)
24. \( \log_8 32 \)
25. \( \log_{11} 9 \)
26. \( \log_2 18 \)
27. \( \log_9 6 \)
28. \( \log_7 \sqrt{8} \)

29. HORTICULTURE Siberian irises flourish when the concentration of hydrogen ions \( [H^+] \) in the soil is not less than \( 1.58 \times 10^{-8} \) mole per liter. What is the pH of the soil in which these irises will flourish?

30. ACIDITY The pH of vinegar is 2.9 and the pH of milk is 6.6. How many times greater is the hydrogen ion concentration of vinegar than of milk?

31. BIOLOGY There are initially 1000 bacteria in a culture. The number of bacteria doubles each hour. The number of bacteria \( N \) present after \( t \) hours is \( N = 1000(2)^t \). How long will it take the culture to increase to 50,000 bacteria?

32. SOUND An equation for loudness \( L \) in decibels is given by \( L = 10 \log R \), where \( R \) is the sound’s relative intensity. An air-raid siren can reach 150 decibels and jet engine noise can reach 120 decibels. How many times greater is the relative intensity of the air-raid siren than that of the jet engine noise?
Practice

**Base e and Natural Logarithms**

Use a calculator to evaluate each expression to four decimal places.

1. $e^{1.5}$
2. $\ln 8$
3. $\ln 3.2$
4. $e^{-0.6}$
5. $e^{4.2}$
6. $\ln 1$
7. $e^{-2.5}$
8. $\ln 0.037$

Write an equivalent exponential or logarithmic equation.

9. $\ln 50 = x$
10. $\ln 36 = 2x$
11. $\ln 6 = 1.7918$
12. $\ln 9.3 = 2.2300$
13. $e^x = 8$
14. $e^5 = 10x$
15. $e^{-x} = 4$
16. $e^2 = x + 1$

Evaluate each expression.

17. $e^{\ln 12}$
18. $e^{\ln 3x}$
19. $\ln e^{-1}$
20. $\ln e^{-2y}$

Solve each equation or inequality.

21. $e^x < 9$
22. $e^{-x} = 31$
23. $e^x = 1.1$
24. $e^x = 5.8$
25. $2e^x - 3 = 1$
26. $5e^x + 1 \geq 7$
27. $4 + e^x = 19$
28. $-3e^x + 10 < 8$
29. $e^{3x} = 8$
30. $e^{-4x} = 5$
31. $e^{0.5x} = 6$
32. $2e^{5x} = 24$
33. $e^{2x} + 1 = 55$
34. $e^{3x} - 5 = 32$
35. $9 + e^{2x} = 10$
36. $e^{-3x} + 7 \geq 15$
37. $\ln 4x = 3$
38. $\ln (-2x) = 7$
39. $\ln 2.5x = 10$
40. $\ln (x - 6) = 1$
41. $\ln (x + 2) = 3$
42. $\ln (x + 3) = 5$
43. $\ln 3x + \ln 2x = 9$
44. $\ln 5x + \ln x = 7$

**INVESTING** For Exercises 45 and 46, use the formula for continuously compounded interest, $A = Pe^{rt}$, where $P$ is the principal, $r$ is the annual interest rate, and $t$ is the time in years.

45. If Sarita deposits $1000 in an account paying 3.4% annual interest compounded continuously, what is the balance in the account after 5 years?

46. How long will it take the balance in Sarita’s account to reach $2000?

**47. RADIOACTIVE DECAY** The amount of a radioactive substance $y$ that remains after $t$ years is given by the equation $y = ae^{kt}$, where $a$ is the initial amount present and $k$ is the decay constant for the radioactive substance. If $a = 100, y = 50, and k = -0.035$, find $t$. 

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Glencoe Algebra 2
Solve each problem.

1. **INVESTING** The formula \( A = P \left(1 + \frac{r}{2}\right)^{2t} \) gives the value of an investment after \( t \) years in an account that earns an annual interest rate \( r \) compounded twice a year. Suppose $500 is invested at 6% annual interest compounded twice a year. In how many years will the investment be worth $1000?

2. **BACTERIA** How many hours will it take a culture of bacteria to increase from 20 to 2000 if the growth rate per hour is 85%?

3. **RADIOACTIVE DECAY** A radioactive substance has a half-life of 32 years. Find the constant \( k \) in the decay formula for the substance.

4. **DEPRECIATION** A piece of machinery valued at $250,000 depreciates at a fixed rate of 12% per year. After how many years will the value have depreciated to $100,000?

5. **INFLATION** For Dave to buy a new car comparably equipped to the one he bought years ago would cost $12,500. Since Dave bought the car, the inflation rate for cars like his has been at an average annual rate of 5.1%. If Dave originally paid $8400 for the car, how long ago did he buy it?

6. **RADIOACTIVE DECAY** Cobalt, an element used to make alloys, has several isotopes. One of these, cobalt-60, is radioactive and has a half-life of 5.7 years. Cobalt-60 is used to trace the path of nonradioactive substances in a system. What is the value of \( k \) for Cobalt-60?

7. **WHALES** Modern whales appeared 5–10 million years ago. The vertebrae of a whale discovered by paleontologists contain roughly 0.25% as much carbon-14 as they would have contained when the whale was alive. How long ago did the whale die? Use \( k = 0.00012 \).

8. **POPULATION** The population of rabbits in an area is modeled by the growth equation \( P(t) = 8e^{0.26t} \), where \( P \) is in thousands and \( t \) is in years. How long will it take for the population to reach 25,000?

9. **DEPRECIATION** A computer system depreciates at an average rate of 4% per month. If the value of the computer system was originally $12,000, in how many months is it worth $7350?

10. **BIOLOGY** In a laboratory, a culture increases from 30 to 195 organisms in 5 hours. What is the hourly growth rate in the growth formula \( y = a(1 + r)^t \)?
Find the next four terms of each arithmetic sequence.

1. 5, 8, 11, ...
2. −4, −6, −8, ...
3. 100, 93, 86, ...
4. −24, −19, −14, ...
5. $\frac{7}{2}, 6, \frac{17}{2}, 11, ...
6. 4.8, 4.1, 3.4, ...

Find the first five terms of each arithmetic sequence described.

7. $a_1 = 7, d = 7$
8. $a_1 = −8, d = 2$
9. $a_1 = −12, d = −4$
10. $a_1 = \frac{1}{2}, d = \frac{1}{2}$
11. $a_1 = \frac{5}{6}, d = −\frac{1}{3}$
12. $a_1 = 10.2, d = −5.8$

Find the indicated term of each arithmetic sequence.

13. $a_1 = 5, d = 3, n = 10$
14. $a_1 = 9, d = 3, n = 29$
15. $a_{18}$ for −6, −7, −8, ....
16. $a_{37}$ for 124, 119, 114, ....
17. $a_1 = \frac{9}{5}, d = −\frac{3}{5}, n = 10$
18. $a_1 = 14.25, d = 0.15, n = 31$

Complete the statement for each arithmetic sequence.

19. 166 is the ___th term of 30, 34, 38, ...
20. 2 is the ___th term of $\frac{3}{5}, \frac{4}{5}, 1, ...$

Write an equation for the $n$th term of each arithmetic sequence.

21. −5, −3, −1, 1, ...
22. −8, −11, −14, −17, ...
23. 1, −1, −3, −5, ...
24. −5, 3, 11, 19, ...

Find the arithmetic means in each sequence.

25. −5, ___ , ___, ___, 11
26. 82, ___ , ___, ___, ___, 18

EDUCATION Trevor Koba has opened an English Language School in Isehara, Japan. He began with 26 students. If he enrolls 3 new students each week, in how many weeks will he have 101 students?

27. SALARIES Yolanda interviewed for a job that promised her a starting salary of $32,000 with a $1250 raise at the end of each year. What will her salary be during her sixth year if she accepts the job?
11-2 Practice

**Arithmetic Series**

Find $S_n$ for each arithmetic series described.

1. $a_1 = 16, a_n = 98, n = 13$
2. $a_1 = 3, a_n = 36, n = 12$
3. $a_1 = -5, a_n = -26, n = 8$
4. $a_1 = 5, n = 10, a_n = -13$
5. $a_1 = 6, n = 15, a_n = -22$
6. $a_1 = -20, n = 25, a_n = 148$
7. $a_1 = 13, d = -6, n = 21$
8. $a_1 = 5, d = 4, n = 11$
9. $a_1 = 5, d = 2, a_n = 33$
10. $a_1 = -121, d = 3, a_n = 5$
11. $d = 0.4, n = 10, a_n = 3.8$
12. $d = -\frac{2}{3}, n = 16, a_n = 44$

Find the sum of each arithmetic series.

13. $5 + 7 + 9 + 11 + \ldots + 27$
14. $-4 + 1 + 6 + 11 + \ldots + 91$
15. $13 + 20 + 27 + \ldots + 272$
16. $89 + 86 + 83 + 80 + \ldots + 20$
17. $\sum_{n=1}^{4} (1 - 2n)$
18. $\sum_{j=1}^{6} (5 + 3n)$
19. $\sum_{n=1}^{5} (9 - 4n)$
20. $\sum_{k=4}^{10} (2k + 1)$
21. $\sum_{n=3}^{8} (5n - 10)$
22. $\sum_{n=1}^{101} (4 - 4n)$

Find the first three terms of each arithmetic series described.

23. $a_1 = 14, a_n = -85, S_n = -1207$
24. $a_1 = 1, a_n = 19, S_n = 100$
25. $n = 16, a_n = 15, S_n = -120$
26. $n = 15, a_n = 5\frac{4}{5}, S_n = 45$

27. **STACKING**  A health club rolls its towels and stacks them in layers on a shelf. Each layer of towels has one less towel than the layer below it. If there are 20 towels on the bottom layer and one towel on the top layer, how many towels are stacked on the shelf?

28. **BUSINESS**  A merchant places $1 in a jackpot on August 1, then draws the name of a regular customer. If the customer is present, he or she wins the $1 in the jackpot. If the customer is not present, the merchant adds $2 to the jackpot on August 2 and draws another name. Each day the merchant adds an amount equal to the day of the month. If the first person to win the jackpot wins $496, on what day of the month was her or his name drawn?
Find the next two terms of each geometric sequence.

1. \(-15, -30, -60, \ldots\)
2. \(80, 40, 20, \ldots\)
3. \(90, 30, 10, \ldots\)
4. \(-1458, 486, -162, \ldots\)
5. \(\frac{1}{4}, \frac{3}{8}, \frac{9}{16}, \ldots\)
6. \(216, 144, 96, \ldots\)

Find the first five terms of each geometric sequence described.

7. \(a_1 = -1, r = -3\)
8. \(a_1 = 7, r = -4\)
9. \(a_1 = -\frac{1}{3}, r = 2\)
10. \(a_1 = 12, r = \frac{2}{3}\)

Find the indicated term of each geometric sequence.

11. \(a_1 = 5, r = 3, n = 6\)
12. \(a_1 = 20, r = -3, n = 6\)
13. \(a_1 = -4, r = -2, n = 10\)
14. \(a_8 \text{ for } -\frac{1}{250}, -\frac{1}{50}, -\frac{1}{10}, \ldots\)
15. \(a_{12} \text{ for } 96, 48, 24, \ldots\)
16. \(a_1 = 8, r = \frac{1}{2}, n = 9\)
17. \(a_1 = -3125, r = -\frac{1}{5}, n = 9\)
18. \(a_1 = 3, r = \frac{1}{10}, n = 8\)

Write an equation for the \(n^{th}\) term of each geometric sequence.

19. \(1, 4, 16, \ldots\)
20. \(-1, -5, -25, \ldots\)
21. \(1, \frac{1}{2}, \frac{1}{4}, \ldots\)
22. \(-3, -6, -12, \ldots\)
23. \(7, -14, 28, \ldots\)
24. \(-5, -30, -180, \ldots\)

Find the geometric means in each sequence.

25. \(3, ? , ? , ? , 768\)
26. \(5, ? , ? , ? , 1280\)
27. \(144, ? , ? , ? , 9\)
28. \(37,500, ? , ? , ? , ? , -12\)

29. **BIOLOGY** A culture initially contains 200 bacteria. If the number of bacteria doubles every 2 hours, how many bacteria will be in the culture at the end of 12 hours?

30. **LIGHT** If each foot of water in a lake screens out 60% of the light above, what percent of the light passes through 5 feet of water?

31. **INVESTING** Raul invests $1000 in a savings account that earns 5% interest compounded annually. How much money will he have in the account at the end of 5 years?
Find \( S_n \) for each geometric series described.

1. \( a_1 = 2, a_6 = 64, r = 2 \)
2. \( a_1 = 160, a_6 = 5, r = \frac{1}{2} \)
3. \( a_1 = -3, a_n = -192, r = -2 \)
4. \( a_1 = -81, a_n = -16, r = -\frac{2}{3} \)
5. \( a_1 = -3, a_n = 3072, r = -4 \)
6. \( a_1 = 54, a_6 = \frac{2}{9}, r = \frac{1}{3} \)
7. \( a_1 = 5, r = 3, n = 9 \)
8. \( a_1 = -6, r = -1, n = 21 \)
9. \( a_1 = -6, r = -3, n = 7 \)
10. \( a_1 = -9, r = \frac{2}{3}, n = 4 \)
11. \( a_1 = \frac{1}{3}, r = 3, n = 10 \)
12. \( a_1 = 16, r = -1.5, n = 6 \)

Find the sum of each geometric series.

13. \( 162 + 54 + 18 + \ldots \) to 6 terms
14. \( 2 + 4 + 8 + \ldots \) to 8 terms
15. \( 64 - 96 + 144 - \ldots \) to 7 terms
16. \( \frac{1}{9} - \frac{1}{3} + 1 - \ldots \) to 6 terms

17. \( \sum_{n=1}^{8} (-3)^n - 1 \)
18. \( \sum_{n=1}^{9} 5(-2)^n - 1 \)
19. \( \sum_{n=1}^{5} -1(4)^n - 1 \)
20. \( \sum_{n=1}^{6} \left(\frac{1}{2}\right)^n - 1 \)
21. \( \sum_{n=1}^{10} 2560\left(\frac{1}{2}\right)^n - 1 \)
22. \( \sum_{n=1}^{4} 9\left(\frac{2}{3}\right)^n - 1 \)

Find the indicated term for each geometric series described.

23. \( S_n = 1023, a_n = 768, r = 4; a_1 \)
24. \( S_n = 10,160, a_n = 5120, r = 2; a_1 \)
25. \( S_n = -1365, n = 12, r = -2; a_1 \)
26. \( S_n = 665, n = 6, r = 1.5; a_1 \)

27. CONSTRUCTION A pile driver drives a post 27 inches into the ground on its first hit. Each additional hit drives the post \( \frac{2}{3} \) the distance of the prior hit. Find the total distance the post has been driven after 5 hits.

28. COMMUNICATIONS Hugh Moore e-mails a joke to 5 friends on Sunday morning. Each of these friends e-mails the joke to 5 of her or his friends on Monday morning, and so on. Assuming no duplication, how many people will have heard the joke by the end of Saturday, not including Hugh?
Find the sum of each infinite geometric series, if it exists.

1. \( a_1 = 35, r = \frac{2}{7} \)
2. \( a_1 = 26, r = \frac{1}{2} \)
3. \( a_1 = 98, r = -\frac{3}{4} \)
4. \( a_1 = 42, r = \frac{6}{5} \)
5. \( a_1 = 112, r = -\frac{3}{5} \)
6. \( a_1 = 500, r = \frac{1}{5} \)
7. \( a_1 = 135, r = -\frac{1}{2} \)
8. \( 18 - 6 + 2 - \ldots \)
9. \( 2 + 6 + 18 + \ldots \)
10. \( 6 + 4 + \frac{8}{3} + \ldots \)
11. \( \frac{4}{25} + \frac{2}{5} + 1 + \ldots \)
12. \( 10 + 1 + 0.1 + \ldots \)
13. \( 100 + 20 + 4 + \ldots \)
14. \( -270 + 135 - 67.5 + \ldots \)
15. \( 0.5 + 0.25 + 0.125 + \ldots \)
16. \( \frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \ldots \)
17. \( 0.8 + 0.08 + 0.008 + \ldots \)
18. \( \frac{1}{12} - \frac{1}{6} + \frac{1}{3} - \ldots \)
19. \( 3 + \frac{9}{7} + \frac{27}{49} + \ldots \)
20. \( 0.3 - 0.003 + 0.00003 - \ldots \)
21. \( 0.06 + 0.006 + 0.0006 + \ldots \)
22. \( \frac{2}{3} - 2 + 6 - \ldots \)
23. \( \sum_{n=1}^{\infty} 3 \left( \frac{1}{4} \right)^n - 1 \)
24. \( \sum_{n=1}^{\infty} \frac{2}{3} \left( -\frac{3}{4} \right)^n - 1 \)
25. \( \sum_{n=1}^{\infty} 18 \left( \frac{2}{3} \right)^n - 1 \)
26. \( \sum_{n=1}^{\infty} 5 (-0.1)^n - 1 \)

Write each repeating decimal as a fraction.

27. \( \overline{0.6} \)
28. \( 0.09 \)
29. \( 0.\overline{43} \)
30. \( 0.\overline{27} \)
31. \( 0.\overline{243} \)
32. \( 0.\overline{84} \)
33. \( 0.\overline{990} \)
34. \( 0.\overline{150} \)

35. **PENDULUMS** On its first swing, a pendulum travels 8 feet. On each successive swing, the pendulum travels \( \frac{4}{5} \) the distance of its previous swing. What is the total distance traveled by the pendulum when it stops swinging?

36. **ELASTICITY** A ball dropped from a height of 10 feet bounces back \( \frac{9}{10} \) of that distance. With each successive bounce, the ball continues to reach \( \frac{9}{10} \) of its previous height. What is the total vertical distance (both up and down) traveled by the ball when it stops bouncing? (Hint: Add the total distance the ball falls to the total distance it rises.)
Practice

Recursion and Special Sequences

Find the first five terms of each sequence.

1. \( a_1 = 3, a_{n+1} = a_n + 5 \)
2. \( a_1 = -7, a_{n+1} = a_n + 8 \)
3. \( a_1 = -3, a_{n+1} = 3a_n + 2 \)
4. \( a_1 = -8, a_{n+1} = 10 - a_n \)
5. \( a_1 = 4, a_{n+1} = n - a_n \)
6. \( a_1 = -3, a_{n+1} = 3a_n \)
7. \( a_1 = 4, a_{n+1} = -3a_n + 4 \)
8. \( a_1 = 2, a_{n+1} = -4a_n - 5 \)
9. \( a_1 = 3, a_2 = 1, a_{n+1} = a_n - a_{n-1} \)
10. \( a_1 = -1, a_2 = 5, a_{n+1} = 4a_n - 1 - a_n \)
11. \( a_1 = 2, a_2 = -3, a_{n+1} = 5a_n - 8a_{n-1} \)
12. \( a_1 = -2, a_2 = 1, a_{n+1} = -2a_n + 6a_{n-1} \)

Find the first three iterates of each function for the given initial value.

13. \( f(x) = 3x + 4, x_0 = -1 \)
14. \( f(x) = 10x + 2, x_0 = -1 \)
15. \( f(x) = 8 + 3x, x_0 = 1 \)
16. \( f(x) = 8 - x, x_0 = -3 \)
17. \( f(x) = 4x + 5, x_0 = -1 \)
18. \( f(x) = 5(x + 3), x_0 = -2 \)
19. \( f(x) = -8x + 9, x_0 = 1 \)
20. \( f(x) = -4x^2, x_0 = -1 \)
21. \( f(x) = x^2 - 1, x_0 = 3 \)
22. \( f(x) = 2x^2; x_0 = 5 \)

23. Inflation

Iterating the function \( c(x) = 1.05x \) gives the future cost of an item at a constant 5% inflation rate. Find the cost of a $2000 ring in five years at 5% inflation.

**Fractals**

For Exercises 24–27, use the following information.

Replacing each side of the square shown with the combination of segments below it gives the figure to its right.

24. What is the perimeter of the original square?

25. What is the perimeter of the new shape?

26. If you repeat the process by replacing each side of the new shape by a proportional combination of 5 segments, what will the perimeter of the third shape be?

27. What function \( f(x) \) can you iterate to find the perimeter of each successive shape if you continue this process?
11-7 Practice

The Binomial Theorem

Evaluate each expression.

1. \(7!\)  
2. \(11!\)  
3. \(\frac{9!}{5!}\)  
4. \(\frac{20!}{18!}\)  
5. \(\frac{8!}{6!2!}\)  
6. \(\frac{8!}{5!3!}\)  
7. \(\frac{12!}{6!6!}\)  
8. \(\frac{41!}{3!38!}\)

Expand each power.

9. \((n + v)^5\)  
10. \((x - y)^4\)  
11. \((x + y)^6\)  
12. \((r + 3)^5\)  
13. \((m - 5)^5\)  
14. \((x + 4)^4\)  
15. \((3x + y)^4\)  
16. \((2m - y)^4\)  
17. \((w - 3z)^3\)  
18. \((2d + 3)^6\)  
19. \((x + 2y)^5\)  
20. \((2x - y)^5\)  
21. \((a - 3b)^4\)  
22. \((3 - 2z)^4\)  
23. \((3m - 4n)^3\)  
24. \((5x - 2y)^4\)

Find the indicated term of each expansion.

25. seventh term of \((a + b)^{10}\)  
26. sixth term of \((m - n)^{10}\)  
27. ninth term of \((r - s)^{14}\)  
28. tenth term of \((2x + y)^{12}\)  
29. fourth term of \((x - 3y)^{6}\)  
30. fifth term of \((2x - 1)^{9}\)

31. GEOMETRY How many line segments can be drawn between ten points, no three of which are collinear, if you use exactly two of the ten points to draw each segment?

32. PROBABILITY If you toss a coin 4 times, how many different sequences of tosses will give exactly 3 heads and 1 tail or exactly 1 head and 3 tails?
11-8 Practice

Proof and Mathematical Induction

Prove that each statement is true for all positive integers.

1. \(1 + 2 + 4 + 8 + \ldots + 2^n - 1 = 2^n - 1\)

2. \(1 + 4 + 9 + \ldots + n^2 = \frac{n(n + 1)(2n + 1)}{6}\)

3. \(18^n - 1\) is a multiple of 17.

Find a counterexample for each statement.

4. \(1 + 4 + 7 + \ldots + (3n - 2) = n^3 - n^2 + 1\)

5. \(5^n - 2n - 3\) is divisible by 3.

6. \(1 + 3 + 5 + \ldots + (2n - 1) = \frac{n^2 + 3n - 2}{2}\)

7. \(1^3 + 2^3 + 3^3 + \ldots + n^3 = n^4 - n^3 + 1\)
State whether the events are independent or dependent.

1. choosing an ice cream flavor and choosing a topping for the ice cream
2. choosing an offensive player of the game and a defensive player of the game in a professional football game
3. From 15 entries in an art contest, a camp counselor chooses first, second, and third place winners.
4. Jillian is selecting two more courses for her block schedule next semester. She must select one of three morning history classes and one of two afternoon math classes.

Solve each problem.

5. A briefcase lock has 3 rotating cylinders, each containing 10 digits. How many numerical codes are possible?
6. A golf club manufacturer makes irons with 7 different shaft lengths, 3 different grips, 5 different lies, and 2 different club head materials. How many different combinations are offered?
7. There are five different routes that a commuter can take from her home to the office. In how many ways can she make a round trip if she uses a different route coming than going?
8. In how many ways can the four call letters of a radio station be arranged if the first letter must be W or K and no letters repeat?
9. How many 7-digit phone numbers can be formed if the first digit cannot be 0 or 1, and any digit can be repeated?
10. How many 7-digit phone numbers can be formed if the first digit cannot be 0, and any digit can be repeated?
11. How many 7-digit phone numbers can be formed if the first digit cannot be 0 or 1, and if no digit can be repeated?
12. How many 7-digit phone numbers can be formed if the first digit cannot be 0, and if no digit can be repeated?
13. How many 6-character passwords can be formed if the first character is a digit and the remaining 5 characters are letters that can be repeated?
14. How many 6-character passwords can be formed if the first and last characters are digits and the remaining characters are letters? Assume that any character can be repeated.
Evaluate each expression.

1. \( P(8, 6) \)  
2. \( P(9, 7) \)  
3. \( P(3, 3) \)  
4. \( P(4, 3) \)  
5. \( P(4, 1) \)  
6. \( P(7, 2) \)  
7. \( C(8, 2) \)  
8. \( C(11, 3) \)  
9. \( C(20, 18) \)  
10. \( C(9, 9) \)  
11. \( C(3, 1) \)  
12. \( C(9, 3) \cdot C(6, 2) \)

Determine whether each situation involves a permutation or a combination. Then find the number of possibilities.

13. selecting a 4-person bobsled team from a group of 9 athletes  
   - permutation; 126  
14. an arrangement of the letters in the word *Canada*  
   - permutation; 120  
15. arranging 4 charms on a bracelet that has a clasp, a front, and a back  
   - permutation; 24  
16. selecting 3 desserts from 10 desserts that are displayed on a dessert cart in a restaurant  
   - combination; 120  
17. an arrangement of the letters in the word *annually*  
   - permutation; 5040  
18. forming a 2-person sales team from a group of 12 salespeople  
   - combination; 66  
19. making 5-sided polygons by choosing any 5 of 11 points located on a circle to be the vertices  
   - combination; 462  
20. seating 5 men and 5 women alternately in a row, beginning with a woman  
   - permutation; 14,400  
21. STUDENT GROUPS Farmington High is planning its academic festival. All math classes will send 2 representatives to compete in the math bowl. How many different groups of students can be chosen from a class of 16 students?  
   - combination; 120  
22. PHOTOGRAPHY A photographer is taking pictures of a bride and groom and their 6 attendants. If she takes photographs of 3 people in a group, how many different groups can she photograph?  
23. AIRLINES An airline is hiring 5 flight attendants. If 8 people apply for the job, how many different groups of 5 attendants can the airline hire?  
24. SUBSCRIPTIONS A school librarian would like to buy subscriptions to 7 new magazines. Her budget, however, will allow her to buy only 4 new subscriptions. How many different groups of 4 magazines can she choose from the 7 magazines?
12-3 Practice

Probability

A bag contains 1 green, 4 red, and 5 yellow balls. Two balls are selected at random. Find the probability of each selection.

1. \( P(2 \text{ red}) \) 
2. \( P(1 \text{ red and } 1 \text{ yellow}) \) 
3. \( P(1 \text{ green and } 1 \text{ yellow}) \) 
4. \( P(2 \text{ green}) \) 
5. \( P(2 \text{ red and } 1 \text{ yellow}) \) 
6. \( P(1 \text{ red and } 1 \text{ green}) \)

A bank contains 3 pennies, 8 nickels, 4 dimes, and 10 quarters. Two coins are selected at random. Find the probability of each selection.

7. \( P(2 \text{ pennies}) \) 
8. \( P(2 \text{ dimes}) \) 
9. \( P(1 \text{ nickel and } 1 \text{ dime}) \) 
10. \( P(1 \text{ quarter and } 1 \text{ penny}) \) 
11. \( P(1 \text{ quarter and } 1 \text{ nickel}) \) 
12. \( P(2 \text{ dimes and } 1 \text{ quarter}) \)

Henrico visits a home decorating store to choose wallpapers for his new house. The store has 28 books of wallpaper samples, including 10 books of WallPride samples and 18 books of Deluxe Wall Coverings samples. The store will allow Henrico to bring 4 books home for a few days so he can decide which wallpapers he wants to buy. If Henrico randomly chooses 4 books to bring home, find the probability of each selection.

13. \( P(4 \text{ WallPride}) \) 
14. \( P(2 \text{ WallPride and } 2 \text{ Deluxe}) \) 
15. \( P(1 \text{ WallPride and } 3 \text{ Deluxe}) \) 
16. \( P(3 \text{ WallPride and } 1 \text{ Deluxe}) \)

For Exercises 17–20, use the table that shows the range of verbal SAT scores for freshmen at a small liberal arts college. If a freshman student is chosen at random, find each probability. Express as decimals rounded to the nearest thousandth.

17. \( P(400–449) \) 
18. \( P(550–559) \) 
19. \( P(\text{at least } 650) \)

Find the odds of an event occurring, given the probability of the event.

20. \( \frac{4}{11} \) 
21. \( \frac{12}{13} \) 
22. \( \frac{5}{99} \) 
23. \( \frac{1}{1000} \) 
24. \( \frac{5}{16} \) 
25. \( \frac{3}{95} \) 
26. \( \frac{9}{70} \) 
27. \( \frac{8}{15} \)

Find the probability of an event occurring, given the odds of the event.

28. 2:23 
29. 2:5 
30. 15:1 
31. 9:7 
32. 11:14 
33. 1000:1 
34. 12:17 
35. 8:13
12-4 Practice

Multiplying Probabilities

A die is rolled three times. Find each probability.

1. \(P(\text{three 4s})\)  
2. \(P(\text{no 4s})\)

3. \(P(2, \text{then 3, then 1})\)  
4. \(P(\text{three different even numbers})\)

5. \(P(\text{any number, then 5, then 5})\)  
6. \(P(\text{even number, then odd number, then 1})\)

There are 3 nickels, 2 dimes, and 5 quarters in a purse. Three coins are selected in succession at random. Find the probability.

7. \(P(\text{nickel, then dime, then quarter}), \text{if no replacement occurs}\)

8. \(P(\text{nickel, then dime, then quarter}), \text{if replacement occurs}\)

9. \(P(2 \text{ nickels, then 1 quarter}), \text{if no replacement occurs}\)

10. \(P(3 \text{ dimes}), \text{if replacement occurs}\)

11. \(P(3 \text{ dimes}), \text{if no replacement occurs}\)

For Exercises 12 and 13, determine whether the events are independent or dependent. Then find each probability.

12. Serena is creating a painting. She wants to use 2 more colors. She chooses randomly from 6 shades of red, 10 shades of green, 4 shades of yellow, 4 shades of purple, and 6 shades of blue. What is the probability that she chooses 2 shades of green?

13. Kershel’s mother is shopping at a bakery. The owner offers Kershel a cookie from a jar containing 22 chocolate chip cookies, 18 sugar cookies, and 15 oatmeal cookies. Without looking, Kershel selects one, drops it back in, and then randomly selects another. What is the probability that neither selection was a chocolate chip cookie?

14. METEOROLOGY The Fadeeva’s are planning a 3-day vacation to the mountains. A long-range forecast reports that the probability of rain each day is 10%. Assuming that the daily probabilities of rain are independent, what is the probability that there is no rain on the first two days, but that it rains on the third day?

RANDOM NUMBERS For Exercises 15 and 16, use the following information.

Anita has a list of 20 jobs around the house to do, and plans to do 3 of them today. She assigns each job a number from 1 to 20, and sets her calculator to generate random numbers from 1 to 20, which can reoccur. Of the jobs, 3 are outside, and the rest are inside.

15. Sketch a tree diagram showing all of the possibilities that the first three numbers generated correspond to inside jobs or outside jobs. Use it to find the probability that the first two numbers correspond to inside jobs, and the third to an outside job.

16. What is the probability that the number generated corresponds to an outside job three times in a row?
An urn contains 7 white marbles and 5 blue marbles. Four marbles are selected without replacement. Find each probability.

1. \(P(4 \text{ white or 4 blue})\)
2. \(P(\text{exactly 3 white})\)
3. \(P(\text{at least 3 white})\)
4. \(P(\text{fewer than 3 white})\)
5. \(P(3 \text{ white or 3 blue})\)
6. \(P(\text{no white or no blue})\)

Jason and Maria are playing a board game in which three dice are tossed to determine a player's move. Find each probability.

7. \(P(\text{two 5s})\)
8. \(P(\text{three 5s})\)
9. \(P(\text{at least two 5s})\)
10. \(P(\text{no 5s})\)
11. \(P(\text{one 5})\)
12. \(P(\text{one 5 or two 5s})\)

Determine whether the events are mutually exclusive or inclusive. Then find the probability.

13. A clerk chooses 4 CD players at random for floor displays from a shipment of 24 CD players. If 15 of the players have a blue case and the rest have a red case, what is the probability of choosing 4 players with a blue case or 4 players with a red case?

14. A department store employs 28 high school students, all juniors and seniors. Six of the 12 seniors are females and 12 of the juniors are males. One student employee is chosen at random. What is the probability of selecting a senior or a female?

15. A restaurant has 5 pieces of apple pie, 4 pieces of chocolate cream pie, and 3 pieces of blueberry pie. If Janine selects a piece of pie at random for dessert, what is the probability that she selects either apple or chocolate cream?

16. At a statewide meeting, there are 20 school superintendents, 13 principals, and 6 assistant principals. If one of these people is chosen at random, what is the probability that he or she is either a principal or an assistant principal?

17. An airline has one bank of 13 telephones at a reservations office. Of the 13 operators who work there, 8 take reservations for domestic flights and 5 take reservations for international flights. Seven of the operators taking domestic reservations and 3 of the operators taking international reservations are female. If an operator is chosen at random, what is the probability that the person chosen takes domestic reservations or is a male?

18. **MUSIC** Forty senior citizens were surveyed about their music preferences. The results are displayed in the Venn diagram. If a senior citizen from the survey group is selected at random, what is the probability that he or she likes only country and western music? What is the probability that he or she likes classical and/or country, but not 1940’s pop?
Find the variance and standard deviation of each set of data to the nearest tenth.

1. \{47, 61, 93, 22, 82, 22, 37\}
2. \{10, 10, 54, 39, 96, 91, 91, 18\}
3. \{1, 2, 3, 3, 4, 4, 4, 5, 5, 5, 5\}
4. \{1100, 725, 850, 335, 700, 800, 950\}
5. \{3.4, 7.1, 8.5, 5.1, 4.7, 6.3, 9.9, 8.4, 3.6\}
6. \{2.8, 0.5, 1.9, 0.8, 1.9, 1.5, 3.3, 2.6, 0.7, 2.5\}

7. HEALTH CARE Eight physicians with 15 patients on a hospital floor see these patients an average of 18 minutes a day. The 22 nurses on the same floor see the patients an average of 3 hours a day. As a hospital administrator, would you quote the mean, median, or mode as an indicator of the amount of daily medical attention the patients on this floor receive? Explain.

For Exercises 8–10, use the frequency table that shows the percent of public school teachers in the U. S. in 1999 who used computers or the Internet at school for various administrative and teaching activities.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Percent Using Computer or Internet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Create instructional materials</td>
<td>39</td>
</tr>
<tr>
<td>Administrative record keeping</td>
<td>34</td>
</tr>
<tr>
<td>Communicate with colleagues</td>
<td>23</td>
</tr>
<tr>
<td>Gather information for planning lessons</td>
<td>16</td>
</tr>
<tr>
<td>Multimedia classroom presentations</td>
<td>8</td>
</tr>
<tr>
<td>Access research and best practices for teaching</td>
<td>8</td>
</tr>
<tr>
<td>Communicate with parents or students</td>
<td>8</td>
</tr>
<tr>
<td>Access model lesson plans</td>
<td>6</td>
</tr>
</tbody>
</table>

Source: National Assessment of Educational Progress

8. Find the mean, median, and mode of the data.
9. Suppose you believe teachers use computers or the Internet too infrequently. Which measure would you quote as the “average?” Explain.
10. Suppose you believe teachers use computers or the Internet too often. Which measure would you quote as the “average?” Explain.

For Exercises 11 and 12, use the frequency table that shows the number of games played by 24 American League baseball players between opening day, 2001 and September 8, 2001.

<table>
<thead>
<tr>
<th>No. of Games</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>141</td>
<td>4</td>
</tr>
<tr>
<td>140</td>
<td>3</td>
</tr>
<tr>
<td>139</td>
<td>4</td>
</tr>
<tr>
<td>138</td>
<td>5</td>
</tr>
<tr>
<td>137</td>
<td>2</td>
</tr>
<tr>
<td>136</td>
<td>3</td>
</tr>
<tr>
<td>135</td>
<td>3</td>
</tr>
</tbody>
</table>

11. Find the mean, median, mode, and standard deviation of the number of games played to the nearest tenth.
12. For how many players is the number of games within one standard deviation of the mean?

Source: Major League Baseball
Determine whether the data in each table appear to be positively skewed, negatively skewed, or normally distributed.

1. Time Spent at a Museum Exhibit

<table>
<thead>
<tr>
<th>Minutes</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–25</td>
<td>27</td>
</tr>
<tr>
<td>26–50</td>
<td>46</td>
</tr>
<tr>
<td>51–75</td>
<td>89</td>
</tr>
<tr>
<td>75–100</td>
<td>57</td>
</tr>
<tr>
<td>100+</td>
<td>24</td>
</tr>
</tbody>
</table>

2. Average Age of High School Principals

<table>
<thead>
<tr>
<th>Age in Years</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>31–35</td>
<td>3</td>
</tr>
<tr>
<td>36–40</td>
<td>8</td>
</tr>
<tr>
<td>41–45</td>
<td>15</td>
</tr>
<tr>
<td>46–50</td>
<td>32</td>
</tr>
<tr>
<td>51–55</td>
<td>40</td>
</tr>
<tr>
<td>56–60</td>
<td>38</td>
</tr>
<tr>
<td>60+</td>
<td>4</td>
</tr>
</tbody>
</table>

For Exercises 3 and 4, use the frequency table that shows the number of hours worked per week by 100 high school seniors.

3. Make a histogram of the data.

4. Do the data appear to be positively skewed, negatively skewed, or normally distributed? Explain.

**TESTING** For Exercises 5–10, use the following information.

The scores on a test administered to prospective employees are normally distributed with a mean of 100 and a standard deviation of 15.

5. About what percent of the scores are between 70 and 130?
6. About what percent of the scores are between 85 and 130?
7. About what percent of the scores are over 115?
8. About what percent of the scores are lower than 85 or higher than 115?
9. If 80 people take the test, how many would you expect to score higher than 130?
10. If 75 people take the test, how many would you expect to score lower than 85?

**TEMPERATURE** The daily July surface temperature of a lake at a resort has a mean of 82° and a standard deviation of 4.2°. If you prefer to swim when the temperature is at least 77.8°, about what percent of the days does the temperature meet your preference?
Find each probability if a coin is tossed 6 times.

1. \( P(\text{exactly 3 tails}) \)
2. \( P(\text{exactly 5 tails}) \)
3. \( P(0 \text{ tails}) \)
4. \( P(\text{at least 4 heads}) \)
5. \( P(\text{at least 4 tails}) \)
6. \( P(\text{at most 2 tails}) \)

The probability of Chris making a free throw is \( \frac{2}{3} \). If she shoots 5 times, find each probability.

7. \( P(\text{all missed}) \)
8. \( P(\text{all made}) \)
9. \( P(\text{exactly 2 made}) \)
10. \( P(\text{exactly 1 missed}) \)
11. \( P(\text{at least 3 made}) \)
12. \( P(\text{at most 2 made}) \)

When Tarin and Sam play a certain board game, the probability that Tarin will win a game is \( \frac{3}{4} \). If they play 5 games, find each probability.

13. \( P(\text{Sam wins only once}) \)
14. \( P(\text{Tarin wins exactly twice}) \)
15. \( P(\text{Sam wins exactly 3 games}) \)
16. \( P(\text{Sam wins at least 1 game}) \)
17. \( P(\text{Tarin wins at least 3 games}) \)
18. \( P(\text{Tarin wins at most 2 games}) \)

19. SAFETY  In August 2001, the American Automobile Association reported that 73% of Americans use seat belts. In a random selection of 10 Americans in 2001, what is the probability that exactly half of them use seat belts?  

20. HEALTH  For Exercises 20 and 21, use the following information. 
In 2001, the American Heart Association reported that 50 percent of the Americans who receive heart transplants are ages 50–64 and 20 percent are ages 35–49.  

20. In a randomly selected group of 10 heart transplant recipients, what is the probability that at least 8 of them are ages 50–64? 

21. In a randomly selected group of 5 heart transplant recipients, what is the probability that 2 of them are ages 35–49?
Determine whether each situation would produce a random sample. Write yes or no and explain your answer.

1. calling every twentieth registered voter to determine whether people own or rent their homes in your community

No; registered voters may be more likely to be homeowners, causing the survey to underrepresent renters.

2. predicting local election results by polling people in every twentieth residence in all the different neighborhoods of your community

Yes; since all neighborhoods are represented proportionally, the views of the community as a whole should be well represented.

3. to find out why not many students are using the library, a school’s librarian gives a questionnaire to every tenth student entering the library

No; she is polling only the students who are coming to the library, and will obtain no input from those who aren’t using the library.

4. testing overall performance of tires on interstate highways only

No; for overall performance, tires should be tested on many kinds of surfaces, and under many types of conditions.

5. selecting every 50th hamburger from a fast-food restaurant chain and determining its fat content to assess the fat content of hamburgers served in fast-food restaurant chains throughout the country

No; the selected hamburgers are a random sample of the hamburgers served in one chain, and may represent the fat content for that chain, but will not necessarily represent the fat content of hamburgers served in other fast-food restaurant chains.

6. assigning all shift workers in a manufacturing plant a unique identification number, and then placing the numbers in a hat and drawing 30 at random to determine the annual average salary of the workers

Yes; because the numbers are randomly chosen from among all shift workers, all workers have the same chance of being selected.

Find the margin of sampling error to the nearest percent.

7. \( p = 26\% \), \( n = 100 \)

8. \( p = 55\% \), \( n = 100 \)

9. \( p = 75\% \), \( n = 500 \)

10. \( p = 14\% \), \( n = 500 \)

11. \( p = 96\% \), \( n = 1000 \)

12. \( p = 21\% \), \( n = 1000 \)

13. \( p = 34\% \), \( n = 1000 \)

14. \( p = 49\% \), \( n = 1500 \)

15. \( p = 65\% \), \( n = 1500 \)

16. COMPUTING According to a poll of 500 teenagers, 43\% said that they use a personal computer at home. What is the margin of sampling error?

17. TRUST A survey of 605 people, ages 13–33, shows that 68\% trust their parents more than their best friends to tell them the truth. What is the margin of sampling error?

18. PRODUCTIVITY A study by the University of Illinois in 1995 showed an increase in productivity by 10\% of the employees who wore headsets and listened to music of their choice while they were working. The margin of sampling error for the study was about 7\%. How many employees participated in the study?
1. Find the values of the six trigonometric functions for angle $\theta$.

\[
\sin \theta = \frac{1}{5}, \quad \cos \theta = \frac{7}{8}, \quad \tan \theta = \frac{1}{3}.
\]

\[
\sin \theta = \frac{15}{17}, \quad \cos \theta = \frac{16}{20}, \quad \tan \theta = \frac{3}{3}.
\]

\[
\sin \theta = \frac{2}{3}, \quad \cos \theta = \frac{7}{4}, \quad \tan \theta = \frac{3}{2}.
\]

Write an equation involving $\sin$, $\cos$, or $\tan$ that can be used to find $x$. Then solve the equation. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

4. $\tan x = \frac{7}{3}$

5. $\sin x = \frac{19.2}{17}$

6. $\cos x = \frac{15.3}{32}$

7. Solve $\triangle ABC$ by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

10. $A = 35^\circ, \ a = 12$

11. $B = 71^\circ, \ b = 25$

12. $B = 36^\circ, \ c = 8$

13. $a = 4, \ b = 7$

14. $A = 17^\circ, \ c = 3.2$

15. $b = 52, \ c = 95$

16. **SURVEYING** John stands 150 meters from a water tower and sights the top at an angle
13-2 Practice

Angles and Angle Measure

Draw an angle with the given measure in standard position.

1. 210°

2. 305°

3. 580°

4. 135°

5. -450°

6. -560°

Rewrite each degree measure in radians and each radian measure in degrees.

7. 18°

8. 6°

9. 870°

10. 347°

11. -72°

12. -820°

13. -250°

14. -165°

15. 4π/2

16. 5π/2

17. 13π/5

18. 13π/30

19. -9π/2

20. -7π/12

21. -3π/8

22. -3π/16

Find one angle with positive measure and one angle with negative measure coterminal with each angle.

23. 65°

24. 80°

25. 285°

26. 110°

27. -37°

28. -93°

29. 2π/5

30. 5π/6

31. 17π/6

32. -3π/2

33. -π/4

34. -5π/12

35. TIME Find both the degree and radian measures of the angle through which the hour hand on a clock rotates from 5 A.M. to 10 A.M.

36. ROTATION A truck with 16-inch radius wheels is driven at 77 feet per second (52.5 miles per hour). Find the measure of the angle through which a point on the outside of the wheel travels each second. Round to the nearest degree and nearest radian.
13-3 Practice

Trigonometric Functions of General Angles

Find the exact values of the six trigonometric functions of \( \theta \) if the terminal side of \( \theta \) in standard position contains the given point.

1. \((6, 8)\)  
2. \((-20, 21)\)  
3. \((-2, -5)\)

Find the reference angle for the angle with the given measure.

4. \(236^\circ\)  
5. \(\frac{13\pi}{8}\)  
6. \(-210^\circ\)  
7. \(-\frac{7\pi}{4}\)

Find the exact value of each trigonometric function.

8. \(\tan 135^\circ\)  
9. \(\cot 210^\circ\)  
10. \(\cot (-90^\circ)\)  
11. \(\cos 405^\circ\)

12. \(\tan \frac{5\pi}{3}\)  
13. \(\csc \left(-\frac{3\pi}{4}\right)\)  
14. \(\cot 2\pi\)  
15. \(\tan \frac{13\pi}{6}\)

Suppose \( \theta \) is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of \( \theta \).

16. \(\tan \theta = -\frac{12}{5}, \text{Quadrant IV}\)  
17. \(\sin \theta = \frac{2}{3}, \text{Quadrant III}\)

18. LIGHT Light rays that “bounce off” a surface are reflected by the surface. If the surface is partially transparent, some of the light rays are bent or refracted as they pass from the air through the material. The angles of reflection \( \theta_1 \) and of refraction \( \theta_2 \) in the diagram at the right are related by the equation \( \sin \theta_1 = n \sin \theta_2 \). If \( \theta_1 = 60^\circ \) and \( n = \sqrt{3} \), find the measure of \( \theta_2 \).

19. FORCE A cable running from the top of a utility pole to the ground exerts a horizontal pull of 800 Newtons and a vertical pull of \(800\sqrt{3} \) Newtons. What is the sine of the angle \( \theta \) between the cable and the ground? What is the measure of this angle?
13-4  Practice

**Law of Sines**

Find the area of \( \triangle ABC \) to the nearest tenth.

1. \[
\begin{align*}
11 \text{ yd} & \quad 9 \text{ yd} \\
B & \quad 46^\circ \\
A & \quad C
\end{align*}
\]

4. \( C = 32^\circ, a = 12.6 \text{ m}, b = 8.9 \text{ m} \)

5. \( B = 27^\circ, a = 14.9 \text{ cm}, c = 18.6 \text{ cm} \)

6. \( A = 17.4^\circ, b = 12 \text{ km}, c = 14 \text{ km} \)

7. \( A = 34^\circ, b = 19.4 \text{ ft}, c = 8.6 \text{ ft} \)

Solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

8. \( A = 50^\circ, B = 30^\circ, c = 9 \)

9. \( A = 56^\circ, B = 38^\circ, a = 12 \)

10. \( A = 80^\circ, C = 14^\circ, a = 40 \)

11. \( B = 47^\circ, C = 112^\circ, b = 13 \)

12. \( A = 72^\circ, a = 8, c = 6 \)

13. \( A = 25^\circ, C = 107^\circ, b = 12 \)

Determine whether each triangle has no solution, one solution, or two solutions. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

14. \( A = 29^\circ, a = 6, b = 13 \)

15. \( A = 70^\circ, a = 25, b = 20 \)

16. \( A = 113^\circ, a = 21, b = 25 \)

17. \( A = 110^\circ, a = 20, b = 8 \)

18. \( A = 66^\circ, a = 12, b = 7 \)

19. \( A = 54^\circ, a = 5, b = 8 \)

20. \( A = 45^\circ, a = 15, b = 18 \)

21. \( A = 60^\circ, a = 4\sqrt{3}, b = 8 \)

22. **WILDLIFE** Sarah Phillips, an officer for the Department of Fisheries and Wildlife, checks boaters on a lake to make sure they do not disturb two osprey nesting sites. She leaves a dock and heads due north in her boat to the first nesting site. From here, she turns 5° north of due west and travels an additional 2.14 miles to the second nesting site. She then travels 6.7 miles directly back to the dock. How far from the dock is the first osprey nesting site? Round to the nearest tenth.
13-5 Practice

Law of Cosines

Determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

1. \( \triangle ABC \)
   - \( A \approx 80^\circ \)
   - \( B \approx 12 \)
   - \( C \approx 7 \)

2. \( \triangle ABC \)
   - \( A \approx 3 \)
   - \( B \approx 4 \)
   - \( C \approx 6 \)

3. \( \triangle ABC \)
   - \( A \approx 80^\circ \)
   - \( B \approx 30 \)
   - \( C \approx 40^\circ \)

4. \( a = 16, b = 20, C = 54^\circ \)
   - \( c = \) ?

5. \( B = 71^\circ, c = 6, a = 11 \)
   - \( b = \) ?

6. \( A = 37^\circ, a = 20, b = 18 \)
   - \( c = \) ?

7. \( C = 35^\circ, a = 18, b = 24 \)
   - \( c = \) ?

8. \( a = 8, b = 6, c = 9 \)
   - \( A = \) ?

9. \( A = 23^\circ, b = 10, c = 12 \)
   - \( a = \) ?

10. \( a = 4, b = 5, c = 8 \)
    - \( A = \) ?

11. \( B = 46.6^\circ, C = 112^\circ, b = 13 \)
    - \( a = \) ?

12. \( A = 46.3^\circ, a = 35, b = 30 \)
    - \( c = \) ?

13. \( a = 16.4, b = 21.1, c = 18.5 \)
    - \( A = \) ?

14. \( C = 43.5^\circ, b = 8, c = 6 \)
    - \( A = \) ?

15. \( A = 78.3^\circ, b = 7, c = 11 \)
    - \( a = \) ?

16. SATELLITES Two radar stations 2.4 miles apart are tracking an airplane. The straight-line distance between Station A and the plane is 7.4 miles. The straight-line distance between Station B and the plane is 6.9 miles. What is the angle of elevation from Station A to the plane? Round to the nearest degree.

17. DRAFTING Marion is using a computer-aided drafting program to produce a drawing for a client. She begins a triangle by drawing a segment 4.2 inches long from point A to point B. From B, she moves 42° degrees counterclockwise from the segment connecting A and B and draws a second segment that is 6.4 inches long, ending at point C. To the nearest tenth, how long is the segment from C to A?
13-6 Practice
Circular Functions

The given point $P$ is located on the unit circle. Find $\sin \theta$ and $\cos \theta$.

1. $P\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
2. $P\left(\frac{20}{29}, -\frac{21}{29}\right)$
3. $P(0.8, 0.6)$

4. $P(0, -1)$
5. $P\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
6. $P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Find the exact value of each function.
7. $\cos \frac{7\pi}{4}$
8. $\sin (-30^\circ)$
9. $\sin \left(-\frac{2\pi}{3}\right)$
10. $\cos (-330^\circ)$

11. $\cos 600^\circ$
12. $\sin \frac{9\pi}{2}$
13. $\cos 7\pi$
14. $\cos \left(-\frac{11\pi}{4}\right)$

15. $\sin (-225^\circ)$
16. $\sin 585^\circ$
17. $\cos \left(-\frac{10\pi}{3}\right)$
18. $\sin 840^\circ$

Determine the period of each function.

19. [Graph of a periodic function with period $2\pi$]

20. [Graph of a periodic function with period $\pi$]

21. FERRIS WHEELS A Ferris wheel with a diameter of 100 feet completes 2.5 revolutions per minute. What is the period of the function that describes the height of a seat on the outside edge of the Ferris Wheel as a function of time?
13-7 Practice

Inverse Trigonometric Functions

Write each equation in the form of an inverse function.

1. \( \beta = \cos \alpha \)
2. \( \tan \beta = \alpha \)
3. \( y = \tan 120^\circ \)

4. \( -\frac{1}{2} = \cos x \)
5. \( \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2} \)
6. \( \cos \frac{\pi}{3} = \frac{1}{2} \)

Solve each equation by finding the value of \( x \) to the nearest degree.

7. \( \arcsin 1 = x \)
8. \( \cos^{-1} \frac{\sqrt{3}}{2} = x \)
9. \( x = \tan^{-1} \left( -\frac{\sqrt{3}}{3} \right) \)

10. \( x = \arccos \frac{\sqrt{2}}{2} \)
11. \( x = \arctan (-\sqrt{3}) \)
12. \( \sin^{-1} \left( -\frac{1}{2} \right) = x \)

Find each value. Write angle measures in radians. Round to the nearest hundredth.

13. \( \cos^{-1} \left( -\frac{\sqrt{3}}{2} \right) \)
14. \( \sin^{-1} \left( -\frac{\sqrt{2}}{2} \right) \)
15. \( \arctan \left( -\frac{\sqrt{3}}{3} \right) \)

16. \( \tan \left( \cos^{-1} \frac{1}{2} \right) \)
17. \( \cos \left[ \sin^{-1} \left( -\frac{3}{5} \right) \right] \)
18. \( \cos \left[ \arctan \left( -1 \right) \right] \)

19. \( \tan \left( \sin^{-1} \frac{12}{13} \right) \)
20. \( \sin \left( \arctan \frac{\sqrt{3}}{3} \right) \)
21. \( \cos^{-1} \left( \tan \frac{3\pi}{4} \right) \)

22. \( \sin^{-1} \left( \cos \frac{\pi}{3} \right) \)
23. \( \sin \left( 2 \cos^{-1} \frac{15}{17} \right) \)
24. \( \cos \left( 2 \sin^{-1} \frac{\sqrt{3}}{2} \right) \)

25. **PULLEYS** The equation \( x = \cos^{-1} 0.95 \) describes the angle through which pulley \( A \) moves, and \( y = \cos^{-1} 0.17 \) describes the angle through which pulley \( B \) moves. Both angles are greater than 270° and less than 360°. Which pulley moves through a greater angle?

26. **FLYWHEELS** The equation \( y = \arctan 1 \) describes the counterclockwise angle through which a flywheel rotates in 1 millisecond. Through how many degrees has the flywheel rotated after 25 milliseconds?
Practice
Graphing Trigonometric Functions

Find the amplitude, if it exists, and period of each function. Then graph each function.

1. \( y = -4 \sin \theta \) 
2. \( y = \cot \frac{1}{2} \theta \) 
3. \( y = \cos 5\theta \)

4. \( y = \csc \frac{3}{4} \theta \) 
5. \( y = 2 \tan \frac{1}{2} \theta \) 
6. \( 2y = \sin \theta \)

**FORCE** For Exercises 7 and 8, use the following information.
An anchoring cable exerts a force of 500 Newtons on a pole. The force has the horizontal and vertical components \( F_x \) and \( F_y \). (A force of one Newton (N), is the force that gives an acceleration of 1 m/sec² to a mass of 1 kg.)

7. The function \( F_x = 500 \cos \theta \) describes the relationship between the angle \( \theta \) and the horizontal force. What are the amplitude and period of this function?

8. The function \( F_y = 500 \sin \theta \) describes the relationship between the angle \( \theta \) and the vertical force. What are the amplitude and period of this function?

**WEATHER** For Exercises 9 and 10, use the following information.
The function \( y = 60 + 25 \sin \frac{\pi}{6} t \), where \( t \) is in months and \( t = 0 \) corresponds to April 15, models the average high temperature in degrees Fahrenheit in Centerville.

9. Determine the period of this function. What does this period represent?

10. What is the maximum high temperature and when does this occur?
State the vertical shift, amplitude, period, and phase shift for each function. Then graph the function.

1. \( y = \frac{1}{2} \tan \left( \theta - \frac{\pi}{2} \right) \)
2. \( y = 2 \cos (\theta + 30^\circ) + 3 \)
3. \( y = 3 \csc (2\theta + 60^\circ) - 2.5 \)

ECOLOGY For Exercises 4–6, use the following information.
The population of an insect species in a stand of trees follows the growth cycle of a particular tree species. The insect population can be modeled by the function
\( y = 40 + 30 \sin 6t \), where \( t \) is the number of years since the stand was first cut in November, 1920.

4. How often does the insect population reach its maximum level?
5. When did the population last reach its maximum?
6. What condition in the stand do you think corresponds with a minimum insect population?

BLOOD PRESSURE For Exercises 7–9, use the following information.
Jason’s blood pressure is 110 over 70, meaning that the pressure oscillates between a maximum of 110 and a minimum of 70. Jason’s heart rate is 45 beats per minute. The function that represents Jason’s blood pressure \( P \) can be modeled using a sine function with no phase shift.

7. Find the amplitude, midline, and period in seconds of the function.
8. Write a function that represents Jason’s blood pressure \( P \) after \( t \) seconds.
9. Graph the function.
Find the value of each expression.

1. \( \sin \theta \), if \( \cos \theta = \frac{5}{13} \) and \( 0^\circ \leq \theta < 90^\circ \)
2. \( \sec \theta \), if \( \sin \theta = -\frac{15}{17} \) and \( 180^\circ < \theta < 270^\circ \)

3. \( \cot \theta \), if \( \cos \theta = \frac{3}{10} \) and \( 270^\circ < \theta < 360^\circ \)
4. \( \sin \theta \), if \( \cot \theta = \frac{1}{2} \) and \( 0^\circ \leq \theta < 90^\circ \)

5. \( \cot \theta \), if \( \csc \theta = -\frac{3}{2} \) and \( 180^\circ < \theta < 270^\circ \)
6. \( \sec \theta \), if \( \csc \theta = -8 \) and \( 270^\circ < \theta < 360^\circ \)

7. \( \sec \theta \), if \( \tan \theta = 4 \) and \( 180^\circ < \theta < 270^\circ \)
8. \( \sin \theta \), if \( \tan \theta = -\frac{1}{2} \) and \( 270^\circ < \theta < 360^\circ \)

9. \( \cot \theta \), if \( \tan \theta = \frac{2}{5} \) and \( 0^\circ \leq \theta < 90^\circ \)
10. \( \cot \theta \), if \( \cos \theta = \frac{1}{3} \) and \( 270^\circ < \theta < 360^\circ \)

Simplify each expression.

11. \( \csc \theta \tan \theta \)
12. \( \frac{\sin^2 \theta}{\cot^2 \theta} \)
13. \( \sin^2 \theta \cot^2 \theta \)

14. \( \cot^2 \theta + 1 \)
15. \( \frac{\csc^2 \theta - \cot^2 \theta}{1 - \cos^2 \theta} \)
16. \( \frac{\csc \theta - \sin \theta}{\cos \theta} \)

17. \( \sin \theta + \cos \theta \cot \theta \)
18. \( \frac{\cos \theta}{1 - \sin \theta} - \frac{\cos \theta}{1 + \sin \theta} \)
19. \( \sec^2 \theta \cos^2 \theta - \tan^2 \theta \)

20. AERIAL PHOTOGRAPHY
The illustration shows a plane taking an aerial photograph of point A. Because the point is directly below the plane, there is no distortion in the image. For any point B not directly below the plane, however, the increase in distance creates distortion in the photograph. This is because as the distance from the camera to the point being photographed increases, the exposure of the film reduces by \((\sin \theta)(\csc \theta - \sin \theta)\). Express \((\sin \theta)(\csc \theta - \sin \theta)\) in terms of \(\cos \theta\) only.

21. TSUNAMIS
The equation \( y = a \sin \theta t \) represents the height of the waves passing a buoy at a time \( t \) in seconds. Express \( a \) in terms of \( \csc \theta t \).
14-4 Practice
Verifying Trigonometric Identities

Verify that each of the following is an identity.

1. \( \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta} = \sec^2 \theta \)

2. \( \frac{\cos^2 \theta}{1 - \sin^2 \theta} = 1 \)

3. \((1 + \sin \theta)(1 - \sin \theta) = \cos^2 \theta \)

4. \(\tan^4 \theta + 2 \tan^2 \theta + 1 = \sec^4 \theta \)

5. \(\cos^2 \theta \cot^2 \theta = \cot^2 \theta - \cos^2 \theta \)

6. \((\sin^2 \theta)(\csc^2 \theta + \sec^2 \theta) = \sec^2 \theta \)

7. PROJECTILES The square of the initial velocity of an object launched from the ground is \( v^2 = \frac{2gh}{\sin^2 \theta} \), where \( \theta \) is the angle between the ground and the initial path, \( h \) is the maximum height reached, and \( g \) is the acceleration due to gravity. Verify the identity \( \frac{2gh}{\sin^2 \theta} = \frac{2gh \sec^2 \theta}{\sec^2 \theta - 1} \).

8. LIGHT The intensity of a light source measured in candles is given by \( I = ER^2 \sec \theta \), where \( E \) is the illuminance in foot candles on a surface, \( R \) is the distance in feet from the light source, and \( \theta \) is the angle between the light beam and a line perpendicular to the surface. Verify the identity \( ER^2(1 + \tan^2 \theta) \cos \theta = ER^2 \sec \theta \).
Find the exact value of each expression.

1. \(\cos 75^\circ\)  
2. \(\cos 375^\circ\)  
3. \(\sin (-165^\circ)\)

4. \(\sin (-105^\circ)\)  
5. \(\sin 150^\circ\)  
6. \(\cos 240^\circ\)

7. \(\sin 225^\circ\)  
8. \(\sin (-75^\circ)\)  
9. \(\sin 195^\circ\)

Verify that each of the following is an identity.

10. \(\cos (180^\circ - \theta) = -\cos \theta\)

11. \(\sin (360^\circ + \theta) = \sin \theta\)

12. \(\sin (45^\circ + \theta) - \sin (45^\circ - \theta) = \sqrt{2} \sin \theta\)

13. \(\cos \left(x - \frac{\pi}{6}\right) + \sin \left(x - \frac{\pi}{3}\right) = \sin x\)

14. **SOLAR ENERGY** On March 21, the maximum amount of solar energy that falls on a square foot of ground at a certain location is given by \(E \sin (90^\circ - \phi)\), where \(\phi\) is the latitude of the location and \(E\) is a constant. Use the difference of angles formula to find the amount of solar energy, in terms of \(\cos \phi\), for a location that has a latitude of \(\phi\).

**ELECTRICITY** In Exercises 15 and 16, use the following information.

In a certain circuit carrying alternating current, the formula \(i = 2 \sin (120t)\) can be used to find the current \(i\) in amperes after \(t\) seconds.

15. Rewrite the formula using the sum of two angles.

16. Use the sum of angles formula to find the exact current at \(t = 1\) second.
**Practice**

**Double-Angle and Half-Angle Formulas**

Find the exact values of \( \sin 2\theta \), \( \cos 2\theta \), \( \sin \frac{\theta}{2} \), and \( \cos \frac{\theta}{2} \) for each of the following.

1. \( \cos \theta = \frac{5}{13}, \quad 0^\circ < \theta < 90^\circ \)
2. \( \sin \theta = \frac{8}{17}, \quad 90^\circ < \theta < 180^\circ \)

3. \( \cos \theta = \frac{1}{4}, \quad 270^\circ < \theta < 360^\circ \)
4. \( \sin \theta = -\frac{2}{3}, \quad 180^\circ < \theta < 270^\circ \)

Find the exact value of each expression by using the half-angle formulas.

5. \( \tan 105^\circ \) 
6. \( \tan 15^\circ \) 
7. \( \cos 67.5^\circ \) 
8. \( \sin \left( -\frac{\pi}{8} \right) \)

Verify that each of the following is an identity.

9. \( \sin^2 \frac{\theta}{2} = \frac{\tan \theta - \sin \theta}{2 \tan \theta} \)

10. \( \sin 4\theta = 4 \cos 2\theta \sin \theta \cos \theta \)

11. **AERIAL PHOTOGRAPHY** In aerial photography, there is a reduction in film exposure for any point \( X \) not directly below the camera. The reduction \( E_0 \) is given by \( E_0 = E_0 \cos^4 \theta \), where \( \theta \) is the angle between the perpendicular line from the camera to the ground and the line from the camera to point \( X \), and \( E_0 \) is the exposure for the point directly below the camera. Using the identity \( 2 \sin^2 \theta = 1 - \cos 2\theta \), verify that \( E_0 \cos^4 \theta = E_0 \left( \frac{1}{2} + \frac{\cos 2\theta}{2} \right)^2 \).

12. **IMAGING** A scanner takes thermal images from altitudes of 300 to 12,000 meters. The width \( W \) of the swath covered by the image is given by \( W = 2H' \tan \theta \), where \( H' \) is the height and \( \theta \) is half the scanner's field of view. Verify that \( \frac{2H' \sin 2\theta}{1 + \cos 2\theta} = 2H' \tan \theta \).
Practice

Solving Trigonometric Equations

Find all solutions of each equation for the given interval.

1. \( \sin 2\theta = \cos \theta \), \( 90^\circ \leq \theta < 180^\circ \)
2. \( \sqrt{2} \cos \theta = \sin 2\theta \), \( 0^\circ \leq \theta < 360^\circ \)
3. \( \cos 4\theta = \cos 2\theta \), \( 180^\circ \leq \theta < 360^\circ \)
4. \( \cos \theta + \cos (90 - \theta) = 0 \), \( 0 \leq \theta < 2\pi \)
5. \( 2 + \cos \theta = 2 \sin^2 \theta \), \( \pi \leq \theta \leq \frac{3\pi}{2} \)
6. \( \tan^2 \theta + \sec \theta = 1 \), \( \frac{\pi}{2} \leq \theta < \pi \)

Solve each equation for all values of \( \theta \) if \( \theta \) is measured in radians.

7. \( \cos^2 \theta = \sin^2 \theta \)
8. \( \cot \theta = \cot^3 \theta \)
9. \( \sqrt{2} \sin^3 \theta = \sin^2 \theta \)
10. \( \cos^2 \theta \sin \theta = \sin \theta \)
11. \( 2 \cos 2\theta = 1 - 2 \sin^2 \theta \)
12. \( \sec^2 \theta = 2 \)

Solve each equation for all values of \( \theta \) if \( \theta \) is measured in degrees.

13. \( \sin^2 \theta \cos \theta = \cos \theta \)
14. \( \csc^2 \theta - 3 \csc \theta + 2 = 0 \)
15. \( \frac{3}{1 + \cos \theta} = 4(1 - \cos \theta) \)
16. \( \sqrt{2} \cos^2 \theta = \cos^2 \theta \)

Solve each equation for all values of \( \theta \).

17. \( 4 \sin^2 \theta = 3 \)
18. \( 4 \sin^2 \theta - 1 = 0 \)
19. \( 2 \sin^2 \theta - 3 \sin \theta = -1 \)
20. \( \cos 2\theta + \sin \theta - 1 = 0 \)

21. WAVES Waves are causing a buoy to float in a regular pattern in the water. The vertical position of the buoy can be described by the equation \( h = 2 \sin x \). Write an expression that describes the position of the buoy when its height is at its midline.

22. ELECTRICITY The electric current in a certain circuit with an alternating current can be described by the formula \( i = 3 \sin 240t \), where \( i \) is the current in amperes and \( t \) is the time in seconds. Write an expression that describes the times at which there is no current.