13-2 Introduction to Matrices

What You'll Learn

- Organize data in matrices.
- Solve problems by adding or subtracting matrices or by multiplying by a scalar.

Vocabulary

- matrix
- dimensions
- row
- column
- element
- scalar multiplication

How are matrices used to organize data?

To determine the best type of aircraft to use for certain flights, the management of an airline company considers the following aircraft operating statistics.

Aircraft	Number of Seats	Airborne Speed (mph)	Possible Flight Distance (miles)	Fuel per Hour (gallons)	Operating Cost per Hour (dollars)
B747-100	462	512	2297	3517	7224
DC-10-10	297	496	1402	2311	5703
MD-11	259	527	3073	2464	6539
A300-600	228	475	1372	1505	4783

Source: Air Transport Association of America

The table has rows and columns of information. When we concentrate only on the numerical information, we see an array with 4 rows and 5 columns.

$$\begin{bmatrix} 462 & 512 & 2297 & 3517 & 7224 \\ 297 & 496 & 1402 & 2311 & 5703 \\ 259 & 527 & 3073 & 2464 & 6539 \\ 228 & 475 & 1372 & 1505 & 4783 \end{bmatrix}$$

This array of numbers is called a matrix.

Study Tip

Reading Math
A matrix is sometimes called an *ordered array*.

ORGANIZE DATA IN MATRICES If you have ever used a spreadsheet program on the computer, you have worked with matrices. A **matrix** is a rectangular arrangement of numbers in rows and columns. A matrix is usually described by its **dimensions**, or the number of **rows** and **columns**, with the number of rows stated first. Each entry in a matrix is called an **element**.

Example 1 Name Dimensions of Matrices

State the dimensions of each matrix. Then identify the position of the circled element in each matrix.

a. $[11 \quad \textcircled{15} \quad 24]$

This matrix has 1 row and 3 columns. Therefore, it is a 1-by-3 matrix.

The circled element is in the first row and the second column.

b. $\begin{bmatrix} -4 & 2 \\ 0 & 1 \\ \textcircled{3} & -6 \end{bmatrix}$

This matrix has 3 rows and 2 columns. Therefore, it is a 3-by-2 matrix.

The circled element is in the third row and the first column.

Two matrices are *equal* only if they have the same dimensions and each element of one matrix is equal to the corresponding element in the other matrix.

$$\begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix} \qquad \begin{bmatrix} 2 & 4 \\ 1 & 7 \end{bmatrix} \neq \begin{bmatrix} 2 & 3 \\ 1 & 7 \end{bmatrix} \qquad \begin{bmatrix} 4 & 8 \\ 1 & -3 \end{bmatrix} \neq \begin{bmatrix} 4 & 8 & 0 \\ 1 & -3 & 0 \end{bmatrix}$$

MATRIX OPERATIONS If two matrices have the same dimensions, you can add or subtract them. To do this, add or subtract corresponding elements of the two matrices.

Example 2 Add Matrices

If $A = \begin{bmatrix} 3 & -4 & 7 \\ -1 & 6 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 7 & -4 & -2 \\ 1 & 6 & -3 \end{bmatrix}$, and $C = \begin{bmatrix} 3 & 6 \\ -4 & 5 \end{bmatrix}$, find each sum. If the sum does not exist, write *impossible*.

a. $A + B$

$$A + B = \begin{bmatrix} 3 & -4 & 7 \\ -1 & 6 & 0 \end{bmatrix} + \begin{bmatrix} 7 & -4 & -2 \\ 1 & 6 & -3 \end{bmatrix} \qquad \text{Substitution}$$

$$= \begin{bmatrix} 3+7 & -4+(-4) & 7+(-2) \\ -1+1 & 6+6 & 0+(-3) \end{bmatrix} \qquad \text{Definition of matrix addition}$$

$$= \begin{bmatrix} 10 & -8 & 5 \\ 0 & 12 & -3 \end{bmatrix} \qquad \text{Simplify.}$$

b. $B + C$

$$B + C = \begin{bmatrix} 7 & -4 & -2 \\ 1 & 6 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -4 & 5 \end{bmatrix} \qquad \text{Substitution}$$

Since B is a 2-by-3 matrix and C is a 2-by-2 matrix, the matrices do not have the same dimensions. Therefore, it is impossible to add these matrices.

More About. . .

College Football

Each year the National Football Foundation awards the MacArthur Bowl to the number one college football team. The bowl is made of about 400 ounces of silver and represents a stadium with rows of seats.

Source: *ESPN Information Please® Sports Almanac*

Addition and subtraction of matrices can be used to solve real-world problems.

Example 3 Subtract Matrices

COLLEGE FOOTBALL **The Division I-A college football teams with the five best records during the 1990s are listed below.**

Overall Record

	Wins	Losses	Ties
Florida State	109	13	1
Nebraska	108	16	1
Marshall	114	25	0
Florida	102	22	1
Tennessee	99	22	2

Bowl Record

	Wins	Losses	Ties
Florida State	8	2	0
Nebraska	5	5	0
Marshall	2	1	0
Florida	5	4	0
Tennessee	6	4	0

Use subtraction of matrices to determine the regular season records of these teams during the decade.

$$\begin{bmatrix} 109 & 13 & 1 \\ 108 & 16 & 1 \\ 114 & 25 & 0 \\ 102 & 22 & 1 \\ 99 & 22 & 2 \end{bmatrix} - \begin{bmatrix} 8 & 2 & 0 \\ 5 & 5 & 0 \\ 2 & 1 & 0 \\ 5 & 4 & 0 \\ 6 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 109-8 & 13-2 & 1-0 \\ 108-5 & 16-5 & 1-0 \\ 114-2 & 25-1 & 0-0 \\ 102-5 & 22-4 & 1-0 \\ 99-6 & 22-4 & 2-0 \end{bmatrix}$$

$$= \begin{bmatrix} 101 & 11 & 1 \\ 103 & 11 & 1 \\ 112 & 24 & 0 \\ 97 & 18 & 1 \\ 93 & 18 & 2 \end{bmatrix}$$

So, the regular season records of the teams can be described as follows.

$$\begin{array}{l} \\ \\ \textbf{Florida State} \\ \textbf{Nebraska} \\ \textbf{Marshall} \\ \textbf{Florida} \\ \textbf{Tennessee} \end{array} \begin{array}{c} \textbf{Regular Season Record} \\ \begin{array}{ccc} \textbf{Wins} & \textbf{Losses} & \textbf{Ties} \end{array} \\ \begin{bmatrix} 101 & 11 & 1 \\ 103 & 11 & 1 \\ 112 & 24 & 0 \\ 97 & 18 & 1 \\ 93 & 18 & 2 \end{bmatrix} \end{array}$$

You can multiply any matrix by a constant called a *scalar*. This is called **scalar multiplication**. When scalar multiplication is performed, each element is multiplied by the scalar and a new matrix is formed.

Key Concept *Scalar Multiplication of a Matrix*

$$m\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} ma & mb & mc \\ md & me & mf \end{bmatrix}$$

Example 4 Perform Scalar Multiplication

If $T = \begin{bmatrix} -4 & 2 \\ 0 & 1 \\ 3 & -6 \end{bmatrix}$, find $3T$.

$3T = 3\begin{bmatrix} -4 & 2 \\ 0 & 1 \\ 3 & -6 \end{bmatrix}$ Substitution

$= \begin{bmatrix} 3(-4) & 3(2) \\ 3(0) & 3(1) \\ 3(3) & 3(-6) \end{bmatrix}$ Definition of scalar multiplication

$= \begin{bmatrix} -12 & 6 \\ 0 & 3 \\ 9 & -18 \end{bmatrix}$ Simplify.

Check for Understanding

Concept Check

1. **Describe** the difference between a 2-by-4 matrix and a 4-by-2 matrix.

2. **OPEN ENDED** Write two matrices whose sum is $\begin{bmatrix} 0 & 4 & 5 & -3 \\ 1 & -1 & 4 & 9 \end{bmatrix}$.

3. **FIND THE ERROR** Hiroshi and Estrella are finding $-5\begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix}$.

Hiroshi

$$-5\begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 10 & 5 \end{bmatrix}$$

Estrella

$$-5\begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 5 & -15 \\ 10 & -25 \end{bmatrix}$$

Who is correct? Explain your reasoning.

www.algebra1.com/extra_examples

Guided Practice

State the dimensions of each matrix. Then, identify the position of the circled element in each matrix.

4. $\begin{bmatrix} 4 & \enclose{circle}{0} & 2 \\ 5 & -1 & -3 \\ 6 & 2 & 7 \end{bmatrix}$

5. $\begin{bmatrix} \enclose{circle}{3} & -3 & 1 & 9 \end{bmatrix}$

6. $\begin{bmatrix} 5 \\ 2 \\ \enclose{circle}{1} \\ -3 \end{bmatrix}$

7. $\begin{bmatrix} 0.6 & \enclose{circle}{4.2} \\ -1.7 & 1.05 \\ 0.625 & -2.1 \end{bmatrix}$

If $A = \begin{bmatrix} 20 & -10 \\ 12 & 19 \end{bmatrix}$, $B = \begin{bmatrix} 15 & 14 \\ -10 & 6 \end{bmatrix}$, and $C = \begin{bmatrix} -5 & 7 \end{bmatrix}$, find each sum, difference, or product. If the sum or difference does not exist, write *impossible*.

8. $A + C$ **9.** $B - A$ **10.** $2A$ **11.** $-4C$

Application

PIZZA SALES **For Exercises 12–16, use the following tables that list the number of pizzas sold at Sylvia's Pizza one weekend.**

Study Tip

Reading Math
The data for Exercises 12–16 could be classified as **bivariate,** because there are two variables, size and thickness of the crust.

FRIDAY	Small	Medium	Large
Thin Crust	12	10	3
Thick Crust	11	8	8
Deep Dish	14	8	10

SATURDAY	Small	Medium	Large
Thin Crust	13	12	11
Thick Crust	1	5	10
Deep Dish	8	11	2

SUNDAY	Small	Medium	Large
Thin Crust	11	8	6
Thick Crust	1	8	11
Deep Dish	10	15	11

12. Create a matrix for each day's data. Name the matrices F, R, and N, respectively.

13. Does F equal R? Explain.

14. Create matrix T to represent $F + R + N$.

15. What does T represent?

16. Which type of pizza had the most sales during the entire weekend?

Practice and Apply

Homework Help

For Exercises	See Examples
17–26	1
27–38	2–4
39–48	3

Extra Practice
See page 849.

State the dimensions of each matrix. Then, identify the position of the circled element in each matrix.

17. $\begin{bmatrix} \enclose{circle}{2} & 1 \\ 5 & -8 \end{bmatrix}$

18. $\begin{bmatrix} -36 & 3 \\ \enclose{circle}{25} & -1 \\ 11 & 14 \end{bmatrix}$

19. $\begin{bmatrix} 1 \\ 0 \\ \enclose{circle}{-1} \end{bmatrix}$

20. $\begin{bmatrix} -3 & 56 & -21 \\ 60 & \enclose{circle}{112} & -65 \end{bmatrix}$

21. $\begin{bmatrix} -4 & 0 & -2 \\ 5 & 1 & \enclose{circle}{12} \\ -6 & 3 & -7 \end{bmatrix}$

22. $\begin{bmatrix} 1 & -2 \\ 3 & 4 \\ 1 & 5 \\ \enclose{circle}{-1} & 7 \end{bmatrix}$

23. $\begin{bmatrix} -5 & 3 & 1 \\ 4 & 0 & \enclose{circle}{2} \end{bmatrix}$

24. $\begin{bmatrix} -6 & 3 \\ \enclose{circle}{5} & -4 \end{bmatrix}$

25. Create a 2-by-3 matrix with 2 in the first row and first column and 5 in the second row and second column. The rest of the elements should be ones.

26. Create a 3-by-2 matrix with 8 in the second row and second column and 4 in the third row and second column. The rest of the elements should be zeros.

If $A = \begin{bmatrix} -1 & 5 & 9 \\ 0 & -4 & -2 \\ 3 & 7 & 6 \end{bmatrix}$, $B = \begin{bmatrix} -12 & 7 & -16 \\ 5 & 10 & 13 \\ 20 & 11 & 8 \end{bmatrix}$, $C = \begin{bmatrix} 34 & 91 & 63 \\ 81 & 79 & 60 \end{bmatrix}$, **and** $D = \begin{bmatrix} -52 & 9 & 70 \\ -49 & -8 & 45 \end{bmatrix}$, **find each sum, difference, or product. If the sum or difference does not exist, write *impossible*.**

27. $A + B$ 28. $C + D$ 29. $C - D$ 30. $B - A$

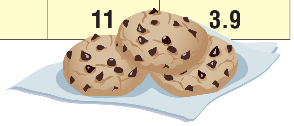
31. $5A$ 32. $2C$ 33. $A + C$ 34. $B + D$

35. $2B + A$ 36. $4A - B$ 37. $2C - 3D$ 38. $5D + 2C$

FOOD **For Exercises 39–41, use the table that shows the nutritional value of food.**

Food	Calories	Protein (grams)	Fat (grams)	Saturated Fat (grams)
Fish Stick	70	6	3	0.8
Vegetable Soup (1 cup)	70	2	2	0.3
Soft Drink (12 oz)	160	0	0	0
Chocolate-Chip Cookie	185	2	11	3.9

Source: U.S.Department of Agriculture



39. If $F = [70 \quad 6 \quad 3 \quad 0.8]$ is a matrix representing the nutritional value of a fish stick, create matrices V, S, and C to represent vegetable soup, soft drink, and chocolate chip cookie, respectively.

40. Suppose Lakeisha has two fish sticks for lunch. Write a matrix representing the nutritional value of the fish sticks.

41. Suppose Lakeisha has two fish sticks, a cup of vegetable soup, a 12-ounce soft drink, and a chocolate chip cookie. Write a matrix representing the nutritional value of her lunch.

FUND-RAISING **For Exercises 42–44, use the table that shows the last year's sales of T-shirts for the student council fund-raiser.**

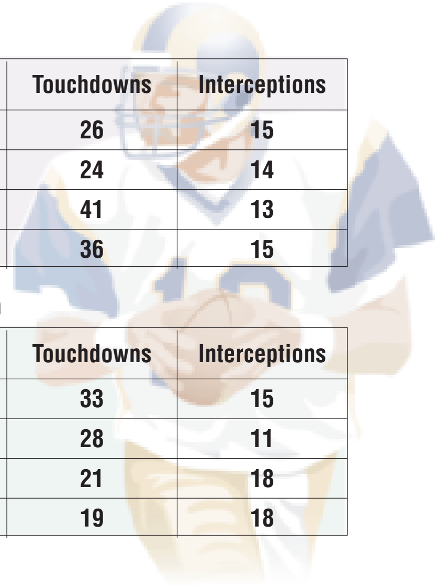
Color	XS	S	M	L	XL
Red	18	28	32	24	21
White	24	30	45	47	25
Blue	17	19	26	30	28

42. Create a matrix to show the number of T-shirts sold last year according to size and color. Label this matrix N.

43. The student council anticipates a 20% increase in T-shirt sales this year. What value of the scalar r should be used so that rN results in a matrix that estimates the number of each size and color T-shirts needed this year?

44. Calculate rN, rounding appropriately, to show estimates for this year's sales.

FOOTBALL For Exercises 45–48, use the table that shows the passing performance of four National Football League quarterbacks.



1999 Regular Season

Quarterback	Attempts	Completions	Passing Yards	Touchdowns	Interceptions
Peyton Manning	533	331	4135	26	15
Rich Gannon	515	304	3840	24	14
Kurt Warner	499	325	4353	41	13
Steve Beuerlein	571	343	4436	36	15

2000 Regular Season

Quarterback	Attempts	Completions	Passing Yards	Touchdowns	Interceptions
Peyton Manning	571	357	4413	33	15
Rich Gannon	473	284	3430	28	11
Kurt Warner	347	235	3429	21	18
Steve Beuerlein	533	324	3730	19	18

Source: ESPN

45. Create matrix A for the 1999 data and matrix B for the 2000 data.

46. What are the dimensions of each matrix in Exercise 45?

47. Calculate $T = A + B$.

48. What does matrix T represent?

49. **CRITICAL THINKING** Suppose M and N are each 3-by-3 matrices. Determine whether each statement is *sometimes*, *always*, or *never* true.

a. $M = N$ **b.** $M + N = N + M$

c. $M - N = N - M$ **d.** $5M = M$

e. $M + N = M$ **f.** $5M = N$

50. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are matrices used to organize data?

Include the following in your answer:

- a comparison of a table and a matrix, and
- description of some real-world data that could be organized in a matrix.

Standardized Test Practice

51. Which of the following is equal to $\begin{bmatrix} 3 & 4 & 5 \\ -6 & -1 & 8 \end{bmatrix}$?

(A) $\begin{bmatrix} -1 & 8 & 3 \\ -4 & 0 & 5 \end{bmatrix} + \begin{bmatrix} 4 & -4 & 2 \\ 2 & -1 & -2 \end{bmatrix}$

(B) $\begin{bmatrix} 7 & -1 & 2 \\ 3 & 4 & -5 \end{bmatrix} + \begin{bmatrix} -4 & -3 & 3 \\ -3 & -5 & -3 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & -3 & 5 \\ 7 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 7 & 0 \\ -13 & 1 & 8 \end{bmatrix}$

(D) $\begin{bmatrix} 5 & 9 & -2 \\ 3 & 7 & 5 \end{bmatrix} + \begin{bmatrix} -2 & -5 & -3 \\ 3 & -8 & 3 \end{bmatrix}$

52. Suppose M and N are each 2-by-2 matrices. If $M + N = M$, which of the following is true?

(A) $N = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(B) $N = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(C) $N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(D) $N = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Graphing Calculator

MATRIX OPERATIONS **You can use a graphing calculator to perform matrix operations. Use the EDIT command on the MATRX menu of a TI-83 Plus to enter each of the following matrices.**

$$A = \begin{bmatrix} 7.9 & 5.4 & -6.8 \\ -5.9 & 4.4 & -7.7 \end{bmatrix}, B = \begin{bmatrix} -7.2 & -5.8 & 9.1 \\ 4.3 & -8.4 & 5.3 \end{bmatrix}, C = \begin{bmatrix} 9.8 & -1.2 & 5.2 \\ -7.8 & 5.1 & -9.0 \end{bmatrix}$$

Use these stored matrices to find each sum, difference, or product.

53. $A + B$ **54.** $C - B$ **55.** $B + C - A$ **56.** $1.8A$ **57.** $0.4C$

Maintain Your Skills

Mixed Review

PRINTING **For Exercises 58 and 59, use the following information.**
To determine the quality of calendars printed at a local shop, the last 10 calendars printed each day are examined. *(Lesson 13-1)*

58. Identify the sample.

59. State whether it is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple*, *stratified*, or *systematic*. If biased, classify as *convenience* or *voluntary response*.

Solve each equation. *(Lesson 12-9)*

60. $\frac{-4}{a+1} + \frac{3}{a} = 1$ **61.** $\frac{3}{x} + \frac{4x}{x-3} = 4$ **62.** $\frac{d+3}{d+5} + \frac{2}{d-9} = \frac{5}{2d+10}$

Find the *n*th term of each geometric sequence. *(Lesson 10-7)*

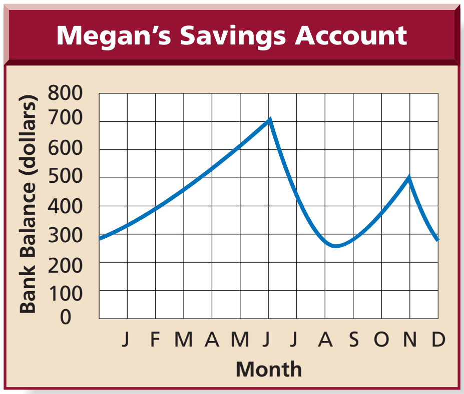
63. $a_1 = 4, n = 5, r = 3$ **64.** $a_1 = -2, n = 3, r = 7$ **65.** $a_1 = 4, n = 5, r = -2$

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*. *(Lesson 9-3)*

66. $b^2 + 7b + 12$ **67.** $a^2 + 2ab - 3b^2$ **68.** $d^2 + 8d - 15$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **For Exercises 69 and 70, use the graph that shows the amount of money in Megan's savings account.** *(To review **interpreting graphs**, see Lesson 1-9.)*



69. Describe what is happening to Megan's bank balance. Give possible reasons why the graph rises and falls at particular points.

70. Describe the elements in the domain and range.

Practice Quiz 1

Lessons 13-1 and 13-2

Identify each sample, suggest a population from which it was selected, and state whether it is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple*, *stratified*, or *systematic*. If biased, classify as *convenience* or *voluntary response*. *(Lesson 13-1)*

1. Every other household in a neighborhood is surveyed to determine how to improve the neighborhood park.

2. Every other household in a neighborhood is surveyed to determine the favorite candidate for the state's governor.

Find each sum, difference, or product. *(Lesson 13-2)*

3. $\begin{bmatrix} -8 & 3 \\ -4 & -9 \end{bmatrix} + \begin{bmatrix} 5 & -7 \\ -1 & 0 \end{bmatrix}$ **4.** $\begin{bmatrix} -9 & 6 & 4 \\ -1 & 3 & 2 \end{bmatrix} - \begin{bmatrix} 7 & -2 & 8 \\ 5 & -3 & 1 \end{bmatrix}$ **5.** $3\begin{bmatrix} 8 & -3 & -4 & 5 \\ 6 & -1 & 2 & 10 \end{bmatrix}$