Algebraic Properties

Mississippi Standard: Apply algebraic properties in problem-solving.

Review the properties in the table below. These properties can be applied when problem-solving.

### Key Concept

#### Properties

<table>
<thead>
<tr>
<th>Commutative Property</th>
<th>Associative Property</th>
<th>Distributive Property</th>
<th>Additive Identity</th>
<th>Zero Property of Multiplication</th>
<th>Multiplicative Identity</th>
</tr>
</thead>
<tbody>
<tr>
<td>The order in which numbers are added or multiplied does not change the sum or product.</td>
<td>The way in which numbers are grouped when added or multiplied does not change the sum or product.</td>
<td>To multiply a sum by a number, multiply each addend by the number outside the parentheses.</td>
<td>The sum of any number and 0 is the number.</td>
<td>The product of any number and 0 is 0.</td>
<td>The product of any number and 1 is the number.</td>
</tr>
</tbody>
</table>

#### Examples

- **Commutative Property**
  - $6 + 7 = 7 + 6$
  - $3 \cdot 8 = 8 \cdot 3$
  - $a + b = b + a$
  - $a \cdot b = b \cdot a$

- **Associative Property**
  - $(2 + 7) + 4 = 2 + (7 + 4)$
  - $(3 \cdot 4) \cdot 5 = 3 \cdot (4 \cdot 5)$
  - $(a + b) + c = a + (b + c)$
  - $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

- **Distributive Property**
  - $2(7 + 4) = 2 \cdot 7 + 2 \cdot 4$
  - $(5 + 6)3 = 5 \cdot 3 + 6 \cdot 3$
  - $a(b + c) = ab + ac$
  - $(b + c)a = ba + ca$

- **Additive Identity**
  - $7 + 0 = 7$
  - $a + 0 = a$

- **Zero Property of Multiplication**
  - $9 \cdot 0 = 0$
  - $a \cdot 0 = 0$

- **Multiplicative Identity**
  - $3 \cdot 1 = 3$
  - $a \cdot 1 = a$

### Identify Properties

Name the property shown by each statement.

1. $(7 + 3x) + 2x = 7 + (3x + 2x)$  **Associative Property**
2. $0 \cdot 5a = 0$  **Zero Property of Multiplication**

### Use Properties to Simplify Expressions

Simplify each expression. Justify each step.

3. $4 + (x + 13)$
   - $4 + (x + 13) = 4 + (13 + x)$  **Commutative Property**
   - $(4 + 13) + x$  **Associative Property**
   - $17 + x$  **Add 4 and 13.**
Apply Properties to Problem Solving

MUSEUMS Three friends are going to the science museum. The cost of admission is $x each. It will cost an additional $4 to view a movie on the 3-D screen. Write and simplify an expression that represents the total cost for the three friends. The cost of admission plus the movie can be represented by $(x + 4)$.

Multiply this cost by the number of friends, $3(x + 4)$.

$$3(x + 4) = 3(x) + 3(4) \quad \text{Distributive Property}$$

$$= 3x + 12 \quad \text{Multiply.}$$

So, the total cost for the three friends is $3x + 12$.

MUSEUMS Refer to Example 5. A fourth friend will meet the group of friends at the museum but will not go to the movie. Write and simplify an expression that represents the total cost for the four friends. The cost for the fourth friend is $x$. Add this to $3x + 12$.

$$3x + 12 + x = 3x + x + 12 \quad \text{Commutative Property}$$

$$= 4x + 12 \quad \text{Add.}$$

So, the total cost for the four friends is $4x + 12$.

Exercises

Name the property shown by each statement.

1. $3n \cdot m = m \cdot 3n$
2. $0 + 18d = 18d$
3. $(7y \cdot 8) \cdot 10y = 7y \cdot (8 \cdot 10y)$
4. $20xy \cdot 1 = 20xy$
5. $3(6a + 7b) = 3 \cdot 6a + 3 \cdot 7b$
6. $8^2 \cdot 0 = 0$

Simplify each expression. Justify each step.

7. $1 + (6 + x)$
8. $5(6a)$
9. $11 \cdot (6 \cdot n)$
10. $5(x + 8)$
11. $15(4w)$
12. $9(x - 2)$
13. $9 + 2y + 11 + 5y$
14. $4(x + 7) + 2x$
15. $11n + 7(2 + 3n)$

16. ANIMALS A zebra can run up to 40 miles per hour. An elephant can run up to $x$ miles per hour. Write and simplify an expression to find how many more miles a zebra will run in six hours than an elephant.

17. CELL PHONES Seven friends have similar cell phone plans. The price of each plan is $x$. Three of the seven friends pay an extra $4 per month for unlimited text messaging. Write and simplify an expression that represents the total cost of the seven plans.
Euler’s Formula

**Mississippi Standard:** Construct three-dimensional figures using manipulatives and generalize the relationships among vertices, faces, and edges (such as Euler’s Formula).

Recall that a *three-dimensional figure* has a length, width, and depth (or height). The flat surfaces of a three-dimensional figure are the **faces**. The line segments where the faces meet are the **edges**. The points where the edges intersect are the **vertices**.

The number of faces, vertices, and edges of a three-dimensional figure are related by Euler’s Formula.

### Key Concept

<table>
<thead>
<tr>
<th>Words</th>
<th>Euler’s Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>In a three-dimensional figure, the sum of the faces $F$ and vertices $V$ is equal to two more than the number of edges $E$.</td>
<td>$F + V = E + 2$</td>
</tr>
</tbody>
</table>

You can verify Euler’s Formula for three-dimensional figures. For example, the rectangular prism above has 6 faces, 8 vertices, and 12 edges.

\[ F + V = E + 2 \quad \text{Euler’s Formula} \]

\[ 6 + 8 \neq 12 + 2 \quad \text{Substitute 6 for } F, 8 \text{ for } V, \text{ and 12 for } E. \]

\[ 14 = 14 \checkmark \quad \text{Add. The sentence is true.} \]

Since $14 = 14$, the formula is true for the rectangular prism.

**Verify Euler’s Formula**

**Example**

Determine whether Euler’s Formula is true for the figure below.

The figure has 5 faces, 6 vertices, and 9 edges.

\[ F + V = E + 2 \quad \text{Euler’s Formula} \]

\[ 5 + 6 \neq 9 + 2 \quad \text{Substitute 5 for } F, 6 \text{ for } V, \text{ and 9 for } E. \]

\[ 11 = 11 \checkmark \quad \text{Add. The sentence is true.} \]

Yes, the formula is true for the figure shown.
**Use Euler’s Formula**

A three-dimensional figure has 5 faces and 4 vertices. Use Euler’s Formula to find the number of edges in the figure.

Use Euler’s Formula and solve for $E$.

$F + V = E + 2$  \hspace{1cm} \text{Euler’s Formula}

$5 + 4 = E + 2$  \hspace{1cm} \text{Substitute 5 for $F$ and 4 for $V$.}

$9 = E + 2$  \hspace{1cm} \text{Add 5 and 4.}

$7 = E$  \hspace{1cm} \text{Subtract 2 from each side.}

The figure has 7 edges.

**Exercises**

Verify Euler’s Formula for each figure.

1. ![Figure 1]

2. ![Figure 2]

3. ![Figure 3]

4. ![Figure 4]

5. ![Figure 5]

6. ![Figure 6]

Use Euler’s Formula to find the missing number of faces, vertices, or edges for each three-dimensional figure described.

7. $F = ?$  \hspace{1cm} 8. $F = 6$  \hspace{1cm} 9. $F = 7$

   $V = 4$  \hspace{1cm}  $V = ?$  \hspace{1cm}  $V = 7$

   $E = 6$  \hspace{1cm}  $E = 12$  \hspace{1cm}  $E = ?$

10. $F = 2$  \hspace{1cm} 11. $F = 8$  \hspace{1cm} 12. $F = ?$

    $V = 1$  \hspace{1cm}  $V = 18$  \hspace{1cm}  $V = 7$

    $E = ?$  \hspace{1cm}  $E = ?$  \hspace{1cm}  $E = 11$

13. $F = 11$  \hspace{1cm} 14. $F = ?$  \hspace{1cm} 15. $F = 10$

    $V = 11$  \hspace{1cm}  $V = 14$  \hspace{1cm}  $V = ?$

    $E = ?$  \hspace{1cm}  $E = 21$  \hspace{1cm}  $E = 16$

16. **DIAMONDS** A princess cut diamond has 5 faces and 8 edges. How many vertices are there on a princess cut diamond?
Dilations

Mississippi Standard: Explain the relationships between corresponding parts of the pre-image and image of a dilation.

Creating an image by enlarging or reducing a figure is called a dilation. The image is the figure resulting from the dilation. The pre-image is the original figure. The pre-image and image are similar figures. You can examine the corresponding vertices and corresponding sides to describe a relationship between the pre-image and image of a dilation.

Describe a Dilation

Figure \(ABCD\) is dilated to create figure \(A'B'C'D'\).

Describe how the image changed from the pre-image.

Examine the corresponding vertices of the pre-image and the image.

\[
\begin{align*}
A(1, 3) &\rightarrow A'(2, 6) \\
B(5, 3) &\rightarrow B'(10, 6) \\
C(5, 1) &\rightarrow C'(10, 2) \\
D(1, 1) &\rightarrow D'(2, 2)
\end{align*}
\]

The vertices of the pre-image, \(ABCD\), were doubled to create the image, \(A'B'C'D'\).

Examine the lengths of the corresponding sides of the pre-image and image.

\[
\begin{align*}
\overline{AB} = 4 &\rightarrow \overline{AB'} = 8 \\
\overline{BC} = 2 &\rightarrow \overline{BC'} = 4 \\
\overline{AD} = 2 &\rightarrow \overline{AD'} = 4
\end{align*}
\]

The sides of the pre-image, \(ABCD\), were doubled to create the image, \(A'B'C'D'\).

In Example 1, the vertices and sides of the pre-image were doubled to create the image. Therefore, the scale factor is 2. The scale factor refers to the ratio of corresponding parts of similar figures. If the scale factor is greater than 1, the dilation is an enlargement. If the scale factor is less than 1, the dilation is a reduction.

Find and Classify a Scale Factor

Segment \(X'Y'\) is a dilation of segment \(XY\). Find the scale factor of the dilation, and classify it as an enlargement or as a reduction.

Write a ratio of the \(x\)- or \(y\)-coordinate of one vertex of the image to the \(x\)- or \(y\)-coordinate of the corresponding vertex of the pre-image. Use the \(y\)-coordinates of \(X(0, 2)\) and \(X'(0, 5)\).

\[
\frac{y\text{-coordinate of point } X'}{y\text{-coordinate of point } X} = \frac{5}{2}
\]

The scale factor is \(\frac{5}{2}\). Since the scale factor is greater than 1, the dilation is an enlargement.
Exercises

Describe how the corresponding parts of the image and pre-image are related.

1. In each figure, the green figure is a dilation of the blue figure. Find the scale factor of each dilation and classify it as an enlargement or as a reduction.

5. GEOMETRY Segment $C'D'$ with endpoints $C'(2, 2)$ and $D'(5, 1)$ is a dilation of segment $CD$ with endpoints $C(4, 4)$ and $D(10, 2)$. Find the scale factor of the dilation, and classify it as an enlargement or as a reduction.

8. COMPUTERS Carlos graphed a triangle with vertices $A(1, 1)$, $B(1, 3)$, and $C(3, 1)$ using geometry software. If he enlarges the triangle by a scale factor of $3$, what will be the vertices of the dilation?

9. PHOTOS To place a photo in a frame, Emma must reduce the photo by a scale factor of $\frac{1}{4}$. Find the dimensions of the reduced photo if the original is 8 inches by 10 inches.

DESIGN For Exercises 11 and 12, use the following information.

Dante’s logo design won a competition for creating the logo for the new town library. The logo, which is 4 inches wide and 7 inches long will be enlarged and used on bags for the library. On the bags, the logo will be 10 inches wide.

11. What is the scale factor for this enlargement?

12. What will be the length of the logo on the bags?