Using Your North Carolina StudyText

*North Carolina StudyText, Math BC, Volume 1*, is a practice workbook designed to help you master the North Carolina Essential Standards for High School Math BC. By mastering the mathematics standards, you will be prepared to do well on your end-of-course (EOC) test. This StudyText is divided into two sections.

**Chapter Resources**

- Each chapter contains four pages for each key lesson in your *North Carolina Geometry* Student Edition. Your teacher may ask you to complete one or more of these worksheets as an assignment.

**Mastering the EOC**

This section of StudyText is composed of three parts. Each part can help you study for your EOC test.

- The **Diagnostic Test** can help you determine which standards you might need to review before taking the EOC test. Each question lists the standard that it is assessing. Your teacher may assign review pages based on the questions that you did not answer correctly.

- **Practice by Standard** gives you more practice problems to help you become a better test-taker. The problems are organized by the North Carolina High School Math BC Essential Standards. You can also use these pages as a general review before you take the EOC test.

- The **Practice Test** can be used to simulate what an EOC test might be like so that you will be better prepared to take it in the spring.
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Mastering the EOC, Geometry / Math BC, Volume 1

   Diagnostic Test ........................................ A1
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Points, Lines, and Planes

Name Points, Lines, and Planes In geometry, a point is a location, a line contains points, and a plane is a flat surface that contains points and lines. If points are on the same line, they are collinear. If points on are the same plane, they are coplanar.

Example

Use the figure to name each of the following.

a. a line containing point $A$

The line can be named as $\ell$. Also, any two of the three points on the line can be used to name it. $AB$, $AC$, or $BC$.

b. a plane containing point $D$

The plane can be named as plane $\mathcal{N}$ or can be named using three noncollinear points in the plane, such as plane $ABD$, plane $ACD$, and so on.

Exercises

Refer to the figure.

1. Name a line that contains point $A$.

2. What is another name for line $m$?

3. Name a point not on $AC$.

4. What is another name for line $\ell$?

5. Name a point not on line $\ell$ or line $m$.

Draw and label a figure for each relationship.

6. $\overrightarrow{AB}$ is in plane $Q$.

7. $\overrightarrow{ST}$ intersects $\overrightarrow{AB}$ at $P$.

8. Point $X$ is collinear with points $A$ and $P$.

9. Point $Y$ is not collinear with points $T$ and $P$.

10. Line $\ell$ contains points $X$ and $Y$. 
Points, Lines, and Planes

Points, Lines, and Planes in Space  
Space is a boundless, three-dimensional set of all points. It contains lines and planes. The intersection of two or more geometric figures is the set of points they have in common.

Example

a. Name the intersection of the planes \( O \) and \( \mathcal{N} \).  
The planes intersect at line \( \overrightarrow{AB} \).

b. Does \( \overrightarrow{AB} \) intersect point \( D \)? Explain.  
No. \( AB \) is coplanar with \( D \), but \( D \) is not on the line \( \overrightarrow{AB} \).

Exercises

Refer to the figure.

1. Name the intersection of plane \( \mathcal{N} \) and line \( \overrightarrow{AE} \).

2. Name the intersection of \( \overrightarrow{BC} \) and \( \overrightarrow{DC} \).

3. Does \( \overrightarrow{DC} \) intersect \( \overrightarrow{AE} \)? Explain.

Refer to the figure.

4. Name the three line segments that intersect at point \( A \).

5. Name the line of intersection of planes \( GAB \) and \( FEH \).

6. Do planes \( GFE \) and \( HBC \) intersect? Explain.
Refer to the figure.

1. Name a line that contains points T and P.

2. Name a line that intersects the plane containing points Q, N, and P.

3. Name the plane that contains $\overrightarrow{TN}$ and $\overrightarrow{QR}$.

Draw and label a figure for each relationship.

4. $\overrightarrow{AK}$ and $\overrightarrow{CG}$ intersect at point M in plane $T$.

5. A line contains $L(-4, -4)$ and $M(2, 3)$. Line $q$ is in the same coordinate plane but does not intersect $\overrightarrow{LM}$. Line $q$ contains point N.

Refer to the figure.

6. How many planes are shown in the figure?

7. Name three collinear points.


VISUALIZATION Name the geometric term(s) modeled by each object.

9. STOP

10. tip of pin

11.

12. a car antenna

13. a library card
1-1 Word Problem Practice

Points, Lines, and Planes

1. STREETS The map shows some of the roads in downtown Little Rock. Lines are used to represent streets and points are used to represent intersections. Four of the street intersections are labeled. What street corresponds to line AB?

2. FLYING Marsha plans to fly herself from Gainsville to Miami. She wants to model her flight path using a straight line connecting the two cities on the map. Sketch her flight path on the map shown below.

3. MAPS Nathan’s mother wants him to go to the post office and the supermarket. She tells him that the post office, the supermarket and their home are collinear, and the post office is between the supermarket and their home. Make a map showing the three locations based on this information.

4. ARCHITECTURE An architect models the floor, walls, and ceiling of a building with planes. To locate one of the planes that will represent a wall, the architect starts by marking off two points in the plane that represents the floor. What further information can the architect give to specify the plane that will represent the wall?

5. CONSTRUCTION Mr. Riley gave his students some rods to represent lines and some clay to show points of intersection. Below is the figure Lynn constructed with all of the points of intersection and some of the lines labeled.

a. What is the intersection of lines k and n?

b. Name the lines that intersect at point C.

c. Are there 3 points that are collinear and coplanar? If so, name them.
Linear Measure

Measure Line Segments  A part of a line between two endpoints is called a line segment. The lengths of $\overline{MN}$ and $\overline{RS}$ are written as $MN$ and $RS$. All measurements are approximations dependant upon the smallest unit of measure available on the measuring instrument.

**Example 1** Find the length of $\overline{MN}$.

The long marks are centimeters, and the shorter marks are millimeters. There are 10 millimeters for each centimeter. The length of $\overline{MN}$ is about 34 millimeters.

**Example 2** Find the length of $\overline{RS}$.

The long marks are inches and the short marks are quarter inches. Point S is closer to the $1\frac{3}{4}$ inch mark. The length of $\overline{RS}$ is about $1\frac{3}{4}$ inches.

**Exercises**

Find the length of each line segment or object.

1. $\overline{AB}$
   - cm 1 2 3

2. $\overline{ST}$
   - in. 1 2

3. Pencil
   - in. 1 2

4. Object
   - cm 1 2 3

5. Pentagon
   - cm 1 2 3 4 5 6

6. Heart
   - in. 1 2
**Linear Measure**

**Calculate Measures** On \( PQ \), to say that point \( M \) is between points \( P \) and \( Q \) means \( P, Q, \) and \( M \) are collinear and \( PM + MQ = PQ \).

On \( AC \), \( AB = BC = 3 \text{ cm} \). We can say that the segments are **congruent segments**, or \( AB \cong BC \). Slashes on the figure indicate which segments are congruent.

**Example 1** Find \( EF \).

![Diagram showing points E, D, and F with measurements 1.2 cm and 1.9 cm.]

Point \( D \) is between \( E \) and \( F \). Calculate \( EF \) by adding \( ED \) and \( DF \).

\[
ED + DF = EF \quad \text{Betweenness of points}
\]

\[
1.2 + 1.9 = EF \quad \text{Substitution}
\]

\[
3.1 = EF \quad \text{Simplify.}
\]

Therefore, \( EF \) is 3.1 centimeters long.

**Example 2** Find \( x \) and \( AC \).

![Diagram showing points A, B, and C with segments marked 2x + 5 and 2x.]

\( B \) is between \( A \) and \( C \).

\[
AB + BC = AC \quad \text{Betweenness of points}
\]

\[
x + 2x = 2x + 5 \quad \text{Substitution}
\]

\[
3x = 2x + 5 \quad \text{Add } x + 2x.
\]

\[
x = 5 \quad \text{Simplify.}
\]

\[
AC = 2x + 5 = 2(5) + 5 = 15
\]

**Exercises**

Find the measurement of each segment. Assume that each figure is not drawn to scale.

1. \( \overline{RT} \)
2. \( \overline{BC} \)
3. \( \overline{XZ} \)
4. \( \overline{WX} \)

**ALGEBRA** Find the value of \( x \) and \( RS \) if \( S \) is between \( R \) and \( T \).

5. \( RS = 5x, ST = 3x, \) and \( RT = 48 \)
6. \( RS = 2x, ST = 5x + 4, \) and \( RT = 32 \)
7. \( RS = 6x, ST =12, \) and \( RT = 72 \)
8. \( RS = 4x, ST = 4x, \) and \( RT = 24 \)

Determine whether each pair of segments is congruent.

9. \( \overline{AB}, \overline{CD} \)
10. \( \overline{XY}, \overline{YZ} \)
Practice

Linear Measure

Find the length of each line segment or object.

1. \( EF \)

\[
\begin{array}{c}
\text{in.} \\
1 \\
2 \\
\end{array}
\]

2. \( \star \star \star \star \star \star \star \star \star \)

\[
\begin{array}{c}
\text{cm} \\
1 \\
2 \\
3 \\
4 \\
5 \\
\end{array}
\]

Find the measurement of each segment. Assume that each figure is not drawn to scale.

3. \( PS \)

\[
\begin{array}{c}
P \\
Q \\
S \\
\end{array}
\]

4. \( AD \)

\[
\begin{array}{c}
A \\
C \\
D \\
\end{array}
\]

5. \( WX \)

\[
\begin{array}{c}
W \\
X \\
Y \\
\end{array}
\]

ALGEBRA Find the value of \( x \) and \( KL \) if \( K \) is between \( J \) and \( L \).

6. \( JK = 6x, KL = 3x, \text{ and } JL = 27 \)

7. \( JK = 2x, KL = x + 2, \text{ and } JL = 5x - 10 \)

Determine whether each pair of segments is congruent.

8. \( TU, SW \)

\[
\begin{array}{c}
T \\
U \\
W \\
\end{array}
\]

9. \( AD, BC \)

\[
\begin{array}{c}
A \\
B \\
C \\
\end{array}
\]

10. \( GF, FE \)

\[
\begin{array}{c}
G \\
F \\
E \\
\end{array}
\]

11. CARPENTRY Jorge used the figure at the right to make a pattern for a mosaic he plans to inlay on a tabletop. Name all of the congruent segments in the figure.
1-2

Word Problem Practice

Linear Measure

1. **MEASURING** Vera is measuring the size of a small hexagonal silver box that she owns. She places a standard 12 inch ruler alongside the box. About how long is one of the sides of the box?

![Hexagonal box]

2. **WALKING** Marshall lives 2300 yards from school and 1500 yards from the pharmacy. The school, pharmacy, and his home are all collinear, as shown in the figure.

   ![Collinear points]

   What is the total distance from the pharmacy to the school?

3. **HIKING TRAIL** A hiking trail is 20 kilometers long. Park organizers want to build 5 rest stops for hikers with one on each end of the trail and the other 3 spaced evenly between. How much distance will separate successive rest stops?

4. **RAILROADS** A straight railroad track is being built to connect two cities. The measured distance of the track between the two cities is 160.5 miles. A mailstop is 28.5 miles from the first city. How far is the mailstop from the second city?

5. **BUILDING BLOCKS** Lucy’s younger brother has three wooden cylinders. They have heights 8 inches, 4 inches, and 6 inches and can be stacked one on top of the other.

   ![Cylinders]

   a. If all three cylinders are stacked one on top of the other, how high will the resulting column be? Does it matter in what order the cylinders are stacked?

   b. What are all the possible heights of columns that can be built by stacking some or all of these cylinders?
Making Conjectures  Inductive reasoning is reasoning that uses information from different examples to form a conclusion or statement called a conjecture.

**Example 1**  Write a conjecture about the next number in the sequence 1, 3, 9, 27, 81.

Look for a pattern:
Each number is a power of 3.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>3</th>
<th>9</th>
<th>27</th>
<th>81</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$3^0$</td>
<td> </td>
<td> </td>
<td> </td>
<td> </td>
</tr>
<tr>
<td>1</td>
<td>$3^1$</td>
<td> </td>
<td> </td>
<td> </td>
<td> </td>
</tr>
<tr>
<td>2</td>
<td>$3^2$</td>
<td> </td>
<td> </td>
<td> </td>
<td> </td>
</tr>
<tr>
<td>3</td>
<td>$3^3$</td>
<td> </td>
<td> </td>
<td> </td>
<td> </td>
</tr>
<tr>
<td>4</td>
<td>$3^4$</td>
<td> </td>
<td> </td>
<td> </td>
<td> </td>
</tr>
</tbody>
</table>

Conjecture: The next number will be $3^5$ or 243.

**Example 2**  Write a conjecture about the number of small squares in the next figure.

Look for a pattern: The sides of the squares have measures 1, 2, and 3 units.

Conjecture: For the next figure, the side of the square will be 4 units, so the figure will have 16 small squares.

**Exercises**

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.

1. $-5, 10, -20, 40$

2. $1, 10, 100, 1000$

3. $1, \frac{6}{5}, \frac{7}{5}, \frac{8}{5}$

Write a conjecture about each value or geometric relationship.

4. $A(-1, -1), B(2, 2), C(4, 4)$

5. $\angle 1$ and $\angle 2$ form a right angle.

6. $\angle ABC$ and $\angle DBE$ are vertical angles.

7. $\angle E$ and $\angle F$ are right angles.
Find Counterexamples  A conjecture is false if there is even one situation in which the conjecture is not true. The false example is called a **counterexample**.

**Example**  Find a counterexample to show the conjecture is false.

If $\overline{AB} \cong \overline{BC}$, then $B$ is the midpoint of $\overline{AC}$.

Is it possible to draw a diagram with $\overline{AB} \cong \overline{BC}$ such that $B$ is not the midpoint? This diagram is a counterexample because point $B$ is not on $\overline{AC}$. The conjecture is false.

**Exercises**

Determine whether each conjecture is **true** or **false**. Give a counterexample for any false conjecture.

1. If points $A$, $B$, and $C$ are collinear, then $AB + BC = AC$.

2. If $\angle R$ and $\angle S$ are supplementary, and $\angle R$ and $\angle T$ are supplementary, then $\angle T$ and $\angle S$ are congruent.

3. If $\angle ABC$ and $\angle DEF$ are supplementary, then $\angle ABC$ and $\angle DEF$ form a linear pair.

4. If $\overline{DE} \perp \overline{EF}$, then $\angle DEF$ is a right angle.
Make a conjecture about the next item in each sequence.

1. 0, 3, 6, ...

2. 5, −10, 15, −20

3. −2, 1, −\(\frac{1}{2}\), \(\frac{1}{4}\), −\(\frac{1}{8}\)

4. 12, 6, 3, 1.5, 0.75

Make a conjecture about each value or geometric relationship.

5. \(\angle ABC\) is a right angle.

6. Point \(S\) is between \(R\) and \(T\).

7. \(P, Q, R,\) and \(S\) are noncollinear and \(PQ \cong QR \cong RS \cong SP\).

8. \(ABCD\) is a parallelogram.

Determine whether each conjecture is true or false. Give a counterexample for any false conjecture.

9. If \(S, T,\) and \(U\) are collinear and \(ST = TU\), then \(T\) is the midpoint of \(SU\).

10. If \(\angle 1\) and \(\angle 2\) are adjacent angles, then \(\angle 1\) and \(\angle 2\) form a linear pair.

11. If \(GH\) and \(JK\) form a right angle and intersect at \(P\), then \(GH \perp JK\).

12. **Allergies** Each spring, Rachel starts sneezing when the pear trees on her street blossom. She reasons that she is allergic to pear trees. Find a counterexample to Rachel’s conjecture.
1. RAMPS Rodney is rolling marbles down a ramp. Every second that passes, he measures how far the marbles travel. He records the information in the table shown below.

<table>
<thead>
<tr>
<th>Second</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (cm)</td>
<td>20</td>
<td>60</td>
<td>100</td>
<td>140</td>
</tr>
</tbody>
</table>

Make a conjecture about how far the marble will roll in the fifth second.

2. PRIMES A prime number is a number other than 1, that is divisible by only itself and 1. Lucille read that prime numbers are very important in cryptography, so she decided to find a systematic way of producing prime numbers. After some experimenting, she conjectured that $2^n - 1$ is a prime for all whole numbers $n > 1$. Find a counterexample to this conjecture.

3. GENEALOGY Miranda is developing a chart that shows her ancestry. She makes the three sketches shown below. The first dot represents herself. The second sketch represents herself and her parents. The third sketch represents herself, her parents, and her grandparents.

Sketch what you think would be the next figure in the sequence.

4. MEDALS Barbara is in charge of the award medals for a sporting event. She has 31 medals to give out to various individuals on 6 competing teams. She asserts that at least one team will end up with more than 5 medals. Do you believe her assertion? If you do, try to explain why you think her assertion is true, and if you do not, explain how she can be wrong.

5. PATTERNS The figure shows a sequence of squares each made out of identical square tiles.

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```

a. Starting from zero tiles, how many tiles do you need to make the first square? How many tiles do you have to add to the first square to get the second square? How many tiles do you have to add to the second square to get the third square?

b. Make a conjecture about the list of numbers you started writing in your answer to Exercise a.

c. Make a conjecture about the sum of the first $n$ odd numbers.
**2-2 Study Guide**

**Logic**

**Determine Truth Values** A statement is any sentence that is either true or false. The truth value of a statement is either true (T) or false (F). A statement can be represented by using a letter. For example,

Statement \( p \): Chicago is a city in Illinois. The truth value of statement \( p \) is true.

Several statements can be joined in a compound statement.

<table>
<thead>
<tr>
<th>Negation:</th>
<th>Statement ( p ) and statement ( q ) joined by the word and is a conjunction.</th>
<th>Statement ( p ) and statement ( q ) joined by the word or is a disjunction.</th>
</tr>
</thead>
<tbody>
<tr>
<td>not ( p ) (Read: not ( p ))</td>
<td>Symbols: ( p \land q ) (Read: ( p ) and ( q ))</td>
<td>Symbols: ( p \lor q ) (Read: ( p ) or ( q ))</td>
</tr>
<tr>
<td>The statements ( p ) and ( \neg p ) have opposite truth values.</td>
<td>The conjunction ( p \land q ) is true only when both ( p ) and ( q ) are true.</td>
<td>The disjunction ( p \lor q ) is true if ( p ) is true, if ( q ) is true, or if both are true.</td>
</tr>
</tbody>
</table>

**Example 1** Write a compound statement for each conjunction. Then find its truth value.

\( p \): An elephant is a mammal.
\( q \): A square has four right angles.

a. \( p \land q \)

Join the statements with and: An elephant is a mammal and a square has four right angles. Both parts of the statement are true so the compound statement is true.

b. \( \neg p \land q \)

\( \neg p \) is the statement “An elephant is not a mammal.” Join \( \neg p \) and \( q \) with the word and: An elephant is not a mammal and a square has four right angles. The first part of the compound statement, \( \neg p \), is false. Therefore the compound statement is false.

**Example 2** Write a compound statement for each disjunction. Then find its truth value.

\( p \): A diameter of a circle is twice the radius.
\( q \): A rectangle has four equal sides.

a. \( p \lor q \)

Join the statements \( p \) and \( q \) with the word or: A diameter of a circle is twice the radius or a rectangle has four equal sides. The first part of the compound statement, \( p \), is true, so the compound statement is true.

b. \( \neg p \lor q \)

Join \( \neg p \) and \( q \) with the word or: A diameter of a circle is not twice the radius or a rectangle has four equal sides. Neither part of the disjunction is true, so the compound statement is false.

<table>
<thead>
<tr>
<th>Exercises</th>
</tr>
</thead>
</table>

Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value.

\( p \): 10 + 8 = 18 \( q \): September has 30 days \( r \): A rectangle has four sides.

1. \( p \) and \( q \)
2. \( p \lor r \)
3. \( q \) or \( r \)
4. \( q \land \neg r \)
Truth Tables  One way to organize the truth values of statements is in a truth table. The truth tables for negation, conjunction, and disjunction are shown at the right.

<table>
<thead>
<tr>
<th>p</th>
<th>\sim p</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p \land q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p \lor q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>q</th>
<th>r</th>
<th>q \lor r</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

### Example 1
Construct a truth table for the compound statement \(q \lor r\).
Use the disjunction table.

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>r</th>
<th>(q \lor r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
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<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
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<td>F</td>
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<td>F</td>
<td>F</td>
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<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

### Example 2
Construct a truth table for the compound statement \(p\) and \((q \lor r)\).
Use the disjunction table for \((q \lor r)\). Then use the conjunction table for \(p\) and \((q \lor r)\).

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>r</th>
<th>(q \lor r)</th>
<th>p \land (q \lor r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
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<td>F</td>
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<td>F</td>
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</tbody>
</table>

### Exercises
Construct a truth table for each compound statement.

1. \(p \lor r\)
2. \(\sim p \lor q\)
3. \(q \land \sim r\)
4. \(\sim p \land \sim r\)
5. \((p \land r) \lor q\)
Use the following statements to write a compound statement for each conjunction
or disjunction. Then find its truth value.

\( p \): 60 seconds = 1 minute
\( q \): Congruent supplementary angles each have a measure of 90.
\( r \): \(-12 + 11 < -1\)

1. \( p \land q \)

2. \( q \lor r \)

3. \( \sim p \lor q \)

4. \( \sim p \land \sim r \)

Complete each truth table.

5. \[
\begin{array}{c|c|c|c|c}
 p & q & \sim p & \sim q & \sim p \lor \sim q \\
 T & T & & & \\
 T & F & & & \\
 F & T & & & \\
 F & F & & & \\
\end{array}
\]

6. \[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c}
 p & q & \sim p & \sim q & \sim p \lor q & \sim p \land (\sim p \lor q) \\
 T & T & & & & \\
 T & F & & & & \\
 F & T & & & & \\
 F & F & & & & \\
\end{array}
\]

Construct a truth table for each compound statement.

7. \( q \lor (p \land \sim q) \)

8. \( \sim q \land (\sim p \lor q) \)

9. **SCHOOL** The Venn diagram shows the number of students in the band who work after school or on the weekends.
   a. How many students work after school and on weekends?
   b. How many students work after school or on weekends?
1. HOCKEY  Carol asked John if his hockey team won the game last night and if he scored a goal. John said “yes.” Carol then asked Peter if he or John scored a goal at the game. Peter said “yes.” What can you conclude about whether or not Peter scored?

2. CHOCOLATE  Nash has a bag of miniature chocolate bars that come in two distinct types: dark and milk. Nash picks a chocolate out of the bag. Consider these statements:
   \( p \): the chocolate bar is dark chocolate
   \( q \): the chocolate bar is milk chocolate
   Is the following statement true?
   \[ \sim (\sim p \land \sim q) \]

3. VIDEO GAMES  Harold is allowed to play video games only if he washes the dishes or takes out the trash. However, if Harold does not do his homework, he is not allowed to play video games under any circumstance. Complete the truth table.
   \( p \): Harold has washed the dishes
   \( q \): Harold has taken out the trash
   \( r \): Harold has done his homework
   \( s \): Harold is allowed to play video games

   \[ \begin{array}{cccc}
   p & q & r & s \\
   T & T & T & T \\
   T & T & F & F \\
   T & F & T & T \\
   T & F & F & F \\
   F & T & T & T \\
   F & T & F & F \\
   F & F & T & F \\
   F & F & F & F \\
   \end{array} \]

4. CIRCUITS  In Earl’s house, the dining room light is controlled by two switches according to the following table.

<table>
<thead>
<tr>
<th>Switch A</th>
<th>Switch B</th>
<th>Light</th>
</tr>
</thead>
<tbody>
<tr>
<td>up</td>
<td>up</td>
<td>off</td>
</tr>
<tr>
<td>up</td>
<td>down</td>
<td>on</td>
</tr>
<tr>
<td>down</td>
<td>up</td>
<td>on</td>
</tr>
<tr>
<td>down</td>
<td>down</td>
<td>off</td>
</tr>
</tbody>
</table>

   If up and on are considered true and down and off are considered false, write an expression that gives the truth value of the light as a function of the truth values of the two switches.

5. READING  Two hundred people were asked what kind of literature they like to read. They could choose among novels, poetry, and plays. The results are shown in the Venn diagram.

   a. How many people said they like all three types of literature?

   b. How many like to read poetry?

   c. What percentage of the people who like plays also like novels and poetry?
If-Then Statements

An if-then statement is a statement such as “If you are reading this page, then you are studying math.” A statement that can be written in if-then form is called a **conditional statement**. The phrase immediately following the word *if* is the **hypothesis**. The phrase immediately following the word *then* is the **conclusion**.

A conditional statement can be represented in symbols as $p \rightarrow q$, which is read “$p$ implies $q$” or “if $p$, then $q$.”

### Example 1

**Identify the hypothesis and conclusion of the conditional statement.**

If $\angle X \cong \angle R$ and $\angle R \cong \angle S$, then $\angle X \cong \angle S$.

- **Hypothesis:** $\angle X \cong \angle R$ and $\angle R \cong \angle S$
- **Conclusion:** $\angle X \cong \angle S$

### Example 2

**Identify the hypothesis and conclusion. Write the statement in if-then form.**

You receive a free pizza with 12 coupons.

If you have 12 coupons, then you receive a free pizza.

- **Hypothesis:** You have 12 coupons
- **Conclusion:** You receive a free pizza

### Exercises

**Identify the hypothesis and conclusion of each conditional statement.**

1. If it is Saturday, then there is no school.
2. If $x - 8 = 32$, then $x = 40$.
3. If a polygon has four right angles, then the polygon is a rectangle.

**Write each statement in if-then form.**

4. All apes love bananas.
5. The sum of the measures of complementary angles is 90.
6. Collinear points lie on the same line.

**Determine the truth value of each conditional statement. If true, explain your reasoning. If false, give a counterexample.**

7. If today is Wednesday, then yesterday was Friday.

8. If $a$ is positive, then $10a$ is greater than $a$. 
Converse, Inverse, and Contrapositive  If you change the hypothesis or conclusion of a conditional statement, you form related conditionals. This chart shows the three related conditionals, converse, inverse, and contrapositive, and how they are related to a conditional statement.

<table>
<thead>
<tr>
<th></th>
<th>Symbols</th>
<th>Formed by</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conditional</strong></td>
<td>$p \rightarrow q$</td>
<td>using the given hypothesis and conclusion</td>
<td>If two angles are vertical angles, then they are congruent.</td>
</tr>
<tr>
<td><strong>Converse</strong></td>
<td>$q \rightarrow p$</td>
<td>exchanging the hypothesis and conclusion</td>
<td>If two angles are congruent, then they are vertical angles.</td>
</tr>
<tr>
<td><strong>Inverse</strong></td>
<td>$\sim p \rightarrow \sim q$</td>
<td>replacing the hypothesis with its negation and replacing the conclusion with its negation</td>
<td>If two angles are not vertical angles, then they are not congruent.</td>
</tr>
<tr>
<td><strong>Contrapositive</strong></td>
<td>$\sim q \rightarrow \sim p$</td>
<td>negating the hypothesis, negating the conclusion, and switching them</td>
<td>If two angles are not congruent, then they are not vertical angles.</td>
</tr>
</tbody>
</table>

Just as a conditional statement can be true or false, the related conditionals also can be true or false. A conditional statement always has the same truth value as its contrapositive, and the converse and inverse always have the same truth value.

**Exercises**

Write the converse, inverse, and contrapositive of each true conditional statement. Determine whether each related conditional is true or false. If a statement is false, find a counterexample.

1. If you live in San Diego, then you live in California.
2. If a polygon is a rectangle, then it is a square.
3. If two angles are complementary, then the sum of their measures is 90.
Identify the hypothesis and conclusion of each conditional statement.

1. If $3x + 4 = -5$, then $x = -3$.

2. If you take a class in television broadcasting, then you will film a sporting event.

Write each statement in if-then form.

3. “Those who do not remember the past are condemned to repeat it.” (George Santayana)

4. Adjacent angles share a common vertex and a common side.

Determine the truth value of each conditional statement. If true, explain your reasoning. If false, give a counterexample.

5. If $a$ and $b$ are negative, then $a + b$ is also negative.

6. If two triangles have equivalent angle measures, then they are congruent.

7. If the moon has purple spots, then it is June.

8. SUMMER CAMP Older campers who attend Woodland Falls Camp are expected to work. Campers who are juniors wait on tables.
   
   a. Write a conditional statement in if-then form.
   
   b. Write the converse of your conditional statement.
1. **TANNING** Maya reads in a paper that people who tan themselves under the Sun for extended periods are at increased risk of skin cancer. From this information, can she conclude that she will not increase her risk of skin cancer if she avoids tanning for extended periods of time?

2. **PARALLELOGRAMS** Clark says that being a parallelogram is equivalent to being a quadrilateral with equal opposite angles. Write his statement in if-then form.

3. **AIR TRAVEL** Ulma is waiting to board an airplane. Over the speakers she hears a flight attendant say “If you are seated in rows 10 to 20, you may now board.” What are the inverse, converse, and the contrapositive of this statement?

4. **MEDICATION** Linda’s medicine bottle says “If you will be driving, then you should not take this medicine.” What are the inverse, converse, and the contrapositive of this statement?

5. **VENN DIAGRAMS** José made this Venn diagram to show how quadrilaterals, rectangles, squares, and rhombi are related. (A rhombus is a quadrilateral with four sides of equal length.)

Let Q be a quadrilateral. For each problem tell whether the statement is true or false. If it is false, provide a counterexample.

a. If Q is a square, then Q a rectangle.

b. If Q is not a rectangle, then Q is not a rhombus.

c. If Q is a rectangle but not a square, then Q is not a rhombus.

d. If Q is not a rhombus, then Q is not a square.
Deductive Reasoning

Law of Detachment  **Deductive reasoning** is the process of using facts, rules, definitions, or properties to reach conclusions. One form of deductive reasoning that draws conclusions from a true conditional \( p \rightarrow q \) and a true statement \( p \) is called the **Law of Detachment**.

| Law of Detachment | If \( p \rightarrow q \) is true and \( p \) is true, then \( q \) is true. |

---

**Example**  Determine whether each conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

a. Given: Two angles supplementary to the same angle are congruent. \( \angle A \) and \( \angle C \) are supplementary to \( \angle B \).
   
   **Conclusion:** \( \angle A \) is congruent to \( \angle C \).
   
   The statement \( \angle A \) and \( \angle C \) are supplementary to \( \angle B \) is the hypothesis of the conditional. Therefore, by the Law of Detachment, the conclusion is true.

b. Given: If Helen is going to work, then she is wearing pearls. Helen is wearing pearls.
   
   **Conclusion:** Helen is going to work.
   
   The given statement *Helen is going to work* satisfies the conclusion of the true conditional. However, knowing that a conditional statement and its conclusion are true does not make the hypothesis true. Helen could be wearing pearls on a date. The conclusion is invalid.

**Exercises**

Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

1. **Given:** If a number is divisible by 6, then the number is divisible by 3.
   
   **18** is divisible by 6.
   
   **Conclusion:** 18 is divisible by 3.

2. **Given:** If a pet is a rabbit, then it eats carrots. Jennie’s pet eats carrots.
   
   **Conclusion:** Jennie’s pet is a rabbit.

3. **Given:** If a hen is a Plymouth Rock, then her eggs are brown.
   
   **Berta** is a Plymouth Rock hen.
   
   **Conclusion:** Berta’s eggs are brown.
Law of Syllogism  Another way to make a valid conclusion is to use the Law of Syllogism. It allows you to draw conclusions from two true statements when the conclusion of one statement is the hypothesis of another.

**Example**  The two conditional statements below are true. Use the Law of Syllogism to find a valid conclusion. State the conclusion.

(1) If a number is a whole number, then the number is an integer.
(2) If a number is an integer, then it is a rational number.

\[ p: \text{A number is a whole number.} \]
\[ q: \text{A number is an integer.} \]
\[ r: \text{A number is a rational number.} \]

The two conditional statements are \( p \rightarrow q \) and \( q \rightarrow r \). Using the Law of Syllogism, a valid conclusion is \( p \rightarrow r \). A statement of \( p \rightarrow r \) is “if a number is a whole number, then it is a rational number.”

**Exercises**

Use the Law of Syllogism to draw a valid conclusion from each set of statements, if possible. If no valid conclusion is possible, write *no valid conclusion*.

1. If a dog eats Superdog Dog Food, he will be happy.
   Rover is happy.

2. If an angle is supplementary to an obtuse angle, then it is acute.
   If an angle is acute, then its measure is less than 90.

3. If the measure of \( \angle A \) is less than 90, then \( \angle A \) is acute.
   If \( \angle A \) is acute, then \( \angle A \equiv \angle B \).

4. If an angle is a right angle, then the measure of the angle is 90.
   If two lines are perpendicular, then they form a right angle.

5. If you study for the test, then you will receive a high grade.
   Your grade on the test is high.
Deductive Reasoning

Determine whether the stated conclusion is valid based on the given information. If not, write invalid. Explain your reasoning.

1. Given: If a point is the midpoint of a segment, then it divides the segment into two congruent segments. \( R \) is the midpoint of \( QS \)
   
   Conclusion: \( QR \cong RS \).

2. Given: If a point is the midpoint of a segment, then it divides the segment into two congruent segments. \( AB \cong BC \)
   
   Conclusion: \( B \) divides \( AC \) into two congruent segments.

Use the Law of Syllogism to draw a valid conclusion from each set of statements, if possible. If no valid conclusion can be drawn, write no valid conclusion.

3. If two angles form a linear pair, then the two angles are supplementary.
   
   If two angles are supplementary, then the sum of their measures is 180.

4. If a hurricane is Category 5, then winds are greater than 155 miles per hour.
   
   If winds are greater than 155 miles per hour, then trees, shrubs, and signs are blown down.

Draw a valid conclusion from the statements, if possible. Then state whether your conclusion was drawn using the Law of Detachment or the Law of Syllogism. If no valid conclusion can be drawn, write no valid conclusion and explain your reasoning.

5. Given: If a whole number is even, then its square is divisible by 4.
   
   The number I am thinking of is an even number.

6. BIOLOGY If an organism is a parasite, then it survives by living on or in a host organism. If a parasite lives in or on a host organism, then it harms its host. What conclusion can you draw if a virus is a parasite?
1. SIGNS Two signs are posted on a haunted house.

   NO ONE UNDER 5 ALLOWED
   NO ONE UNDER 8 ALLOWED WITHOUT A PARENT

   Inside the haunted house, you find a child with his parent. What can you deduce about the age of the child based on the house rules?

2. LOGIC As Laura’s mother rushed off to work, she quickly gave Laura some instructions. “If you need me, try my cell . . . if I don’t answer it means I’m in a meeting, but don’t worry, the meeting won’t last more than 30 minutes and I’ll call you back when it’s over.” Later that day, Laura needed her mother, but her mother was stuck in a meeting and couldn’t answer the phone. Laura concludes that she will have to wait no more than 30 minutes before she gets a call back from her mother. What law of logic did Laura use to draw this conclusion?

3. MUSIC Composer Ludwig van Beethoven wrote 9 symphonies and 5 piano concertos. If you lived in Vienna in the early 1800s, you could attend a concert conducted by Beethoven himself. Write a valid conclusion to the hypothesis If Mozart could not attend a concert conducted by Beethoven, . . .

4. DIRECTIONS Hank has an appointment to see a financial advisor on the fifteenth floor of an office building. When he gets to the building, the people at the front desk tell him that if he wants to go to the fifteenth floor, then he must take the red elevator. While looking for the red elevator, a guard informs him that if he wants to find the red elevator he must find the replica of Michelangelo’s David. When he finally got to the fifteenth floor, his financial advisor greeted him asking, “What did you think of the Michelangelo?” How did Hank’s financial advisor conclude that Hank must have seen the Michelangelo statue?

5. LAWS The law says that if you are under 21, then you are not allowed to drink alcoholic beverages and if you are under 18, then you are not allowed to vote. For each problem give the possible ages of the person described or state that the person cannot exist.

   a. John cannot drink wine legally but is allowed to vote.

   b. Mary cannot vote legally but can drink beer legally.
2-5 Study Guide

Postulates and Paragraph Proofs

Points, Lines, and Planes In geometry, a postulate is a statement that is accepted as true. Postulates describe fundamental relationships in geometry.

<table>
<thead>
<tr>
<th>Postulate 2.1:</th>
<th>Through any two points, there is exactly one line.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Postulate 2.2:</td>
<td>Through any three noncollinear points, there is exactly one plane.</td>
</tr>
<tr>
<td>Postulate 2.3:</td>
<td>A line contains at least two points.</td>
</tr>
<tr>
<td>Postulate 2.4:</td>
<td>A plane contains at least three noncollinear points.</td>
</tr>
<tr>
<td>Postulate 2.5:</td>
<td>If two points lie in a plane, then the entire line containing those points lies in the plane.</td>
</tr>
<tr>
<td>Postulate 2.6:</td>
<td>If two lines intersect, then their intersection is exactly one point.</td>
</tr>
<tr>
<td>Postulate 2.7:</td>
<td>If two planes intersect, then their intersection is a line.</td>
</tr>
</tbody>
</table>

**Example** Determine whether each statement is always, sometimes, or never true.

a. There is exactly one plane that contains points A, B, and C.
   - Sometimes; if A, B, and C are collinear, they are contained in many planes. If they are noncollinear, then they are contained in exactly one plane.

b. Points E and F are contained in exactly one line.
   - Always; the first postulate states that there is exactly one line through any two points.

c. Two lines intersect in two distinct points M and N.
   - Never; the intersection of two lines is one point.

**Exercises**

Determine whether each statement is always, sometimes, or never true.

1. A line contains exactly one point.
2. Noncollinear points R, S, and T are contained in exactly one plane.
3. Any two lines ℓ and m intersect.
4. If points G and H are contained in plane M, then GH is perpendicular to plane M.
5. Planes R and S intersect in point T.
6. If points A, B, and C are noncollinear, then segments AB, BC, and CA are contained in exactly one plane.

In the figure, AC and DE are in plane Q and AC || DE.
State the postulate that can be used to show each statement is true.

7. Exactly one plane contains points F, B, and E.

8. BE lies in plane Q,
Paragraph Proofs  A logical argument that uses deductive reasoning to reach a valid conclusion is called a proof. In one type of proof, a paragraph proof, you write a paragraph to explain why a statement is true.

A statement that can be proved true is called a theorem. You can use undefined terms, definitions, postulates, and already-proved theorems to prove other statements true.

Example  In \( \triangle ABC \), \( \overline{BD} \) is an angle bisector. Write a paragraph proof to show that \( \angle ABD \cong \angle CBD \).

By definition, an angle bisector divides an angle into two congruent angles. Since \( \overline{BD} \) is an angle bisector, \( \angle ABC \) is divided into two congruent angles. Thus, \( \angle ABD \cong \angle CBD \).

Exercises

1. Given that \( \angle A \cong \angle D \) and \( \angle D \cong \angle E \), write a paragraph proof to show that \( \angle A \cong \angle E \).

2. It is given that \( \overline{BC} \cong \overline{EF} \), \( M \) is the midpoint of \( \overline{BC} \), and \( N \) is the midpoint of \( \overline{EF} \). Write a paragraph proof to show that \( BM = EN \).

3. Given that \( S \) is the midpoint of \( \overline{QP} \), \( T \) is the midpoint of \( \overline{PR} \), and \( P \) is the midpoint of \( \overline{ST} \), write a paragraph proof to show that \( QS = TR \).
**Postulates and Paragraph Proofs**

Explain how the figure illustrates that each statement is true. Then state the postulate that can be used to show each statement is true.

1. The planes $J$ and $K$ intersect at line $m$.

2. The lines $\ell$ and $m$ intersect at point $Q$.

Determine whether the following statements are *always*, *sometimes*, or *never* true. Explain.

3. The intersection of two planes contains at least two points.

4. If three planes have a point in common, then they have a whole line in common.

In the figure, line $m$ and $\overrightarrow{TQ}$ lie in plane $A$. State the postulate that can be used to show that each statement is true.

5. Points $L$, and $T$ and line $m$ lie in the same plane.

6. Line $m$ and $\overrightarrow{ST}$ intersect at $T$.

7. In the figure, $E$ is the midpoint of $\overrightarrow{AB}$ and $\overrightarrow{CD}$, and $AB = CD$. Write a paragraph proof to prove that $\overrightarrow{AE} \cong \overrightarrow{ED}$.

8. **LOGIC** Points $A$, $B$, and $C$ are noncollinear. Points $B$, $C$, and $D$ are noncollinear. Points $A$, $B$, $C$, and $D$ are noncoplanar. Describe two planes that intersect in line $BC$. 

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Chapter 2

*North Carolina StudyText, Math BC, Volume 1*
1. **ROOFING** Noel and Kirk are building a new roof. They wanted a roof with two sloping planes that meet along a curved arch. Is this possible?

2. **AIRLINES** An airline company wants to provide service to San Francisco, Los Angeles, Chicago, Dallas, Washington D.C., and New York City. The company’s CEO draws lines between each pair of cities in the list on a map. No three of the cities are collinear. How many lines did the CEO draw?

3. **TRIANGULATION** A sailor spots a whale through her binoculars. She wonders how far away the whale is, but the whale does not show up on the radar system. She sees another boat in the distance and radios the captain asking him to spot the whale and record its direction. Explain how this added information could enable the sailor to pinpoint the location of the whale. Under what circumstance would this idea fail?

4. **POINTS** Carson claims that a line can intersect a plane at only one point and draws this picture to show his reasoning.

5. **FRIENDSHIPS** A small company has 16 employees. The owner of the company became concerned that the employees did not know each other very well. He decided to make a picture of the friendships in the company. He placed 16 points on a sheet of paper in such a way that no 3 were collinear. Each point represented a different employee. He then asked each employee who their friends were and connected two points with a line segment if they represented friends.

   a. What is the maximum number of line segments that can be drawn between pairs among the 16 points?

   b. When the owner finished the picture, he found that his company was split into two groups, one with 10 people and the other with 6. The people within a group were all friends, but nobody from one group was a friend of anybody from the other group. How many line segments were there?
Algebraic Proof

Algebraic Proof  A list of algebraic steps to solve problems where each step is justified is called an algebraic proof. The table shows properties you have studied in algebra.

The following properties are true for any real numbers \(a, b,\) and \(c\).

<table>
<thead>
<tr>
<th>Property</th>
<th>If (a = b), then (a + c = b + c).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition Property of Equality</td>
<td></td>
</tr>
<tr>
<td>Subtraction Property of Equality</td>
<td>If (a = b), the (a - c = b - c).</td>
</tr>
<tr>
<td>Multiplication Property of Equality</td>
<td>If (a = b), then (a \cdot c = b \cdot c).</td>
</tr>
<tr>
<td>Division Property of Equality</td>
<td>If (a = b) and (c \neq 0), then, (\frac{a}{c} = \frac{b}{c}).</td>
</tr>
<tr>
<td>Reflexive Property of Equality</td>
<td>(a = a)</td>
</tr>
<tr>
<td>Symmetric Property of Equality</td>
<td>If (a = b) and (b = a).</td>
</tr>
<tr>
<td>Transitive Property of Equality</td>
<td>If (a = b) and (b = c), then (a = c).</td>
</tr>
<tr>
<td>Substitution Property of Equality</td>
<td>If (a = b) and (b = c), then (a = c).</td>
</tr>
<tr>
<td>Distributive Property</td>
<td>(a(b + c) = ab + ac)</td>
</tr>
</tbody>
</table>

Example

Solve \(6x + 2(x - 1) = 30\). Write a justification for each step.

Algebraic Steps

\[
\begin{align*}
6x + 2(x - 1) &= 30 \\
6x + 2x - 2 &= 30 \\
8x - 2 &= 30 \\
8x - 2 + 2 &= 30 + 2 \\
8x &= 32 \\
\frac{8x}{8} &= \frac{32}{8} \\
x &= 4
\end{align*}
\]

Properties

Original equation or Given
Distributive Property
Substitution Property of Equality
Addition Property of Equality
Substitution Property of Equality
Division Property of Equality
Substitution Property of Equality

Exercises

Complete each proof.

1. Given: \(\frac{4x + 6}{2} = 9\)
   Prove: \(x = 3\)
   Proof:

   Statements | Reasons
   --------- | ---------
   a. \(\frac{4x + 6}{2} = 9\) | a. __________ |
   b. \(-\left(\frac{4x + 6}{2}\right) = 2(9)\) | b. Mult. Prop. |
   c. \(4x + 6 = 18\) | c. __________ |
   d. \(4x + 6 - 6 = 18 - 6\) | d. __________ |
   e. \(4x = __________\) | e. Substitution |
   f. \(\frac{4x}{4} = __________\) | f. Div. Prop. |
   g. __________ | g. Substitution |

2. Given: \(4x + 8 = x + 2\)
   Prove: \(x = -2\)
   Proof:

   Statements | Reasons
   --------- | ---------
   a. \(4x + 8 = x + 2\) | a. __________ |
   b. \(4x + 8 - x = x + 2 - x\) | b. __________ |
   c. \(3x + 8 = 2\) | c. Substitution |
   d. __________ | d. Subtr. Prop. |
   e. __________ | e. Substitution |
   f. \(\frac{3x}{3} = \frac{-6}{3}\) | f. __________ |
   g. __________ | g. Substitution |
Algebraic Proof

Geometric Proof Geometry deals with numbers as measures, so geometric proofs use properties of numbers. Here are some of the algebraic properties used in proofs.

<table>
<thead>
<tr>
<th>Property</th>
<th>Segments</th>
<th>Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive</td>
<td>$AB = AB$</td>
<td>$m\angle 1 = m\angle 1$</td>
</tr>
<tr>
<td>Symmetric</td>
<td>If $AB = CD$, then $CD = AB$.</td>
<td>If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$.</td>
</tr>
<tr>
<td>Transitive</td>
<td>If $AB = CD$ and $CD = EF$, then $AB = EF$.</td>
<td>If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.</td>
</tr>
</tbody>
</table>

**Example** Write a two-column proof to verify this conjecture.

**Given:** $m\angle 1 = m\angle 2$, $m\angle 2 = m\angle 3$

**Prove:** $m\angle 1 = m\angle 3$

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $m\angle 1 = m\angle 2$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $m\angle 2 = m\angle 3$</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. $m\angle 1 = m\angle 3$</td>
<td>3. Transitive Property of Equality</td>
</tr>
</tbody>
</table>

**Exercises**

State the property that justifies each statement.

1. If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$.
2. If $m\angle 1 = 90$ and $m\angle 2 = m\angle 1$, then $m\angle 2 = 90$.
3. If $AB = RS$ and $RS = WY$, then $AB = WY$.
4. If $AB = CD$, then $\frac{1}{2}AB = \frac{1}{2}CD$.
5. If $m\angle 1 + m\angle 2 = 110$ and $m\angle 2 = m\angle 3$, then $m\angle 1 + m\angle 3 = 110$.
6. $RS = RS$
7. If $AB = RS$ and $TU = WY$, then $AB + TU = RS + WY$.
8. If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.
9. If the formula for the area of a triangle is $A = \frac{1}{2}bh$, then $bh$ is equal to 2 times the area of the triangle.
   Write a two-column proof to verify this conjecture.
2-6 Practice

Algebraic Proof

PROOF Write a two-column proof to verify each conjecture.

1. If \( m\angle ABC + m\angle CBD = 90\), \( m\angle ABC = 3x - 5\), and \( m\angle CBD = \frac{x + 1}{2} \), then \( x = 27 \).

2. FINANCE The formula for simple interest is \( I = prt \), where \( I \) is interest, \( p \) is principal, \( r \) is rate, and \( t \) is time. Solve the formula for \( r \) and justify each step.
2-6 Word Problem Practice

Algebraic Proof

1. **DOGS** Jessica and Robert each own the same number of dogs. Robert and Gail also own the same number of dogs. Without knowing how many dogs they own, one can still conclude that Jessica and Gail each own the same number of dogs. What property is used to make this conclusion?

2. **MONEY** Lars and Peter each have the same amount of money in their wallets. They went to the store together and decided to buy some cookies, splitting the cost equally. After buying the cookies, do they still have the same amount of money in their wallets? What property is relevant to help you decide?

3. **MANUFACTURING** A company manufactures small electronic components called diodes. Each diode is worth $1.50. Plant A produced 4443 diodes and Plant B produced 5557 diodes. The supervisor was asked what the total value of all the diodes was. The supervisor immediately responded “$15,000.” The supervisor would not have been able to compute the value so quickly if he had to multiply $1.50 by 4443 and then add this to the result of $1.50 times 5557. Explain how you think the supervisor got the answer so quickly?

4. **FIGURINES** Pete and Rhonda paint figurines. They can both paint 8 figurines per hour. One day, Pete worked 6 hours while Rhonda worked 9 hours. How many figurines did they paint that day? Show how to get the answer using the Distributive Property.

5. **AGE** William’s father is eight years older than 4 times William’s age. William’s father is 36 years old.

   a. Let \( x \) be William’s age. Translate the given information into an algebraic equation involving \( x \).

   b. Fill in the missing steps and justifications for each step in finding the value of \( x \).

<table>
<thead>
<tr>
<th>Statements</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( 4x + 8 = 36 )</td>
<td>1. Original equation</td>
</tr>
<tr>
<td>2. ( \underline{\text{}} )</td>
<td>2. Subtraction Property</td>
</tr>
<tr>
<td>3. ( 4x = 28 )</td>
<td>3. ( \underline{\text{}} )</td>
</tr>
<tr>
<td>4. ( \frac{4x}{4} = \frac{28}{4} )</td>
<td>4. ( \underline{\text{}} )</td>
</tr>
<tr>
<td>5. ( \underline{\text{}} )</td>
<td>5. Substitution Property</td>
</tr>
</tbody>
</table>
2-7 Study Guide

Proving Segment Relationships

Segment Addition  Two basic postulates for working with segments and lengths are the Ruler Postulate, which establishes number lines, and the Segment Addition Postulate, which describes what it means for one point to be between two other points.

<table>
<thead>
<tr>
<th>Ruler Postulate</th>
<th>The points on any line or line segment can be put into one-to-one correspondence with real numbers.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segment Addition Postulate</td>
<td>If A, B, and C are collinear, then point B is between A and C if and only if ( AB + BC = AC ).</td>
</tr>
</tbody>
</table>

Example  Write a two-column proof.

Given: \( Q \) is the midpoint of \( PR \).
\( R \) is the midpoint of \( QS \).

Prove: \( PR = QS \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( Q ) is the midpoint of ( PR ).</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( PQ = QR )</td>
<td>2. Definition of midpoint</td>
</tr>
<tr>
<td>3. ( R ) is the midpoint of ( QS ).</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( QR = RS )</td>
<td>4. Definition of midpoint</td>
</tr>
<tr>
<td>5. ( PQ + QR = QR + RS )</td>
<td>5. Addition Property</td>
</tr>
<tr>
<td>6. ( PQ = RS )</td>
<td>6. Transitive Property</td>
</tr>
<tr>
<td>7. ( PQ + QR = PR, QR + RS = QS )</td>
<td>7. Segment Addition Postulate</td>
</tr>
<tr>
<td>8. ( PR = QS )</td>
<td>8. Substitution</td>
</tr>
</tbody>
</table>

Exercises

Complete each proof.

1. Given: \( BC = DE \)
Prove: \( AB + DE = AC \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( BC = DE )</td>
<td>1.</td>
</tr>
<tr>
<td>3. ( AB + DE = AC )</td>
<td>3.</td>
</tr>
</tbody>
</table>

2. Given: \( Q \) is between \( P \) and \( R \), \( R \) is between \( Q \) and \( S \), \( PR = QS \).

Prove: \( PQ = RS \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( Q ) is between ( P ) and ( R ).</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( PQ + QR = PR )</td>
<td>2.</td>
</tr>
<tr>
<td>3. ( R ) is between ( Q ) and ( S ).</td>
<td>3.</td>
</tr>
<tr>
<td>5. ( PR = QS )</td>
<td>5.</td>
</tr>
<tr>
<td>6. ( PQ + QR = QR + RS )</td>
<td>6.</td>
</tr>
<tr>
<td>7. ( PQ + QR = QR + RS - QR )</td>
<td>7.</td>
</tr>
<tr>
<td>8. _________</td>
<td>8. Substitution</td>
</tr>
</tbody>
</table>
2-7 Study Guide (continued) Proving Segment Relationships

Segment Congruence Remember that segment measures are reflexive, symmetric, and transitive. Since segments with the same measure are congruent, congruent segments are also reflexive, symmetric, and transitive.

<table>
<thead>
<tr>
<th>Property</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive Property</td>
<td>$AB \cong AB$</td>
</tr>
<tr>
<td>Symmetric Property</td>
<td>If $AB \cong CD$, then $CD \cong AB$.</td>
</tr>
<tr>
<td>Transitive Property</td>
<td>If $AB \cong CD$ and $CD \cong EF$, then $AB \cong EF$.</td>
</tr>
</tbody>
</table>

Example Write a two-column proof.

Given: $AB \cong DE$; $BC \cong EF$  
Prove: $AC \cong DF$

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $AB \cong DE$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $AB = DE$</td>
<td>2. Definition of congruence of segments</td>
</tr>
<tr>
<td>3. $BC \cong EF$</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. $BC = EF$</td>
<td>4. Definition of congruence of segments</td>
</tr>
<tr>
<td>5. $AB + BC = DE + EF$</td>
<td>5. Addition Property</td>
</tr>
<tr>
<td>7. $AC = DF$</td>
<td>7. Substitution</td>
</tr>
<tr>
<td>8. $AC \cong DF$</td>
<td>8. Definition of congruence of segments</td>
</tr>
</tbody>
</table>

Exercises

Justify each statement with a property of congruence.

1. If $DE \cong GH$, then $GH \cong DE$.
2. If $AB \cong RS$ and $RS \cong WY$ then $AB \cong WY$.
3. $RS \cong RS$
4. Complete the proof.
   Given: $PR \cong QS$  
   Prove: $PQ \cong RS$

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $PR \cong QS$</td>
<td>a. ___________________</td>
</tr>
<tr>
<td>b. $PR = QS$</td>
<td>b. ___________________</td>
</tr>
<tr>
<td>c. $PQ + QR = PR$</td>
<td>c. ___________________</td>
</tr>
<tr>
<td>d. ___________________</td>
<td>d. Segment Addition Postulate</td>
</tr>
<tr>
<td>e. $PQ + QR = QR + RS$</td>
<td>e. ___________________</td>
</tr>
<tr>
<td>f. ___________________</td>
<td>f. Subtraction Property</td>
</tr>
<tr>
<td>g. ___________________</td>
<td>g. Definition of congruence of segments</td>
</tr>
</tbody>
</table>
2-7 Practice

Proving Segment Relationships

Complete the following proof.

1. Given: $\overline{AB} \cong \overline{DE}$
   $B$ is the midpoint of $\overline{AC}$.
   $E$ is the midpoint of $\overline{DF}$.

   Prove: $\overline{BC} \cong \overline{EF}$

   Proof:

   Statements | Reasons
   --- | ---
   a. $\overline{AB} = \overline{DE}$ | a. Given
   b. $\overline{BC} = \overline{DE}$ | b. $\overline{BC} = \overline{EF}$
   c. Definition of Midpoint
   d. $\overline{EF}$
   e. $\overline{BC} = \overline{EF}$
   f. $\overline{EF}$

2. TRAVEL Refer to the figure. DeAnne knows that the distance from Grayson to Apex is the same as the distance from Redding to Pine Bluff. Prove that the distance from Grayson to Redding is equal to the distance from Apex to Pine Bluff.
2-7 Word Problem Practice

Proving Segment Relationships

1. FAMILY Maria is 11 inches shorter than her sister Nancy. Brad is 11 inches shorter than his brother Chad. If Maria is shorter than Brad, how do the heights of Nancy and Chad compare? What if Maria and Brad are the same height?

2. DISTANCE Martha and Laura live 1400 meters apart. A library is opened between them and is 500 meters from Martha.

How far is the library from Laura?

3. LUMBER Byron works in a lumber yard. His boss just cut a dozen planks and asked Byron to double check that they are all the same length. The planks were numbered 1 through 12. Byron took out plank number 1 and checked that the other planks are all the same length as plank 1. He concluded that they must all be the same length. Explain how you know plank 7 and plank 10 are the same length even though they were never directly compared to each other?

4. NEIGHBORHOODS Karla, John, and Mandy live in three houses that are on the same line. John lives between Karla and Mandy. Karla and Mandy live a mile apart. Is it possible for John to be a mile from both Karla and Mandy?

5. LIGHTS Five lights, A, B, C, D, and E, are lined up in a row. The middle light is the midpoint of the second and fourth light and also the midpoint of the first and last light.

a. Draw a figure to illustrate the situation.

b. Complete this proof.

Given: C is the midpoint of BD and AE.

Prove: AB = DE

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. C is the midpoint of BD and AE.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. BC = CD and</td>
<td>2.</td>
</tr>
<tr>
<td>3. AC = AB + BC, CE = CD + DE</td>
<td>3.</td>
</tr>
<tr>
<td>4. AB = AC − BC</td>
<td>4.</td>
</tr>
<tr>
<td>5.</td>
<td>5. Substitution Property</td>
</tr>
<tr>
<td>7.</td>
<td>7.</td>
</tr>
</tbody>
</table>
Supplementary and Complementary Angles There are two basic postulates for working with angles. The Protractor Postulate assigns numbers to angle measures, and the Angle Addition Postulate relates parts of an angle to the whole angle.

<table>
<thead>
<tr>
<th>Protractor Postulate</th>
<th>Given any angle, the measure can be put into one-to-one correspondence with real numbers between 0 and 180.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle Addition Postulate</td>
<td>( R ) is in the interior of ( \angle PQS ) if and only if ( m\angle PQR + m\angle RQS = m\angle PQS ).</td>
</tr>
</tbody>
</table>

The two postulates can be used to prove the following two theorems.

<table>
<thead>
<tr>
<th>Supplement Theorem</th>
<th>If two angles form a linear pair, then they are supplementary angles. Example: If ( \angle 1 ) and ( \angle 2 ) form a linear pair, then ( m\angle 1 + m\angle 2 = 180 ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complement Theorem</td>
<td>If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles. Example: If ( \overrightarrow{GF} \perp \overrightarrow{GH} ), then ( m\angle 3 + m\angle 4 = 90 ).</td>
</tr>
</tbody>
</table>

**Example 1** If \( \angle 1 \) and \( \angle 2 \) form a linear pair and \( m\angle 2 = 115 \), find \( m\angle 1 \).

\[
\begin{align*}
m\angle 1 + m\angle 2 &= 180 \\
m\angle 1 + 115 &= 180 \\
m\angle 1 &= 65
\end{align*}
\]

**Example 2** If \( \angle 1 \) and \( \angle 2 \) form a right angle and \( m\angle 2 = 20 \), find \( m\angle 1 \).

\[
\begin{align*}
m\angle 1 + m\angle 2 &= 90 \\
m\angle 1 + 20 &= 90 \\
m\angle 1 &= 70
\end{align*}
\]

Exercises

Find the measure of each numbered angle and name the theorem that justifies your work.

1. \( \angle 7 = 5x + 5 \), \( \angle 8 = x - 5 \)

2. \( \angle 5 = 5x \), \( \angle 6 = 4x + 6 \), \( \angle 7 = 10x \), \( \angle 8 = 12x - 12 \)

3. \( \angle 11 = 11x \), \( \angle 13 = 10x + 12 \)
Complete each proof.

1. Given: \( \overline{AB} \perp \overline{BC} \);
   \( \angle 1 \) and \( \angle 3 \) are complementary.
   
   Prove: \( \angle 2 \cong \angle 3 \)

   **Proof:**
   
   **Statements**
   
   a. \( \overline{AB} \perp \overline{BC} \)
   b. 
   c. \( m\angle ABC = 90 \)
   d. \( m\angle ABC = m\angle 1 + m\angle 2 \)
   e. \( 90 = m\angle 1 + m\angle 2 \)
   f. \( \angle 1 \) and \( \angle 2 \) are compl.
   g. 
   h. \( \angle 2 \cong \angle 3 \)

   **Reasons**
   
   a. 
   b. Definition of \( \perp \)
   c. Def. of right angle
   d. 
   e. Substitution
   f. 
   g. Given
   h. 

2. Given: \( \angle 1 \) and \( \angle 2 \) form a linear pair.
   \( m\angle 1 + m\angle 3 = 180 \)
   
   Prove: \( \angle 2 \cong \angle 3 \)

   **Proof:**
   
   **Statements**
   
   a. \( \angle 1 \) and \( \angle 2 \) form a linear pair.
   b. \( m\angle 1 + m\angle 3 = 180 \)
   c. \( \angle 1 \) is suppl. to \( \angle 3 \).
   d. 

   **Reasons**
   
   a. Given
   b. Suppl. Theorem
   c. 
   d. \( \triangle \) suppl. to the same \( \angle \) or \( \cong \triangle \) are \( \cong \).
Find the measure of each numbered angle and name the theorems that justify your work.

1. \( m \angle 1 = x + 10 \)  
   \( m \angle 2 = 3x + 18 \)

2. \( m \angle 4 = 2x - 5 \)  
   \( m \angle 5 = 4x - 13 \)

3. \( m \angle 6 = 7x - 24 \)  
   \( m \angle 7 = 5x + 14 \)

4. Write a two-column proof.
   Given: \( \angle 1 \) and \( \angle 2 \) form a linear pair. \( \angle 2 \) and \( \angle 3 \) are supplementary.
   Prove: \( \angle 1 \equiv \angle 3 \)

5. **STREETS** Refer to the figure. Barton Road and Olive Tree Lane form a right angle at their intersection. Tryon Street forms a 57° angle with Olive Tree Lane. What is the measure of the acute angle Tryon Street forms with Barton Road?
1. **ICOSAHEDRA** For a school project, students are making a giant icosahedron, which is a large solid with many identical triangular faces. John is assigned quality control. He must make sure that the measures of all the angles in all the triangles are the same as each other. He does this by using a precut template and comparing the corner angles of every triangle to the template. How does this assure that the angles in all the triangles will be congruent to each other?

2. **VISTAS** If you look straight ahead at a scenic point, you can see a waterfall. If you turn your head 25° to the left, you will see a famous mountain peak. If you turn your head 35° more to the left, you will see another waterfall. If you are looking straight ahead, through how many degrees must you turn your head to the left in order to see the second waterfall?

3. **TUBES** A tube with a hexagonal cross section is placed on the floor.

   ![Hexagonal Cross Section](image)

   What is the measure of \( \angle 1 \) in the figure given that the angle at one corner of the hexagon is 120°?

4. **PAINTING** Students are painting their rectangular classroom ceiling. They want to paint a line that intersects one of the corners as shown in the figure.

   ![Cross section of pipe](image)

   They want the painted line to make a 15° angle with one edge of the ceiling. Unfortunately, between the line and the edge there is a water pipe making it difficult to measure the angle. They decide to measure the angle to the other edge. Given that the corner is a right angle, what is the measure of the other angle?

5. **BUILDINGS** Clyde looks at a building from point \( E \). \( \angle AEC \) has the same measure as \( \angle BED \).

   ![Building Diagram](image)

   a. The measure of \( \angle AEC \) is equal to the sum of the measures of \( \angle AEB \) and what other angle?
   
   b. The measure of \( \angle BED \) is equal to the sum of the measures of \( \angle CED \) and what other angle?
   
   c. Is it true that \( m \angle AEB \) is equal to \( m \angle CED \)?
Parallel Lines and Transversals

Relationships Between Lines and Planes  When two lines lie in the same plane and do not intersect, they are **parallel**. Lines that do not intersect and are not coplanar are **skew lines**.

In the figure, \( \ell \) is parallel to \( m \), or \( \ell \parallel m \). You can also write \( \overline{PQ} \parallel \overline{RS} \). Similarly, if two planes do not intersect, they are **parallel planes**.

**Example**  Refer to the figure at the right to identify each of the following.

a. all planes parallel to plane \( ABD \)
   - Plane \( EFH \)

b. all segments parallel to \( \overline{CG} \)
   - \( BF, DH, \) and \( AE \)

c. all segments skew to \( \overline{EH} \)
   - \( BF, CG, BD, CD, \) and \( AB \)

**Exercises**

Refer to the figure at the right to identify each of the following.

1. all planes that intersect plane \( OPT \)

2. all segments parallel to \( \overline{NU} \)

3. all segments that intersect \( \overline{MP} \)

Refer to the figure at the right to identify each of the following.

4. all segments parallel to \( \overline{QX} \)

5. all planes that intersect plane \( MHE \)

6. all segments parallel to \( \overline{QR} \)

7. all segments skew to \( \overline{AG} \)
Parallel Lines and Transversals

Angle Relationships A line that intersects two or more other lines at two different points in a plane is called a transversal. In the figure below, line \( t \) is a transversal. Two lines and a transversal form eight angles. Some pairs of the angles have special names. The following chart lists the pairs of angles and their names.

<table>
<thead>
<tr>
<th>Angle Pairs</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \angle 3, \angle 4, \angle 5, \angle 6 )</td>
<td>interior angles</td>
</tr>
<tr>
<td>( \angle 3 ) and ( \angle 5; \angle 4 ) and ( \angle 6 )</td>
<td>alternate interior angles</td>
</tr>
<tr>
<td>( \angle 3 ) and ( \angle 6; \angle 4 ) and ( \angle 5 )</td>
<td>consecutive interior angles</td>
</tr>
<tr>
<td>( \angle 1, \angle 2, \angle 7, \angle 8 )</td>
<td>exterior angles</td>
</tr>
<tr>
<td>( \angle 1 ) and ( \angle 7; \angle 2 ) and ( \angle 8 );</td>
<td>alternate exterior angles</td>
</tr>
<tr>
<td>( \angle 1 ) and ( \angle 5; \angle 2 ) and ( \angle 6 );</td>
<td>corresponding angles</td>
</tr>
<tr>
<td>( \angle 3 ) and ( \angle 7; \angle 4 ) and ( \angle 8 )</td>
<td>gesture press</td>
</tr>
</tbody>
</table>

**Example**

Classify the relationship between each pair of angles as alternate interior, alternate exterior, corresponding, or consecutive interior angles.

a. \( \angle 10 \) and \( \angle 16 \)
    - alternate exterior angles
b. \( \angle 4 \) and \( \angle 12 \)
    - corresponding angles
c. \( \angle 12 \) and \( \angle 13 \)
    - consecutive interior angles
d. \( \angle 3 \) and \( \angle 9 \)
    - alternate interior angles

**Exercises**

Use the figure in the Example for Exercises 1–12.

Identify the transversal connecting each pair of angles.

1. \( \angle 9 \) and \( \angle 13 \)
2. \( \angle 5 \) and \( \angle 14 \)
3. \( \angle 4 \) and \( \angle 6 \)

Classify the relationship between each pair of angles as alternate interior, alternate exterior, corresponding, or consecutive interior angles.

4. \( \angle 1 \) and \( \angle 5 \)
5. \( \angle 6 \) and \( \angle 14 \)
6. \( \angle 2 \) and \( \angle 8 \)

7. \( \angle 3 \) and \( \angle 11 \)
8. \( \angle 12 \) and \( \angle 3 \)
9. \( \angle 4 \) and \( \angle 6 \)

10. \( \angle 6 \) and \( \angle 16 \)
11. \( \angle 11 \) and \( \angle 14 \)
12. \( \angle 10 \) and \( \angle 16 \)
3-1 Practice

Parallel Lines and Transversals

Refer to the figure at the right to identify each of the following.

1. all planes that intersect plane STX
2. all segments that intersect QU
3. all segments that are parallel to XY
4. all segments that are skew to VW

Classify the relationship between each pair of angles as alternate interior, alternate exterior, corresponding, or consecutive interior angles.

5. ∠2 and ∠10
6. ∠7 and ∠13
7. ∠9 and ∠13
8. ∠6 and ∠16
9. ∠3 and ∠10
10. ∠8 and ∠14

Name the transversal that forms each pair of angles. Then identify the special name for the angle pair.

11. ∠2 and ∠12
12. ∠6 and ∠18
13. ∠13 and ∠19
14. ∠11 and ∠7

FURNITURE For Exercises 15–16, refer to the drawing of the end table.
15. Find an example of parallel planes.
16. Find an example of parallel lines.
1. **FIGHTERS** Two fighter aircraft fly at the same speed and in the same direction leaving a trail behind them. Describe the relationship between these two trails.

2. **ESCALATORS** An escalator at a shopping mall runs up several levels. The escalator railing can be modeled by a straight line running past horizontal lines that represent the floors. Describe the relationships of these lines.

3. **DESIGN** Carol designed the picture frame shown below. How many pairs of parallel segments are there among various edges of the frame?

4. **NEIGHBORHOODS** John, Georgia, and Phillip live nearby each other as shown in the map. Describe how their corner angles relate to each other in terms of alternate interior, alternate exterior, corresponding, consecutive interior, or vertical angles.

5. **MAPPING** Use the following figure.

   a. Connor lives at the angle that forms an alternate interior angle with Georgia’s residence. Add Connor to the map.

   b. Quincy lives at the angle that forms a consecutive interior angle with Connors’ residence. Add Quincy to the map.
3-2 Study Guide

Angles and Parallel Lines

Parallel Lines and Angle Pairs When two parallel lines are cut by a transversal, the following pairs of angles are congruent.

- corresponding angles
- alternate interior angles
- alternate exterior angles

Also, consecutive interior angles are supplementary.

Example In the figure, \( \angle 2 = 75 \). Find the measures of the remaining angles.

\[
\begin{align*}
m\angle 1 &= 105 & \angle 1 \text{ and } \angle 2 & \text{ form a linear pair.} \\
m\angle 3 &= 105 & \angle 3 \text{ and } \angle 2 & \text{ form a linear pair.} \\
m\angle 4 &= 75 & \angle 4 \text{ and } \angle 2 & \text{ are vertical angles.} \\
m\angle 5 &= 105 & \angle 5 \text{ and } \angle 3 & \text{ are alternate interior angles.} \\
m\angle 6 &= 75 & \angle 6 \text{ and } \angle 2 & \text{ are corresponding angles.} \\
m\angle 7 &= 105 & \angle 7 \text{ and } \angle 3 & \text{ are corresponding angles.} \\
m\angle 8 &= 75 & \angle 8 \text{ and } \angle 6 & \text{ are vertical angles.}
\end{align*}
\]

Exercises

In the figure, \( m\angle 3 = 102 \). Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

1. \( \angle 5 \) 2. \( \angle 6 \) 3. \( \angle 11 \) 4. \( \angle 7 \) 5. \( \angle 15 \) 6. \( \angle 14 \)

In the figure, \( m\angle 9 = 80 \) and \( m\angle 5 = 68 \). Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

7. \( \angle 12 \) 8. \( \angle 1 \) 9. \( \angle 4 \) 10. \( \angle 3 \) 11. \( \angle 7 \) 12. \( \angle 16 \)
Angles and Parallel Lines

Algebra and Angle Measures

Algebra can be used to find unknown values in angles formed by a transversal and parallel lines.

**Example**

If \( m \angle 1 = 3x + 15 \), \( m \angle 2 = 4x - 5 \), and \( m \angle 3 = 5y \), find the value of \( x \) and \( y \).

\[ p \parallel q, \text{ so } m \angle 1 = m \angle 2 \]

because they are corresponding angles.

\[ m \angle 1 = m \angle 2 \]

\[ 3x + 15 = 4x - 5 \]

\[ 3x + 15 - 3x = 4x - 5 - 3x \]

\[ 15 = x - 5 \]

\[ 15 + 5 = x - 5 + 5 \]

\[ 20 = x \]

\[ r \parallel s, \text{ so } m \angle 2 = m \angle 3 \]

because they are corresponding angles.

\[ m \angle 2 = m \angle 3 \]

\[ 75 = 5y \]

\[ \frac{75}{5} = \frac{5y}{5} \]

\[ 15 = y \]

**Exercises**

Find the value of the variable(s) in each figure. Explain your reasoning.

1. \[ (5x - 5)^\circ \]
   \[ (6y - 4)^\circ \]
   \[ (4x + 10)^\circ \]

2. \[ 90^\circ \]
   \[ (15x + 30)^\circ \]
   \[ (3y + 18)^\circ \]
   \[ 10x^\circ \]

3. \[ (11x + 4)^\circ \]
   \[ (5y + 5)^\circ \]
   \[ 5x^\circ \]
   \[ (13y - 5)^\circ \]

4. \[ 3x^\circ \]
   \[ 2y^\circ \]
   \[ 4y^\circ \]
   \[ (5x - 20)^\circ \]

Find the value of the variable(s) in each figure. Explain your reasoning.

5. \[ (4x + 6)^\circ \]
   \[ x^\circ \]
   \[ 106^\circ \]
   \[ 2y^\circ \]

6. \[ 2x^\circ \]
   \[ 90^\circ \]
   \[ x^\circ \]
   \[ 2y^\circ \]
   \[ z^\circ \]
3-2 Practice

Angles and Parallel Lines

In the figure, $m\angle 2 = 92$ and $m\angle 12 = 74$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

1. $\angle 10$  
2. $\angle 8$  
3. $\angle 9$  
4. $\angle 5$  
5. $\angle 11$  
6. $\angle 13$

Find the value of the variable(s) in each figure. Explain your reasoning.

7. \[
\begin{align*}
3x^\circ & \quad (9x + 12)^\circ \\
(4y - 10)^\circ &
\end{align*}
\]

8. \[
\begin{align*}
(5y - 4)^\circ & \quad (2x + 13)^\circ \\
3y^\circ &
\end{align*}
\]

Find $x$. (Hint: Draw an auxiliary line.)

9. \[
\begin{align*}
\angle 1 & \quad 100^\circ \\
\angle 2 & \quad 50^\circ \\
\end{align*}
\]

10. \[
\begin{align*}
\angle 1 & \quad 144^\circ \\
\angle 2 & \quad 144^\circ \\
\end{align*}
\]

11. PROOF Write a paragraph proof of Theorem 3.3.

Given: $\ell \parallel m$, $m \parallel n$  
Prove: $\angle 1 \equiv \angle 12$

12. FENCING A diagonal brace strengthens the wire fence and prevents it from sagging. The brace makes a $50^\circ$ angle with the wire as shown. Find the value of the variable.
3-2 Word Problem Practice

Angles and Parallel Lines

1. RAMPS A parking garage ramp rises to connect two horizontal levels of a parking lot. The ramp makes a 10° angle with the horizontal. What is the measure of angle 1 in the figure?

2. BRIDGES A double decker bridge has two parallel levels connected by a network of diagonal girders. One of the girders makes a 52° angle with the lower level as shown in the figure. What is the measure of angle 1?

3. CITY ENGINEERING Seventh Avenue runs perpendicular to both 1st and 2nd Streets, which are parallel. However, Maple Avenue makes a 115° angle with 2nd Street. What is the measure of angle 1?

4. PODIUMS A carpenter is building a podium. The side panel of the podium is cut from a rectangular piece of wood.

The rectangle must be sawed along the dashed line in the figure. What is the measure of angle 1?

5. SECURITY An important bridge crosses a river at a key location. Because it is so important, robotic security cameras are placed at the locations of the dots in the figure. Each robot can scan x degrees. On the lower bank, it takes 4 robots to cover the full angle from the edge of the river to the bridge. On the upper bank, it takes 5 robots to cover the full angle from the edge of the river to the bridge.

a. How are the angles that are covered by the robots at the lower and upper banks related? Derive an equation that x satisfies based on this relationship.

b. How wide is the scanning angle for each robot? What are the angles that the bridge makes with the upper and lower banks?
### Proving Lines Parallel

**Identify Parallel Lines** If two lines in a plane are cut by a transversal and certain conditions are met, then the lines must be parallel.

<table>
<thead>
<tr>
<th>If</th>
<th>then</th>
</tr>
</thead>
<tbody>
<tr>
<td>• corresponding angles are congruent,</td>
<td>the lines are parallel.</td>
</tr>
<tr>
<td>• alternate exterior angles are congruent,</td>
<td></td>
</tr>
<tr>
<td>• consecutive interior angles are supplementary,</td>
<td></td>
</tr>
<tr>
<td>• alternate interior angles are congruent, or</td>
<td></td>
</tr>
<tr>
<td>• two lines are perpendicular to the same line,</td>
<td></td>
</tr>
</tbody>
</table>

**Example 1** If \( m\angle 1 = m\angle 2 \), determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

\[ \angle 1 \text{ and } \angle 2 \text{ are corresponding angles of lines } r \text{ and } s. \] Since \( \angle 1 \cong \angle 2 \), \( r \parallel s \) by the Converse of the Corresponding Angles Postulate.

**Example 2** Find \( m\angle ABC \) so that \( m \parallel n \).

\[ m \parallel n. \]

We can conclude that \( m \parallel n \) if alternate interior angles are congruent.

\[ m\angle BAD = m\angle ABC \]

\[ 3x + 10 = 6x - 20 \]
\[ 10 = 3x - 20 \]
\[ 30 = 3x \]
\[ 10 = x \]

\[ m\angle ABC = 6x - 20 \]
\[ = 6(10) - 20 \text{ or } 40 \]

**Exercises**

Find \( x \) so that \( \ell \parallel m \). Identify the postulate or theorem you used.

1. \( \ell \)

2. \( \ell \)

3. \( \ell \)

4. \( \ell \)

5. \( \ell \)

6. \( \ell \)
Proving Lines Parallel

**Example**

Given: $\angle 1 \cong \angle 2$, $\angle 1 \cong \angle 3$

Prove: $AB \parallel DC$

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle 1 \cong \angle 2$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle 2 \cong \angle 3$</td>
<td>2. Transitive Property of $\cong$</td>
</tr>
<tr>
<td>3. $AB \parallel DC$</td>
<td>3. If alt. int. angles are $\cong$, then the lines are $\parallel$.</td>
</tr>
</tbody>
</table>

**Exercises**

1. Complete the proof.

Given: $\angle 1 \cong \angle 5$, $\angle 15 \cong \angle 5$

Prove: $\ell \parallel m$, $r \parallel s$

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle 15 \cong \angle 5$</td>
<td>1. $\angle 15 \cong \angle 5$</td>
</tr>
<tr>
<td>2. $\angle 13 \cong \angle 15$</td>
<td>2. $\angle 13 \cong \angle 15$</td>
</tr>
<tr>
<td>3. $\angle 5 \cong \angle 13$</td>
<td>3. $\angle 5 \cong \angle 13$</td>
</tr>
<tr>
<td>4. $r \parallel s$</td>
<td>4. $r \parallel s$</td>
</tr>
<tr>
<td>5. $\angle 5 \cong \angle 13$</td>
<td>5. Given</td>
</tr>
<tr>
<td>6. $\ell \parallel m$</td>
<td>6. If corr $\Delta$ are $\cong$, then lines $\parallel$.</td>
</tr>
</tbody>
</table>
Proving Lines Parallel

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

1. \( m \angle BCG + m \angle FGC = 180 \)
2. \( \angle CBF \cong \angle GFH \)
3. \( \angle EFB \cong \angle FBC \)
4. \( \angle ACD \cong \angle KBF \)

Find \( x \) so that \( l \parallel m \). Identify the postulate or theorem you used.

5. \( (4x - 6)^\circ \)
6. \( (5x + 18)^\circ \)
7. \( (2x + 12)^\circ \)

8. PROOF Write a two-column proof.
   Given: \( \angle 2 \) and \( \angle 3 \) are supplementary.
   Prove: \( AB \parallel CD \)

9. LANDSCAPING The head gardener at a botanical garden wants to plant rosebushes in parallel rows on either side of an existing footpath. How can the gardener ensure that the rows are parallel?
3-5 Word Problem Practice

Proving Lines Parallel

1. **RECTANGLES** Jim made a frame for a painting. He wants to check to make sure that opposite sides are parallel by measuring the angles at the corners and seeing if they are right angles. How many corners must he check in order to be sure that the opposite sides are parallel?

2. **BOOKS** The two gray books on the bookshelf each make a 70° angle with the base of the shelf.

   What more can you say about these two gray books?

3. **PATTERNS** A rectangle is cut along the slanted, dashed line shown in the figure. The two pieces are rearranged to form another figure. Describe as precisely as you can the shape of the new figure. Explain.

4. **FIREWORKS** A fireworks display is being readied for a celebration. The designers want to have four fireworks shoot out along parallel trajectories. They decide to place two launchers on a dock and the other two on the roof of a building.

   To pull off this display, what should the measure of angle 1 be?

5. **SIGNS** Harold is making a giant letter “A” to put on the rooftop of the “A is for Apple” Orchard Store. The figure shows a sketch of the design.

   a. What should the measures of angles 1 and 2 be so that the horizontal part of the “A” is truly horizontal?

   b. When building the “A,” Harold makes sure that angle 1 is correct, but when he measures angle 2, it is not correct. What does this imply about the “A”?
**Study Guide**

**Perpendiculars and Distance**

**Distance From a Point to a Line**  When a point is not on a line, the distance from the point to the line is the length of the segment that contains the point and is perpendicular to the line.

---

**Example**

Construct the segment that represents the distance from $E$ to $\overrightarrow{AF}$.

Extend $\overrightarrow{AF}$. Draw $\overrightarrow{EG} \perp \overrightarrow{AF}$.

$\overrightarrow{EG}$ represents the distance from $E$ to $\overrightarrow{AF}$.

---

**Exercises**

Construct the segment that represents the distance indicated.

1. $C$ to $\overrightarrow{AB}$

![Diagram of a triangle with points A, B, and C]

2. $D$ to $\overrightarrow{AB}$

![Diagram of a quadrilateral with points A, B, and C]

3. $T$ to $\overrightarrow{RS}$

![Diagram of a quadrilateral with points R, S, U, and T]

4. $S$ to $\overrightarrow{PQ}$

![Diagram of a pentagon with points P, Q, R, S, and T]

5. $S$ to $\overrightarrow{QR}$

![Diagram of a pentagon with points P, Q, R, S, and T]

6. $S$ to $\overrightarrow{RT}$

![Diagram of a pentagon with points P, Q, R, S, and T]
Distance Between Parallel Lines  The distance between parallel lines is the length of a segment that has an endpoint on each line and is perpendicular to them. Parallel lines are everywhere equidistant, which means that all such perpendicular segments have the same length.

Example  Find the distance between the parallel lines ℓ and m with the equations \( y = 2x + 1 \) and \( y = 2x - 4 \), respectively.

To find the point of intersection of \( p \) and \( m \), solve a system of equations.

Line \( m \): \( y = 2x - 4 \)

Line \( p \): \( y = -\frac{1}{2}x + 1 \)

Use substitution.

\[ 2x - 4 = -\frac{1}{2}x + 1 \]
\[ 4x - 8 = -x + 2 \]
\[ 5x = 10 \]
\[ x = 2 \]

Substitute 2 for \( x \) to find the \( y \)-coordinate.

\[ y = -\frac{1}{2}x + 1 \]
\[ y = -\frac{1}{2}(2) + 1 = -1 + 1 = 0 \]

The point of intersection of \( p \) and \( m \) is \( (2, 0) \).

Use the Distance Formula to find the distance between \( (0, 1) \) and \( (2, 0) \).

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ = \sqrt{(2 - 0)^2 + (0 - 1)^2} \]
\[ = \sqrt{5} \]

The distance between \( ℓ \) and \( m \) is \( \sqrt{5} \) units.

Exercises  Find the distance between each pair of parallel lines with the given equations.

1. \( y = 8 \)
   \( y = -3 \)
2. \( y = x + 3 \)
   \( y = x - 1 \)
3. \( y = -2x \)
   \( y = -2x - 5 \)
**Perpendiculars and Distance**

Construct the segment that represents the distance indicated.

1. O to $\overrightarrow{MN}$
2. A to $\overrightarrow{DC}$
3. T to $\overrightarrow{UV}$

**COORDINATE GEOMETRY** Find the distance from $P$ to $\ell$.

4. Line $\ell$ contains points $(-2, 0)$ and $(4, 8)$. Point $P$ has coordinates $(5, 1)$.

5. Line $\ell$ contains points $(3, 5)$ and $(7, 9)$. Point $P$ has coordinates $(2, 10)$.

6. Line $\ell$ contains points $(5, 18)$ and $(9, 10)$. Point $P$ has coordinates $(-4, 26)$.

7. Line $\ell$ contains points $(-2, 4)$ and $(1, -9)$. Point $P$ has coordinates $(14, -6)$.

Find the distance between each pair of parallel lines with the given equation.

8. $y = -x$
   $y = -x - 4$

9. $y = 2x + 7$
   $y = 2x - 3$

10. $y = 3x + 12$
    $y = 3x - 18$

11. Graph the line $y = -x + 1$. Construct a perpendicular segment through the point at $(-2, -3)$. Then find the distance from the point to the line.

12. **CANOEING** Bronson and a friend are going to carry a canoe across a flat field to the bank of a straight canal. Describe the shortest path they can use.
1. **DISTANCE** What does it mean if the distance between a point \( P \) and a line \( \ell \) is zero? What does it mean if the distance between two lines is zero?

2. **DISTANCE** Paul is standing in the schoolyard. The figure shows his distance from various classroom doors lined up along the same wall.

   How far is Paul from the wall itself?

3. **SEASHELLS** Mason is standing on the seashore. He believes that if he makes a wish and throws a seashell back into the ocean, his wish will come true. Mason is standing at the origin of a coordinate plane and the shoreline is represented by the graph of the line \( y = 1.5x + 13 \). Each unit represents 1 meter. How far does Mason need to be able to throw the seashell to throw one into the ocean? Round your answer to the nearest centimeter.

4. **SUPPORTS** Two support beams are modeled by the lines \( y = 2x + 10 \) and \( y = 2x + 15 \). What is the distance between these two lines?

5. **RESCUE** Rachel sees a baseball heading straight for her friend Brad. Brad has no idea that he is about to be hit by a baseball.

   Rachel decides to run and intercept the baseball. Rachel is located at \((-3, 7)\). Brad is at \((-1, -6.25)\). The baseball is currently at \((20, 2.5)\) and closing fast.

   **a.** What is the equation of the line that passes through Brad and the baseball?

   **b.** If Rachel runs along the path of shortest distance to intercept (i.e., along the line perpendicular to the trajectory of the baseball), what are the coordinates of the point where she will end up when she is between Brad and the baseball?

   **c.** What is the shortest distance that Rachel must run in order to get between her friend and the baseball?
4-2 Study Guide

Angles of Triangles

Triangle Angle-Sum Theorem If the measures of two angles of a triangle are known, the measure of the third angle can always be found.

<table>
<thead>
<tr>
<th>Triangle Angle Sum Theorem</th>
<th>The sum of the measures of the angles of a triangle is 180. In the figure at the right, ( m\angle A + m\angle B + m\angle C = 180 ).</th>
</tr>
</thead>
</table>

Example 1 Find \( m\angle T \).

\[ m\angle R + m\angle S + m\angle T = 180 \quad \text{Triangle Angle-Sum Theorem} \]

\[ 25 + 35 + m\angle T = 180 \quad \text{Substitution} \]

\[ 60 + m\angle T = 180 \quad \text{Simplify} \]

\[ m\angle T = 120 \quad \text{Subtract 60 from each side.} \]

Example 2 Find the missing angle measures.

\[ m\angle 1 + m\angle A + m\angle B = 180 \quad \text{Triangle Angle-Sum Theorem} \]

\[ m\angle 1 + 58 + 90 = 180 \quad \text{Substitution} \]

\[ m\angle 1 = 32 \quad \text{Subtract 148 from each side.} \]

\[ m\angle 2 = 32 \quad \text{Vertical angles are congruent.} \]

\[ m\angle 3 + m\angle 2 + m\angle E = 180 \quad \text{Triangle Angle-Sum Theorem} \]

\[ m\angle 3 + 32 + 108 = 180 \quad \text{Substitution} \]

\[ m\angle 3 = 40 \quad \text{Subtract 140 from each side.} \]

Exercises

Find the measure of each numbered angle.

1. \[ P \]
2. \[ Q \]
3. \[ W \]
4. \[ M \]
5. \[ T \]
6. \[ A \]
Angles of Triangles

Exterior Angle Theorem
At each vertex of a triangle, the angle formed by one side and an extension of the other side is called an exterior angle of the triangle. For each exterior angle of a triangle, the remote interior angles are the interior angles that are not adjacent to that exterior angle. In the diagram below, \( \angle B \) and \( \angle A \) are the remote interior angles for exterior \( \angle DCB \).

Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

\[
m\angle 1 = m\angle A + m\angle B
\]

Example 1
Find \( m\angle 1 \).

\[
m\angle 1 = m\angle R + m\angle S
\]

Exterior Angle Theorem

\[
= 60 + 80 = 140
\]

Substitution

Simplify.

Example 2
Find \( x \).

\[
m\angle PQS = m\angle R + m\angle S
\]

Exterior Angle Theorem

\[
78 = 55 + x
\]

Substitution

\[
23 = x
\]

Exercises

Find the measures of each numbered angle.

1.

2.

3.

4.

Find each measure.

5. \( m\angle ABC \)

6. \( m\angle F \)
Angles of Triangles

Find the measure of each numbered angle.

1. 

2. 

Find each measure.

3. \( m \angle 1 \)
4. \( m \angle 2 \)
5. \( m \angle 3 \)

Find each measure.

6. \( m \angle 1 \)
7. \( m \angle 4 \)
8. \( m \angle 3 \)
9. \( m \angle 2 \)
10. \( m \angle 5 \)
11. \( m \angle 6 \)

Find each measure.

12. \( m \angle 1 \)
13. \( m \angle 2 \)

14. CONSTRUCTION The diagram shows an example of the Pratt Truss used in bridge construction. Use the diagram to find \( m \angle 1 \).
4-2 Word Problem Practice

Angles of Triangles

1. PATHS Eric walks around a triangular path. At each corner, he records the measure of the angle he creates.

He makes one complete circuit around the path. What is the sum of the three angle measures that he wrote down?

2. STANDING Sam, Kendra, and Tony are standing in such a way that if lines were drawn to connect the friends they would form a triangle.

If Sam is looking at Kendra he needs to turn his head 40° to look at Tony. If Tony is looking at Sam he needs to turn his head 50° to look at Kendra. How many degrees would Kendra have to turn her head to look at Tony if she is looking at Sam?

3. TOWERS A lookout tower sits on a network of struts and posts. Leslie measured two angles on the tower.

What is the measure of angle 1?

4. ZOOS The zoo lights up the chimpanzee pen with an overhead light at night. The cross section of the light beam makes an isosceles triangle.

The top angle of the triangle is 52 and the exterior angle is 116. What is the measure of angle 1?

5. DRAFTING Chloe bought a drafting table and set it up so that she can draw comfortably from her stool. Chloe measured the two angles created by the legs and the tabletop in case she had to dismantle the table.

a. Which of the four numbered angles can Chloe determine by knowing the two angles formed with the tabletop? What are their measures?

b. What conclusion can Chloe make about the unknown angles before she measures them to find their exact measurements?
Congruence and Corresponding Parts
Triangles that have the same size and same shape are **congruent triangles**. Two triangles are congruent if and only if all three pairs of corresponding angles are congruent and all three pairs of corresponding sides are congruent. In the figure, \( \triangle ABC \cong \triangle RST \).

<table>
<thead>
<tr>
<th>Third Angles Theorem</th>
<th>If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.</th>
</tr>
</thead>
</table>

**Example** If \( \triangle XYZ \cong \triangle RST \), name the pairs of congruent angles and congruent sides.
\[
\angle X \cong \angle R, \quad \angle Y \cong \angle S, \quad \angle Z \cong \angle T \\
XY \cong RS, \quad XZ \cong RT, \quad YZ \cong ST
\]

**Exercises**
Show that the polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

1. 
2. 
3. 
4. 
5. 
6. 

**Suppose** \( \triangle ABC \cong \triangle DEF \).

7. Find the value of \( x \).
8. Find the value of \( y \).
**4-3 Study Guide (continued)**

### Congruent Triangles

**Prove Triangles Congruent** Two triangles are congruent if and only if their corresponding parts are congruent. Corresponding parts include corresponding angles and corresponding sides. The phrase “if and only if” means that both the conditional and its converse are true. For triangles, we say, “Corresponding parts of congruent triangles are congruent,” or CPCTC.

#### Example

**Write a two-column proof.**

**Given:** \( \overline{AB} \cong \overline{CB}, \overline{AD} \cong \overline{CD}, \angle BAD \cong \angle BCD \)

\( \overline{BD} \) bisects \( \angle ABC \).

**Prove:** \( \triangle ABD \cong \triangle CBD \)

**Proof:**

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{AB} \cong \overline{CB}, \overline{AD} \cong \overline{CD} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \overline{BD} \cong \overline{BD} )</td>
<td>2. Reflexive Property of congruence</td>
</tr>
<tr>
<td>3. ( \angle BAD \cong \angle BCD )</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( \angle ABD \cong \angle CBD )</td>
<td>4. Definition of angle bisector</td>
</tr>
<tr>
<td>5. ( \angle BDA \cong \angle BDC )</td>
<td>5. Third Angles Theorem</td>
</tr>
<tr>
<td>6. ( \triangle ABD \cong \triangle CBD )</td>
<td>6. CPCTC</td>
</tr>
</tbody>
</table>

#### Exercises

**Write a two-column proof.**

1. **Given:** \( \angle A \cong \angle C, \angle D \cong \angle B, \overline{AD} \cong \overline{CB}, \overline{AE} \cong \overline{CE}, \)
\( \overline{AC} \) bisects \( \overline{BD} \).

**Prove:** \( \triangle AED \cong \triangle CEB \)

2. **Given:** \( \overline{BD} \) bisects \( \angle ABC \) and \( \angle ADC \),
\( \overline{AB} \cong \overline{CB}, \overline{AB} \cong \overline{AD}, \overline{CB} \cong \overline{DC} \)

**Prove:** \( \triangle ABD \cong \triangle CBD \)

---

*North Carolina StudyText, Math BC, Volume 1*
4-3 Practice

Congruent Triangles

Show that the polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

1. 

2. 

Polygon $ABCD \cong$ polygon $PQRS$.

3. Find the value of $x$.

4. Find the value of $y$.

5. PROOF Write a two-column proof.
   Given: $\angle P \cong \angle R$, $\angle PSQ \cong \angle RSQ$, $PQ \cong RQ$, $PS \cong RS$
   Prove: $\triangle PQS \cong \triangle RQS$

6. QUILTING
   a. Indicate the triangles that appear to be congruent.
   b. Name the congruent angles and congruent sides of a pair of congruent triangles.
4-3 Word Problem Practice

Congruent Triangles

1. PICTURE HANGING Candice hung a picture that was in a triangular frame on her bedroom wall.

One day, it fell to the floor, but did not break or bend. The figure shows the object before and after the fall. Label the vertices on the frame after the fall according to the “before” frame.

2. SIERPINSKI’S TRIANGLE The figure below is a portion of Sierpinski’s Triangle. The triangle has the property that any triangle made from any combination of edges is equilateral. How many triangles in this portion are congruent to the black triangle at the bottom corner?

3. QUILTING Stefan drew this pattern for a piece of his quilt. It is made up of congruent isosceles right triangles. He drew one triangle and then repeatedly drew it all the way around.

What are the missing measures of the angles of the triangle?

4. MODELS Dana bought a model airplane kit. When he opened the box, these two congruent triangular pieces of wood fell out of it.

Identify the triangle that is congruent to \( \triangle ABC \).

5. GEOGRAPHY Igor noticed on a map that the triangle whose vertices are the supermarket, the library, and the post office (\( \triangle SLP \)) is congruent to the triangle whose vertices are Igor’s home, Jacob’s home, and Ben’s home (\( \triangle IJB \)). That is, \( \triangle SLP \cong \triangle IJB \).

a. The distance between the supermarket and the post office is 1 mile. Which path along the triangle \( \triangle IJB \) is congruent to this?

b. The measure of \( \angle LPS \) is 40. Identify the angle that is congruent to this angle in \( \triangle IJB \).
SSS Postulate  You know that two triangles are congruent if corresponding sides are congruent and corresponding angles are congruent. The Side-Side-Side (SSS) Postulate lets you show that two triangles are congruent if you know only that the sides of one triangle are congruent to the sides of the second triangle.

SSS Postulate  If three sides of one triangle are congruent to three sides of a second triangle, then the triangles are congruent.

Example  Write a two-column proof.

Given: \( AB \cong DB \) and \( C \) is the midpoint of \( AD \).
Prove: \( \triangle ABC \cong \triangle DBC \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( AB \cong DB )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( C ) is the midpoint of ( AD )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( AC \cong DC )</td>
<td>3. Midpoint Theorem</td>
</tr>
<tr>
<td>4. ( BC \cong BC )</td>
<td>4. Reflexive Property of ( \cong )</td>
</tr>
<tr>
<td>5. ( \triangle ABC \cong \triangle DBC )</td>
<td>5. SSS Postulate</td>
</tr>
</tbody>
</table>

Exercises

Write a two-column proof.

1. \( \triangle ABC \cong \triangle XYZ \)

Given: \( AB \cong XY, AC \cong XZ, BC \cong YZ \)
Prove: \( \triangle ABC \cong \triangle XYZ \)

2. \( \triangle RST \cong \triangle UTS \)

Given: \( RS \cong UT, RT \cong US \)
Prove: \( \triangle RST \cong \triangle UTS \)
4-4 Study Guide (continued)

Proving Triangles Congruent—SSS, SAS

SAS Postulate  Another way to show that two triangles are congruent is to use the Side-Angle-Side (SAS) Postulate.

| SAS Postulate | If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. |

Example  For each diagram, determine which pairs of triangles can be proved congruent by the SAS Postulate.

a. \(\triangle ABC\), the angle is not “included” by the sides \(AB\) and \(AC\). So the triangles cannot be proved congruent by the SAS Postulate.

b. The right angles are congruent and they are the included angles for the congruent sides. \(\triangle DEF \cong \triangle JGH\) by the SAS Postulate.

c. The included angles, \(\angle 1\) and \(\angle 2\), are congruent because they are alternate interior angles for two parallel lines. \(\triangle PSR \cong \triangle RQP\) by the SAS Postulate.

Exercises

Write the specified type of proof.

1. Write a two column proof.
   **Given:** \(NP = PM, NP \perp PL\)
   **Prove:** \(\triangle NPL \cong \triangle MPL\)

2. Write a two-column proof.
   **Given:** \(AB = CD, AB \parallel CD\)
   **Prove:** \(\triangle ACD \cong \triangle CAB\)

3. Write a paragraph proof.
   **Given:** \(V\) is the midpoint of \(YZ\).
   \(V\) is the midpoint of \(WX\).
   **Prove:** \(\triangle XVZ \cong \triangle WYZ\)
4-4 Practice

Proving Triangles Congruent—SSS, SAS

Determine whether $\triangle DEF \cong \triangle PQR$ given the coordinates of the vertices. Explain.

1. $D(-6, 1), E(1, 2), F(-1, -4), P(0, 5), Q(7, 6), R(5, 0)$

2. $D(-7, -3), E(-4, -1), F(-2, -5), P(2, -2), Q(5, -4), R(0, -5)$

3. Write a flow proof.
   Given: $\overline{RS} \cong \overline{TS}$
   $V$ is the midpoint of $\overline{RT}$.
   Prove: $\triangle RSV \cong \triangle TSV$

Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove congruence, write not possible.

4. 

5. 

6. 

7. INDIRECT MEASUREMENT To measure the width of a sinkhole on his property, Harmon marked off congruent triangles as shown in the diagram. How does he know that the lengths $A'B'$ and $AB$ are equal?
1. **STICKS** Tyson had three sticks of lengths 24 inches, 28 inches, and 30 inches. Is it possible to make two noncongruent triangles using the same three sticks? Explain.

2. **BAKERY** Sonia made a sheet of baklava. She has markings on her pan so that she can cut them into large squares. After she cuts the pastry in squares, she cuts them diagonally to form two congruent triangles. Which postulate could you use to prove the two triangles congruent?

3. **CAKE** Carl had a piece of cake in the shape of an isosceles triangle with angles 26, 77, and 77. He wanted to divide it into two equal parts, so he cut it through the middle of the 26 angle to the midpoint of the opposite side. He says that because he is dividing it at the midpoint of a side, the two pieces are congruent. Is this enough information? Explain.

4. **TILES** Tammy installs bathroom tiles. Her current job requires tiles that are equilateral triangles and all the tiles have to be congruent to each other. She has a big sack of tiles all in the shape of equilateral triangles. Although she knows that all the tiles are equilateral, she is not sure they are all the same size. What must she measure on each tile to be sure they are congruent? Explain.

5. **INVESTIGATION** An investigator at a crime scene found a triangular piece of torn fabric. The investigator remembered that one of the suspects had a triangular hole in their coat. Perhaps it was a match. Unfortunately, to avoid tampering with evidence, the investigator did not want to touch the fabric and could not fit it to the coat directly.

   a. If the investigator measures all three side lengths of the fabric and the hole, can the investigator make a conclusion about whether or not the hole could have been filled by the fabric?

   b. If the investigator measures two sides of the fabric and the included angle and then measures two sides of the hole and the included angle can he determine if it is a match? Explain.
ASA Postulate  The Angle-Side-Angle (ASA) Postulate lets you show that two triangles are congruent.

ASA Postulate

| If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent. |

Example Write a two column proof.

Given: \( AB \parallel CD \)
\( \angle CBD \cong \angle ADB \)

Prove: \( \triangle ABD \cong \triangle CDB \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( AB \parallel CD )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle CBD \cong \angle ADB )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( \angle ABD \cong \angle BDC )</td>
<td>3. Alternate Interior Angles Theorem</td>
</tr>
<tr>
<td>4. ( BD \cong BD )</td>
<td>4. Reflexive Property of Congruence</td>
</tr>
<tr>
<td>5. ( \triangle ABD \cong \triangle CDB )</td>
<td>5. ASA</td>
</tr>
</tbody>
</table>

Exercises

PROOF Write the specified type of proof.

1. Write a two column proof.

   Given: \( \angle S \cong \angle V \), \( T \) is the midpoint of \( SV \). 
   Prove: \( \triangle RTS \cong \triangle UTV \)

2. Write a paragraph proof.

   Given: \( CD \) bisects \( AE \), \( AB \parallel CD \) 
   \( \angle E \cong \angle BCA \) 
   Prove: \( \triangle ABC \cong \triangle CDE \)
4-5 Study Guide (continued)

**Proving Triangles Congruent—ASA, AAS**

**AAS Theorem** Another way to show that two triangles are congruent is the Angle-Angle-Side (AAS) Theorem.

| AAS Theorem | If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent. |

You now have five ways to show that two triangles are congruent.

- definition of triangle congruence
- ASA Postulate
- SSS Postulate
- AAS Theorem
- SAS Postulate

**Example** In the diagram, \( \angle BCA \cong \angle DCA \). Which sides are congruent? Which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the AAS Theorem?

- \( \overline{AC} \cong \overline{AC} \) by the Reflexive Property of congruence. The congruent angles cannot be \( \angle 1 \) and \( \angle 2 \), because \( \overline{AC} \) would be the included side.
- If \( \angle B \cong \angle D \), then \( \triangle ABC \cong \triangle ADC \) by the AAS Theorem.

**Exercises**

**PROOF** Write the specified type of proof.

1. Write a two column proof.
   - **Given:** \( \overline{BC} \parallel \overline{EF} \)
   - \( AB \cong DE \)
   - \( \angle C \cong \angle F \)
   - **Prove:** \( \triangle ABC \cong \triangle DEF \)

2. Write a flow proof.
   - **Given:** \( \angle S \cong \angle U \); \( \overline{TR} \) bisects \( \angle STU \).
   - **Prove:** \( \angle SRT \cong \angle URT \)
4-5 Practice

Proving Triangles Congruent—ASA, AAS

PROOF Write the specified type of proof.

1. Write a flow proof.
   Given: \( S \) is the midpoint of \( QT \).
   \( QR \parallel TU \)
   Prove: \( \triangle QSR \cong \triangle TSU \)

2. Write a paragraph proof.
   Given: \( \angle D \cong \angle F \)
   \( GE \) bisects \( \angle DEF \).
   Prove: \( DG \cong FG \)

ARCHITECTURE For Exercises 3 and 4, use the following information.
An architect used the window design in the diagram when remodeling an art studio. \( AB \) and \( CB \) each measure 3 feet.

3. Suppose \( D \) is the midpoint of \( AC \). Determine whether \( \triangle ABD \cong \triangle CBD \).
   Justify your answer.

4. Suppose \( \angle A \cong \angle C \). Determine whether \( \triangle ABD \cong \triangle CBD \). Justify your answer.
1. **DOOR STOPS** Two door stops have cross-sections that are right triangles. They both have a 20° angle and the length of the side between the 90° and 20° angles are equal. Are the cross-sections congruent? Explain.

2. **MAPPING** Two people decide to take a walk. One person is in Bombay and the other is in Milwaukee. They start by walking straight for 1 kilometer. Then they both turn right at an angle of 110, and continue to walk straight again. After a while, they both turn right again, but this time at an angle of 120. They each walk straight for a while in this new direction until they end up where they started. Each person walked in a triangular path at their location. Are these two triangles congruent? Explain.

3. **CONSTRUCTION** The rooftop of Angelo’s house creates an equilateral triangle with the attic floor. Angelo wants to divide his attic into 2 equal parts. He thinks he should divide it by placing a wall from the center of the roof to the floor at a 90° angle. If Angelo does this, then each section will share a side and have corresponding 90° angles. What else must be explained to prove that the two triangular sections are congruent?

4. **LOGIC** When Carolyn finished her musical triangle class, her teacher gave each student in the class a certificate in the shape of a golden triangle. Each student received a different shaped triangle. Carolyn lost her triangle on her way home. Later she saw part of a golden triangle under a grate. Is enough of the triangle visible to allow Carolyn to determine that the triangle is indeed hers? Explain.

5. **PARK MAINTENANCE** Park officials need a triangular tarp to cover a field shaped like an equilateral triangle 200 feet on a side.

   a. Suppose you know that a triangular tarp has two 60° angles and one side of length 200 feet. Will this tarp cover the field? Explain.

   b. Suppose you know that a triangular tarp has three 60° angles. Will this tarp necessarily cover the field? Explain.
**Study Guide**

### Isosceles and Equilateral Triangles

**Properties of Isosceles Triangles** An isosceles triangle has two congruent sides called the legs. The angle formed by the legs is called the vertex angle. The other two angles are called base angles. You can prove a theorem and its converse about isosceles triangles.

- If two sides of a triangle are congruent, then the angles opposite those sides are congruent. (Isosceles Triangle Theorem)
- If two angles of a triangle are congruent, then the sides opposite those angles are congruent. (Converse of Isosceles Triangle Theorem)

**Example 1** Find $x$, given $BC \cong BA$.

\[ BC = BA, \text{ so } m\angle A = m\angle C \]
\[
5x - 10 = 4x + 5 \quad \text{Isos. Triangle Theorem}
\]
\[
x - 10 = 5 \quad \text{Substitution}
\]
\[
x = 15 \quad \text{Add 10 to each side.}
\]

**Example 2** Find $x$.

\[
m\angle S = m\angle T, \text{ so } SR = TR \quad \text{Converse of Isos. } \triangle \text{ Thm.}
\]
\[
3x - 13 = 2x \quad \text{Substitution}
\]
\[
3x = 2x + 13 \quad \text{Add 13 to each side.}
\]
\[
x = 13 \quad \text{Subtract 2x from each side.}
\]

**Exercises**

**ALGEBRA** Find the value of each variable.

1. \[
\begin{align*}
&\angle P = 40^\circ \quad \angle Q = 2x^\circ \\
&\angle R = 6x + 6^\circ
\end{align*}
\]

2. \[
\begin{align*}
&\angle S = 2x + 6^\circ \\
&\angle V = 3x - 6^\circ
\end{align*}
\]

3. \[
\begin{align*}
&\angle U = 3x^\circ \\
&\angle Z = 3x^\circ
\end{align*}
\]

4. \[
\begin{align*}
&\angle B = 2x^\circ \\
&\angle C = 6x + 6^\circ
\end{align*}
\]

5. \[
\begin{align*}
&m\angle N = 5x^\circ
\end{align*}
\]

7. **PROOF** Write a two-column proof.

Given: $\angle 1 \cong \angle 2$

Prove: $AB \cong CB$
Isosceles and Equilateral Triangles

Properties of Equilateral Triangles An equilateral triangle has three congruent sides. The Isosceles Triangle Theorem leads to two corollaries about equilateral triangles.

1. A triangle is equilateral if and only if it is equiangular.
2. Each angle of an equilateral triangle measures 60°.

Example Prove that if a line is parallel to one side of an equilateral triangle, then it forms another equilateral triangle.

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ΔABC is equilateral; PQ</td>
<td></td>
</tr>
<tr>
<td>2. m∠A = m∠B = m∠C = 60</td>
<td>2. Each ∠ of an equilateral Δ measures 60°.</td>
</tr>
<tr>
<td>3. ∠1 ≅ ∠B, ∠2 ≅ ∠C</td>
<td>3. If</td>
</tr>
<tr>
<td>4. m∠1 = 60, m∠2 = 60</td>
<td>4. Substitution</td>
</tr>
<tr>
<td>5. ΔAPQ is equilateral.</td>
<td>5. If a ∆ is equiangular, then it is equilateral.</td>
</tr>
</tbody>
</table>

Exercises

ALGEBRA Find the value of each variable.

1. \( D \)

2. \( G \)

3. \( L \)

4. \( A \)

5. \( X \)

6. \( J \)

7. PROOF Write a two-column proof.

Given: ΔABC is equilateral; ∠1 ≅ ∠2.

Prove: ∠ADB ≅ ∠CDB
4-6 Practice

Isosceles and Equilateral Triangles

Refer to the figure at the right.

1. If $RV \cong RT$, name two congruent angles.

2. If $RS \cong SV$, name two congruent angles.

3. If $\angle SRT \cong \angle STR$, name two congruent segments.

4. If $\angle STV \cong \angle SVT$, name two congruent segments.

Find each measure.

5. $m \angle KML$  

6. $m \angle HMG$  

7. $m \angle GHM$  

8. If $m \angle HJM = 145$, find $m \angle MHJ$.

9. If $m \angle G = 67$, find $m \angle GHM$.

10. PROOF Write a two-column proof.

   Given: $DE \parallel BC$  
        $\angle 1 \cong \angle 2$  

   Prove: $AB \cong AC$

11. SPORTS A pennant for the sports teams at Lincoln High School is in the shape of an isosceles triangle. If the measure of the vertex angle is $18^\circ$, find the measure of each base angle.
1. **TRIANGLES** At an art supply store, two different triangular rulers are available. One has angles 45°, 45°, and 90°. The other has angles 30°, 60°, and 90°. Which triangle is isosceles?

2. **RULERS** A foldable ruler has two hinges that divide the ruler into thirds. If the ends are folded up until they touch, what kind of triangle results?

3. **HEXAGONS** Juanita placed one end of each of 6 black sticks at a common point and then spaced the other ends evenly around that point. She connected the free ends of the sticks with lines. The result was a regular hexagon. This construction shows that a regular hexagon can be made from six congruent triangles. Classify these triangles. Explain.

4. **PATHS** A marble path is constructed out of several congruent isosceles triangles. The vertex angles are all 20°. What is the measure of angle 1 in the figure?

5. **BRIDGES** Every day, cars drive through isosceles triangles when they go over the Leonard Zakim Bridge in Boston. The ten-lane roadway forms the bases of the triangles.

   a. The angle labeled A in the picture has a measure of 67°. What is the measure of ∠B?

   b. What is the measure of ∠C?

   c. Name the two congruent sides.
Identify Congruence Transformations  A **congruence transformation** is a transformation where the original figure, or preimage, and the transformed figure, or image, figure are still congruent. The three types of congruence transformations are **reflection** (or flip), **translation** (or slide), and **rotation** (or turn).

**Example**  Identify the type of congruence transformation shown as a **reflection**, **translation**, or **rotation**.

<p>| | | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
<td><img src="image3" alt="Diagram" /></td>
</tr>
<tr>
<td>Each vertex and its image are the same distance from the y-axis. This is a reflection.</td>
<td>Each vertex and its image are in the same position, just two units down. This is a translation.</td>
<td>Each vertex and its image are the same distance from the origin. The angles formed by each pair of corresponding points and the origin are congruent. This is a rotation.</td>
</tr>
</tbody>
</table>

**Exercises**  Identify the type of congruence transformation shown as a **reflection**, **translation**, or **rotation**.

1. ![Diagram](image4)  
2. ![Diagram](image5)  
3. ![Diagram](image6)  
4. ![Diagram](image7)  
5. ![Diagram](image8)  
6. ![Diagram](image9)
Verify Congruence You can verify that reflections, translations, and rotations of triangles produce congruent triangles using SSS.

Example Verify congruence after a transformation.

\( \triangle WXY \) with vertices \( W(3, -7), X(6, -7), Y(6, -2) \) is a transformation of \( \triangle RST \) with vertices \( R(2, 0), S(5, 0), T(5, 5) \). Graph the original figure and its image. Identify the transformation and verify that it is a congruence transformation.

Graph each figure. Use the Distance Formula to show the sides are congruent and the triangles are congruent by SSS.

\[
RS = 3, \ ST = 5, \ TR = \sqrt{(5-2)^2 + (5-0)^2} = \sqrt{34}
\]

\[
WX = 3, \ XY = 5, \ YW = \sqrt{(6-3)^2 + (-2-(-7))^2} = \sqrt{34}
\]

\[
RS \cong WX, \ ST \cong XY, \ TR \cong YW
\]

By SSS, \( \triangle RST \cong \triangle WXY \).

Exercises

COORDINATE GEOMETRY Graph each pair of triangles with the given vertices. Then identify the transformation, and verify that it is a congruence transformation.

1. \(A(-3, 1), B(-1, 1), C(-1, 4);\)
   \(D(3, 1), E(1, 1), F(1, 4)\)

2. \(Q(-3, 0), R(-2, 2), S(-1, 0);\)
   \(T(2, -4), U(3, -2), V(4, -4)\)
4-7 Practice

Congruence Transformations

Identify the type of congruence transformation shown as a reflection, translation, or rotation.

3. Identify the type of congruence transformation shown as a reflection, translation, or rotation, and verify that it is a congruence transformation.

4. ΔABC has vertices A(−4, 2), B(−2, 0), C(−4, −2). ΔDEF has vertices D(4, 2), E(2, 0), F(4, −2). Graph the original figure and its image. Then identify the transformation and verify that it is a congruence transformation.

5. STENCILS Carly is planning on stenciling a pattern of flowers along the ceiling in her bedroom. She wants all of the flowers to look exactly the same. What type of congruence transformation should she use? Why?
1. **QUILTING** You and your two friends visit a craft fair and notice a quilt, part of whose pattern is shown below. Your first friend says the pattern is repeated by translation. Your second friend says the pattern is repeated by rotation. Who is correct?

![Quilt Pattern](image)

2. **RECYCLING** The international symbol for recycling is shown below. What type of congruence transformation does this illustrate?

![Recycling Symbol](image)

3. **ANATOMY** Carl notices that when he holds his hands palm up in front of him, they look the same. What type of congruence transformation does this illustrate?

4. **STAMPS** A sheet of postage stamps contains 20 stamps in 4 rows of 5 identical stamps each. What type of congruence transformation does this illustrate?

5. **MOSAICS** José cut rectangles out of tissue paper to create a pattern. He cut out four congruent blue rectangles and then cut out four congruent red rectangles that were slightly smaller to create the pattern shown below.

![Mosaic Pattern](image)

   a. Which congruence transformation does this pattern illustrate?

   b. If the whole square were rotated 90°, would the transformation still be the same?
Triangles and Coordinate Proof

Position and Label Triangles  A coordinate proof uses points, distances, and slopes to prove geometric properties. The first step in writing a coordinate proof is to place a figure on the coordinate plane and label the vertices. Use the following guidelines.

1. Use the origin as a vertex or center of the figure.
2. Place at least one side of the polygon on an axis.
3. Keep the figure in the first quadrant if possible.
4. Use coordinates that make computations as simple as possible.

Example  Position an isosceles triangle on the coordinate plane so that its sides are $a$ units long and one side is on the positive $x$-axis.

Start with $R(0, 0)$. If $RT$ is $a$, then another vertex is $T(a, 0)$. For vertex $S$, the $x$-coordinate is $\frac{a}{2}$. Use $b$ for the $y$-coordinate, so the vertex is $S\left(\frac{a}{2}, b\right)$.

Exercises

Name the missing coordinates of each triangle.

1. $\triangle ABC$ with $A(0, 0)$ and $B(2p, 0)$

2. $\triangle TQR$ with $R(0, 0)$ and $S(2a, 0)$

3. $\triangle GFE$ with $E(?, ?)$ and $G(2g, 0)$

Position and label each triangle on the coordinate plane.

4. isosceles triangle $\triangle RST$ with base $\overline{RS}$

5. isosceles right $\triangle DEF$ with legs $e$ units long

6. equilateral triangle $\triangle EQI$ with vertex $Q(0, \sqrt{3}b)$ and sides 2$b$ units long
Write Coordinate Proofs  Coordinate proofs can be used to prove theorems and to verify properties. Many coordinate proofs use the Distance Formula, Slope Formula, or Midpoint Theorem.

**Example**  Prove that a segment from the vertex angle of an isosceles triangle to the midpoint of the base is perpendicular to the base.

First, position and label an isosceles triangle on the coordinate plane. One way is to use $T(a, 0), R(-a, 0),$ and $S(0, c)$. Then $U(0, 0)$ is the midpoint of $RT$.

**Given:** Isosceles $\triangle RST$; $U$ is the midpoint of base $RT$.

**Prove:** $SU \perp RT$

**Proof:**

$U$ is the midpoint of $RT$ so the coordinates of $U$ are $\left(\frac{-a + a}{2}, \frac{0 + 0}{2}\right) = (0, 0)$. Thus $SU$ lies on the $y$-axis, and $\triangle RST$ was placed so $RT$ lies on the $x$-axis. The axes are perpendicular, so $SU \perp RT$.

**Exercise**

**PROOF** Write a coordinate proof for the statement.

Prove that the segments joining the midpoints of the sides of a right triangle form a right triangle.
4-8 Practice

Triangles and Coordinate Proof

Position and label each triangle on the coordinate plane.

1. equilateral \( \triangle SWY \) with sides \( \frac{1}{4}a \) units long
2. isosceles \( \triangle BLP \) with base \( BL \) 3\( b \) units long
3. isosceles right \( \triangle DGJ \) with hypotenuse \( DJ \) and legs 2\( a \) units long

Name the missing coordinates of each triangle.

4. \( \triangle S(?, ?) \)
5. \( \triangle E(0, ?) \)
6. \( \triangle M(0, ?) \)

NEIGHBORHOODS For Exercises 7 and 8, use the following information.
Karina lives 6 miles east and 4 miles north of her high school. After school she works part time at the mall in a music store. The mall is 2 miles west and 3 miles north of the school.

7. Proof Write a coordinate proof to prove that Karina’s high school, her home, and the mall are at the vertices of a right triangle.

Given: \( \triangle SKM \)
Prove: \( \triangle SKM \) is a right triangle.

8. Find the distance between the mall and Karina’s home.
1. **SHELVES** Martha has a shelf bracket shaped like a right isosceles triangle. She wants to know the length of the hypotenuse relative to the sides. She does not have a ruler, but remembers the Distance Formula. She places the bracket on a coordinate grid with the right angle at the origin. The length of each leg is $a$. What are the coordinates of the vertices making the two acute angles?

2. **FLAGS** A flag is shaped like an isosceles triangle. A designer would like to make a drawing of the flag on a coordinate plane. She positions it so that the base of the triangle is on the $y$-axis with one endpoint located at $(0, 0)$. She locates the tip of the flag at $(a, b)$. What are the coordinates of the third vertex?

3. **BILLIARDS** The figure shows a situation on a billiard table.

What are the coordinates of the cue ball before it is hit and the point where the cue ball hits the edge of the table?

4. **TENTS** The entrance to Matt’s tent makes an isosceles triangle. If placed on a coordinate grid with the base on the $x$-axis and the left corner at the origin, the right corner would be at $(6, 0)$ and the vertex angle would be at $(3, 4)$. Prove that it is an isosceles triangle.

5. **DRAFTING** An engineer is designing a roadway. Three roads intersect to form a triangle. The engineer marks two points of the triangle at $(-5, 0)$ and $(5, 0)$ on a coordinate plane.

   a. Describe the set of points in the coordinate plane that could not be used as the third vertex of the triangle.

   b. Describe the set of points in the coordinate plane that would make the vertex of an isosceles triangle together with the two congruent sides.

   c. Describe the set of points in the coordinate plane that would make a right triangle with the other two points if the right angle is located at $(-5, 0)$. 

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**Bisectors of Triangles**

**Perpendicular Bisector** A perpendicular bisector is a line, segment, or ray that is perpendicular to the given segment and passes through its midpoint. Some theorems deal with perpendicular bisectors.

<table>
<thead>
<tr>
<th>Perpendicular Bisector Theorem</th>
<th>Converse of Perpendicular Bisector Theorem</th>
<th>Circumcenter Theorem</th>
</tr>
</thead>
<tbody>
<tr>
<td>If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.</td>
<td>If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.</td>
<td>The perpendicular bisectors of the sides of a triangle intersect at a point called the circumcenter that is equidistant from the vertices of the triangle.</td>
</tr>
</tbody>
</table>

**Example 1** Find the measure of $FM$.

$FK$ is the perpendicular bisector of $GM$.

$FG = FM$

$2.8 = FM$

**Example 2** $BD$ is the perpendicular bisector of $AC$. Find $x$.

$AD = DC$

$3x + 8 = 5x - 6$

$14 = 2x$

$7 = x$

**Exercises**

Find each measure.

1. $XW$

2. $BF$

Point $P$ is the circumcenter of $\triangle EMK$. List any segment(s) congruent to each segment below.

3. $\overline{MY}$

4. $\overline{KP}$

5. $\overline{MN}$

6. $\overline{ER}$
5-1 Study Guide (continued)

Bisectors of Triangles

Angle Bisectors  Another special segment, ray, or line is an angle bisector, which divides an angle into two congruent angles.

<table>
<thead>
<tr>
<th>Angle Bisector Theorem</th>
<th>If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Converse of Angle Bisector Theorem</td>
<td>If a point in the interior of an angle if equidistant from the sides of the angle, then it is on the bisector of the angle.</td>
</tr>
<tr>
<td>Inceter Theorem</td>
<td>The angle bisectors of a triangle intersect at a point called the inceter that is equidistant from the sides of the triangle.</td>
</tr>
</tbody>
</table>

Example  $\overrightarrow{MR}$ is the angle bisector of $\angle NMP$. Find $x$ if $m\angle 1 = 5x + 8$ and $m\angle 2 = 8x - 16$.

$\overrightarrow{MR}$ is the angle bisector of $\angle NMP$, so $m\angle 1 = m\angle 2$.

$5x + 8 = 8x - 16$

$24 = 3x$

$8 = x$

Exercises

Find each measure.

1. $m\angle ABE$

2. $m\angle YBA$

3. $MK$

4. $m\angle EWL$

Point $U$ is the inceter of $\triangle GHY$. Find each measure.

5. $MU$

6. $m\angle UGM$

7. $m\angle PHU$

8. $HU$
5-1 Practice

Bisectors of Triangles

Find each measure.

1. $TP$

2. $VU$

3. $KN$

4. $m\angle NJZ$

5. $QA$

6. $m\angle MFZ$

Point $L$ is the circumcenter of $\triangle BKT$. List any segment(s) congruent to each segment.

7. $\overline{BN}$

8. $\overline{BL}$

Point $A$ is the incenter of $\triangle LYG$. Find each measure.

9. $m\angle YLA$

10. $m\angle YGA$

11. **SCULPTURE** A triangular entranceway has walls with corner angles of $50^\circ$, $70^\circ$, and $60^\circ$. The designer wants to place a tall bronze sculpture on a round pedestal in a central location equidistant from the three walls. How can the designer find where to place the sculpture?
1. **WIND CHIME** Joanna has a flat wooden triangular piece as part of a wind chime. The piece is suspended by a wire anchored at a point equidistant from the sides of the triangle. Where is the anchor point located?

2. **PICNICS** Marsha and Bill are going to the park for a picnic. The park is triangular. One side of the park is bordered by a river and the other two sides are bordered by busy streets. Marsha and Bill want to find a spot that is equally far away from the river and the streets. At what point in the park should they set up their picnic?

3. **MOVING** Martin has 3 grown children. The figure shows the locations of Martin’s children on a map that has a coordinate plane on it. Martin would like to move to a location that is the same distance from all three of his children. What are the coordinates of the location on the map that is equidistant from all three children?

4. **NEIGHBORHOOD** Amanda is looking at her neighborhood map. She notices that her house along with the homes of her friends, Brian and Cathy, can be the vertices of a triangle. The map is on a coordinate grid. Amanda’s house is at the point (1, 3), Brian’s is at (5, −1), and Cathy’s is at (4, 5). Where would the three friends meet if they each left their houses at the same time and walked to the opposite side of the triangle along the path of shortest distance from their house?

5. **PLAYGROUND** A concrete company is pouring concrete into a triangular form as the center of a new playground.

   a. The foreman measures the triangle and notices that the incenter and the circumcenter are the same. What type of triangle is being created?

   b. Suppose the foreman changes the triangular form so that the circumcenter is outside of the triangle but the incenter is inside the triangle. What type of triangle would be created?
Medians  A median is a line segment that connects a vertex of a triangle to the midpoint of the opposite side. The three medians of a triangle intersect at the centroid of the triangle. The centroid is located two thirds of the distance from a vertex to the midpoint of the side opposite the vertex on a median.

Example  In \( \triangle ABC \), \( U \) is the centroid and \( BU = 16 \). Find \( UK \) and \( BK \).

\[
BU = \frac{2}{3}BK \\
16 = \frac{2}{3}BK \\
24 = BK
\]

\[
BU + UK = BK \\
16 + UK = 24 \\
UK = 8
\]

Exercises
In \( \triangle ABC \), \( AU = 16 \), \( BU = 12 \), and \( CF = 18 \). Find each measure.

1. \( UD \)  
2. \( EU \)

3. \( CU \)  
4. \( AD \)

5. \( UF \)  
6. \( BE \)

In \( \triangle CDE \), \( U \) is the centroid, \( UK = 12 \), \( EM = 21 \), and \( UD = 9 \). Find each measure.

7. \( CU \)  
8. \( MU \)

9. \( CK \)  
10. \( JU \)

11. \( EU \)  
12. \( JD \)
Altitudes An altitude of a triangle is a segment from a vertex to the line containing the opposite side meeting at a right angle. Every triangle has three altitudes which meet at a point called the orthocenter.

Example The vertices of \( \triangle ABC \) are \( A(1, 3) \), \( B(7, 7) \) and \( C(9, 3) \). Find the coordinates of the orthocenter of \( \triangle ABC \).

Find the point where two of the three altitudes intersect.

Find the equation of the altitude from \( A \) to \( BC \).

If \( BC \) has a slope of \(-2\), then the altitude has a slope of \( \frac{1}{2} \).

\[
y - y_1 = m(x - x_1)
\]

Point-slope form

\[
y - 3 = \frac{1}{2}(x - 1)
\]

\[
m = \frac{1}{2}, (x, y) = A(1, 3)
\]

Distributive Property

\[
y = \frac{1}{2}x + \frac{5}{2}
\]

Simplify.

Find the equation of the altitude from \( C \) to \( AB \).

If \( AB \) has a slope of \( \frac{2}{3} \), then the altitude has a slope of \( -\frac{3}{2} \).

\[
y - y_1 = m(x - x_1)
\]

Point-slope form

\[
y - 3 = -\frac{3}{2}(x - 9)
\]

\[
m = -\frac{3}{2}, (x, y) = C(9, 3)
\]

Distributive Property

\[
y = -\frac{3}{2}x + \frac{33}{2}
\]

Simplify.

Solve the system of equations and find where the altitudes meet.

\[
y = \frac{1}{2}x + \frac{5}{2}
\]

\[
y = -\frac{3}{2}x + \frac{33}{2}
\]

Original equations

\[
\frac{1}{2}x + \frac{5}{2} = -\frac{3}{2}x + \frac{33}{2}
\]

Substitute \( \frac{1}{2}x + \frac{5}{2} \) for \( y \).

\[
\frac{5}{2} = -2x + \frac{33}{2}
\]

Subtract \( \frac{1}{2}x \) from each side.

\[
-14 = -2x
\]

Subtract \( \frac{33}{2} \) from each side.

\[
x = 7
\]

Divide each side by \(-2\).

\[
y = \frac{1}{2}x + \frac{5}{2} = \frac{1}{2}(7) + \frac{5}{2} = \frac{7}{2} + \frac{5}{2} = 6
\]

The coordinates of the orthocenter of \( \triangle ABC \) are \( (7, 6) \).

Exercises

COORDINATE GEOMETRY Find the coordinates of the orthocenter of the triangle with the given vertices.

1. \( J(1, 0), H(6, 0), I(3, 6) \)

2. \( S(1, 0), T(4, 7), U(8, -3) \)
Medians and Altitudes of Triangles

In \( \triangle ABC \), \( CP = 30 \), \( EP = 18 \), and \( BF = 39 \). Find each measure.

1. \( PD \)  
2. \( FP \)
3. \( BP \)  
4. \( CD \)
5. \( PA \)  
6. \( EA \)

In \( \triangle MIV \), \( Z \) is the centroid, \( MZ = 6 \), \( YI = 18 \), and \( NZ = 12 \). Find each measure.

7. \( ZR \)  
8. \( YZ \)
9. \( MR \)  
10. \( ZV \)
11. \( NV \)  
12. \( IZ \)

COORDINATE GEOMETRY Find the coordinates of the centroid of the triangle with the given vertices.

13. \( I(3, 1), J(6, 3), K(3, 5) \)
14. \( H(0, 1), U(4, 3), P(2, 5) \)

COORDINATE GEOMETRY Find the coordinates of the orthocenter of the triangle with the given vertices.

15. \( P(-1, 2), Q(5, 2), R(2, 1) \)
16. \( S(0, 0), T(3, 3), U(3, 6) \)

17. MOBILES Nabuko wants to construct a mobile out of flat triangles so that the surfaces of the triangles hang parallel to the floor when the mobile is suspended. How can Nabuko be certain that she hangs the triangles to achieve this effect?
1. **BALANCING** Johanna balanced a triangle flat on her finger tip. What point of the triangle must Johanna be touching?

2. **REFLECTIONS** Part of the working space in Paulette’s loft is partitioned in the shape of a nearly equilateral triangle with mirrors hanging on all three partitions. From which point could someone see the opposite corner behind his or her reflection in any of the three mirrors?

3. **DISTANCES** For what kind of triangle is there a point where the distance to each side is half the distance to each vertex? Explain.

4. **MEDIANS** Look at the right triangle below. What do you notice about the orthocenter and the vertices of the triangle?

5. **PLAZAS** An architect is designing a triangular plaza. For aesthetic purposes, the architect pays special attention to the location of the centroid $C$ and the circumcenter $O$.

   a. Give an example of a triangular plaza where $C = O$. If no such example exists, state that this is **impossible**.

   b. Give an example of a triangular plaza where $C$ is inside the plaza and $O$ is outside the plaza. If no such example exists, state that this is **impossible**.

   c. Give an example of a triangular plaza where $C$ is outside the plaza and $O$ is inside the plaza. If no such example exists, state that this is **impossible**.
Angle Inequalities  Properties of inequalities, including the Transitive, Addition, and Subtraction Properties of Inequality, can be used with measures of angles and segments. There is also a Comparison Property of Inequality.

For any real numbers $a$ and $b$, either $a < b$, $a = b$, or $a > b$.

The Exterior Angle Inequality Theorem can be used to prove this inequality involving an exterior angle.

<table>
<thead>
<tr>
<th>Exterior Angle Inequality Theorem</th>
<th>$m\angle 1 &gt; m\angle A$, $m\angle 1 &gt; m\angle B$</th>
</tr>
</thead>
</table>

**Example**  List all angles of $\triangle EFG$ with measures that are less than $m\angle 1$.

The measure of an exterior angle is greater than the measure of either remote interior angle. So $m\angle 3 < m\angle 1$ and $m\angle 4 < m\angle 1$.

**Exercises**

Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition.

1. measures are less than $m\angle 1$
2. measures are greater than $m\angle 3$
3. measures are less than $m\angle 1$
4. measures are greater than $m\angle 1$
5. measures are less than $m\angle 7$
6. measures are greater than $m\angle 2$
7. measures are greater than $m\angle 5$
8. measures are less than $m\angle 4$
9. measures are less than $m\angle 1$
10. measures are greater than $m\angle 4$
Angle-Side Relationships: When the sides of triangles are not congruent, there is a relationship between the sides and angles of the triangles.

- If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.
- If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.

**Example 1**

List the angles in order from smallest to largest measure.

![Diagram of triangle with sides 6 cm, 7 cm, and 9 cm labeled R, S, T.]

\[ \angle R, \angle S, \angle T \]

**Example 2**

List the sides in order from shortest to longest.

![Diagram of triangle with angles 20°, 125°, and 35° labeled A, B, C.]

\[ \text{CB, AB, AC} \]

**Exercises**

List the angles and sides in order from smallest to largest.

1. ![Diagram of triangle with sides 35 cm, 48 cm, and 23.7 cm labeled T, S, R.]
2. ![Diagram of triangle with angles 60°, 40°, and 90° labeled T, R, S.]
3. ![Diagram of triangle with sides 3.8 cm, 4.0 cm, and 4.3 cm labeled B, A, C.]
4. ![Diagram of triangle with sides 11 cm, 14 cm, and 12 cm labeled S, T, U.]
5. ![Diagram of triangle with sides 8 cm, 5 cm, and 4 cm labeled B, C, A.]
6. ![Diagram of triangle with sides 20 cm, 12 cm, and 18 cm labeled Q, P, R.]
7. ![Diagram of triangle with angles 35°, 120°, and 25° labeled C, D, E.]
8. ![Diagram of triangle with angles 56°, 58°, and 58° labeled Y, Z, X.]
9. ![Diagram of triangle with angles 60°, 54°, and 56° labeled R, S, T.]

**Inequalities in One Triangle**

If \( AC > AB \), then \( m\angle B > m\angle C \).

If \( m\angle A > m\angle C \), then \( BC > AB \).
5-3 Practice

Inequalities in One Triangle

Use the figure at the right to determine which angle has the greatest measure.

1. \( \angle 1, \angle 3, \angle 4 \)
2. \( \angle 4, \angle 8, \angle 9 \)
3. \( \angle 2, \angle 3, \angle 7 \)
4. \( \angle 7, \angle 8, \angle 10 \)

Use the Exterior Angle Inequality Theorem to list all angles that satisfy the stated condition.

5. measures are less than \( m\angle 1 \)
6. measures are less than \( m\angle 3 \)
7. measures are greater than \( m\angle 7 \)
8. measures are greater than \( m\angle 2 \)

Use the figure at the right to determine the relationship between the measures of the given angles.

9. \( m\angle QRW, m\angle RWQ \)
10. \( m\angle RTW, m\angle TWR \)
11. \( m\angle RST, m\angle TRS \)
12. \( m\angle WQR, m\angle QRW \)

Use the figure at the right to determine the relationship between the lengths of the given sides.

13. \( \overline{DH}, \overline{GH} \)
14. \( \overline{DE}, \overline{DG} \)
15. \( \overline{EG}, \overline{FG} \)
16. \( \overline{DE}, \overline{EG} \)

17. SPORTS The figure shows the position of three trees on one part of a disc golf course. At which tree position is the angle between the trees the greatest?
1. DISTANCE Carl and Rose live on the same straight road. From their balconies they can see a flagpole in the distance. The angle that each person’s line of sight to the flagpole makes with the road is the same. How do their distances from the flagpole compare?

2. OBTUSE TRIANGLES Don notices that the side opposite the right angle in a right triangle is always the longest of the three sides. Is this also true of the side opposite the obtuse angle in an obtuse triangle? Explain.

3. STRING Jake built a triangular structure with three black sticks. He tied one end of a string to vertex $M$ and the other end to a point on the stick opposite $M$, pulling the string taut. Prove that the length of the string cannot exceed the longer of the two sides of the structure.

4. SQUARES Matthew has three different squares. He arranges the squares to form a triangle as shown. Based on the information, list the squares in order from the one with the smallest perimeter to the one with the largest perimeter.

5. CITIES Stella is going to Texas to visit a friend. As she was looking at a map to see where she might want to go, she noticed the cities Austin, Dallas, and Abilene formed a triangle. She wanted to determine how the distances between the cities were related, so she used a protractor to measure two angles.
   a. Based on the information in the figure, which of the two cities are nearest to each other?

   b. Based on the information in the figure, which of the two cities are farthest apart from each other?
Indirect Algebraic Proof One way to prove that a statement is true is to temporarily assume that what you are trying to prove is false. By showing this assumption to be logically impossible, you prove your assumption false and the original conclusion true. This is known as an indirect proof.

Steps for Writing an Indirect Proof
1. Assume that the conclusion is false by assuming the opposite is true.
2. Show that this assumption leads to a contradiction of the hypothesis or some other fact.
3. Point out that the assumption must be false, and therefore, the conclusion must be true.

Example

Given: \(3x + 5 > 8\)
Prove: \(x > 1\)

Step 1
Assume that \(x\) is not greater than 1. That is, \(x = 1\) or \(x < 1\).

Step 2
Make a table for several possibilities for \(x = 1\) or \(x < 1\).
When \(x = 1\) or \(x < 1\), then \(3x + 5\) is not greater than \(8\).

Step 3
This contradicts the given information that \(3x + 5 > 8\). The assumption that \(x\) is not greater than 1 must be false, which means that the statement “\(x > 1\)” must be true.

Exercises

State the assumption you would make to start an indirect proof of each statement.

1. If \(2x > 14\), then \(x > 7\).
2. For all real numbers, if \(a + b > c\), then \(a > c - b\).

Complete the indirect proof.

Given: \(n\) is an integer and \(n^2\) is even.
Prove: \(n\) is even.

3. Assume that __________________________
4. Then \(n\) can be expressed as \(2a + 1\) by __________________________
5. \(n^2 = _____________\) Substitution
6. = __________________ Multiply.
7. = __________________ Simplify.
8. = \(2(2a^2 + 2a) + 1\) __________________________
9. \(2(2a^2 + 2a) + 1\) is an odd number. This contradicts the given that \(n^2\) is even, so the assumption must be __________________________
10. Therefore, __________________________
Indirect Proof with Geometry  To write an indirect proof in geometry, you assume that the conclusion is false. Then you show that the assumption leads to a contradiction. The contradiction shows that the conclusion cannot be false, so it must be true.

**Example**

Given: \( m\angle C = 100 \)
Prove: \( \angle A \) is not a right angle.

**Step 1** Assume that \( \angle A \) is a right angle.

**Step 2** Show that this leads to a contradiction. If \( \angle A \) is a right angle, then \( m\angle A = 90 \) and \( m\angle C + m\angle A = 100 + 90 = 190 \). Thus the sum of the measures of the angles of \( \triangle ABC \) is greater than 180.

**Step 3** The conclusion that the sum of the measures of the angles of \( \triangle ABC \) is greater than 180 is a contradiction of a known property. The assumption that \( \angle A \) is a right angle must be false, which means that the statement “\( \angle A \) is not a right angle” must be true.

**Exercises**

State the assumption you would make to start an indirect proof of each statement.

1. If \( m\angle A = 90 \), then \( m\angle B = 45 \).

2. If \( \overrightarrow{AV} \) is not congruent to \( \overrightarrow{VE} \), then \( \triangle AVE \) is not isosceles.

Complete the indirect proof.

Given: \( \angle 1 \cong \angle 2 \) and \( \overline{DG} \) is not congruent to \( \overline{FG} \).
Prove: \( \overline{DE} \) is not congruent to \( \overline{FE} \).

3. Assume that _______ Assume the conclusion is false.

4. \( \overline{EG} \cong \overline{EG} \) __________________________________________________________________________

5. \( \triangle EDG \cong \triangle EFG \) __________________________________________________________________________

6. __________________________________________________________________________

7. This contradicts the given information, so the assumption must
   be __________________________________________________________________________

8. Therefore, __________________________________________________________________________
**5-4 Practice**

**Indirect Proof**

State the assumption you would make to start an indirect proof of each statement.

1. \(BD\) bisects \(\angle ABC\).

2. \(RT = TS\)

Write an indirect proof of each statement.

3. **Given**: \(-4x + 2 < -10\)  
   **Prove**: \(x > 3\)

4. **Given**: \(m\angle 2 + m\angle 3 \neq 180\)  
   **Prove**: \(a \parallel b\)

5. **PHYSICS** Sound travels through air at about 344 meters per second when the temperature is 20°C. If Enrique lives 2 kilometers from the fire station and it takes 5 seconds for the sound of the fire station siren to reach him, how can you prove indirectly that it is not 20°C when Enrique hears the siren?
1. **CANOES** Thirty-five students went on a canoeing expedition. They rented 17 canoes for the trip. Use an indirect proof to show that at least one canoe had more than two students in it.

2. **AREA** The area of the United States is about 6,000,000 square miles. The area of Hawaii is about 11,000 square miles. Use an indirect proof to show that at least one of the fifty states has an area greater than 120,000 square miles.

3. **CONSECUTIVE NUMBERS** David was trying to find a common factor other than 1 between various pairs of consecutive integers. Write an indirect proof to show David that two consecutive integers do not share a common factor other than 1.

4. **WORDS** The words *accomplishment*, *counterexample*, and *extemporaneous* all have 14 letters. Use an indirect proof to show that any word with 14 letters must use a repeated letter or have two letters that are consecutive in the alphabet.

5. **LATTICE TRIANGLES** A lattice point is a point whose coordinates are both integers. A lattice triangle is a triangle whose vertices are lattice points. It is a fact that a lattice triangle has an area of at least 0.5 square units.

   a. Suppose \( \triangle ABC \) has a lattice point in its interior. Show that the lattice triangle can be partitioned into three smaller lattice triangles.

   b. Prove indirectly that a lattice triangle with area 0.5 square units contains no lattice point. (Being on the boundary does not count as inside.)
**The Triangle Inequality**

If you take three straws of lengths 8 inches, 5 inches, and 1 inch and try to make a triangle with them, you will find that it is not possible. This illustrates the Triangle Inequality Theorem.

<table>
<thead>
<tr>
<th>Triangle Inequality Theorem</th>
<th>The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image" alt="Triangle Diagram" /></td>
</tr>
<tr>
<td>a + b &gt; c</td>
<td></td>
</tr>
<tr>
<td>b + c &gt; a</td>
<td></td>
</tr>
<tr>
<td>a + c &gt; b</td>
<td></td>
</tr>
</tbody>
</table>

**Example**

The measures of two sides of a triangle are 5 and 8. Find a range for the length of the third side.

By the Triangle Inequality Theorem, all three of the following inequalities must be true.

\[
5 + x > 8 \quad 8 + x > 5 \quad 5 + 8 > x \\
x > 3 \quad x > -3 \quad 13 > x
\]

Therefore \(x\) must be between 3 and 13.

**Exercises**

Is it possible to form a triangle with the given side lengths? If not, explain why not.

1. 3, 4, 6
2. 6, 9, 15
3. 8, 8, 8
4. 2, 4, 5
5. 4, 8, 16
6. 1.5, 2.5, 3

Find the range for the measure of the third side of a triangle given the measures of two sides.

7. 1 cm and 6 cm
8. 12 yd and 18 yd
9. 1.5 ft and 5.5 ft
10. 82 m and 8 m

11. Suppose you have three different positive numbers arranged in order from least to greatest. What single comparison will let you see if the numbers can be the lengths of the sides of a triangle?
Proofs Using The Triangle Inequality Theorem  You can use the Triangle Inequality Theorem as a reason in proofs.

Complete the following proof.

Given: \( \triangle ABC \cong \triangle DEC \)
Prove: \( AB + DE > AD - BE \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \triangle ABC \cong \triangle DEC )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( AB + BC &gt; AC )</td>
<td>2. Triangle Inequality Theorem</td>
</tr>
<tr>
<td>( DE + EC &gt; CD )</td>
<td></td>
</tr>
<tr>
<td>3. ( AB &gt; AC - BC )</td>
<td>3. Subtraction</td>
</tr>
<tr>
<td>( DE &gt; CD - EC )</td>
<td></td>
</tr>
<tr>
<td>4. ( AB + DE &gt; AC - BC + CD - EC )</td>
<td>4. Addition</td>
</tr>
<tr>
<td>5. ( AB + DE &gt; AC + CD - BC - EC )</td>
<td>5. Commutative</td>
</tr>
<tr>
<td>6. ( AB + DE &gt; AC + CD - (BC + EC) )</td>
<td>6. Distributive</td>
</tr>
<tr>
<td>7. ( AC + CD = AD )</td>
<td>7. Segment Addition Postulate</td>
</tr>
<tr>
<td>( BC + EC = BE )</td>
<td></td>
</tr>
<tr>
<td>8. ( AB + DE &gt; AD - BE )</td>
<td>8. Substitution</td>
</tr>
</tbody>
</table>

Exercises

PROOF  Write a two column proof.

Given: \( PL \parallel MT \)
\( K \) is the midpoint of \( PT \).
Prove: \( PK + KM > PL \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( PL \parallel MT )</td>
<td>1.</td>
</tr>
<tr>
<td>2. ( \angle P \cong \angle T )</td>
<td>2.</td>
</tr>
<tr>
<td>3. ( K ) is the midpoint of ( PT ).</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( PK = KT )</td>
<td>4.</td>
</tr>
<tr>
<td>5.</td>
<td>5. Vertical Angles Theorem</td>
</tr>
<tr>
<td>6. ( \triangle PKL \cong \triangle TKM )</td>
<td>6.</td>
</tr>
<tr>
<td>7.</td>
<td>7. Triangle Inequality Theorem</td>
</tr>
<tr>
<td>8.</td>
<td>8. CPCTC</td>
</tr>
<tr>
<td>9. ( PK + KM &gt; PL )</td>
<td>9.</td>
</tr>
</tbody>
</table>
Lesson 5-5

**The Triangle Inequality**

Is it possible to form a triangle with the given side lengths? If not explain why not.

1. 9, 12, 18
2. 8, 9, 17
3. 14, 14, 19
4. 23, 26, 50
5. 32, 41, 63
6. 2.7, 3.1, 4.3
7. 0.7, 1.4, 2.1
8. 12.3, 13.9, 25.2

Find the range for the measure of the third side of a triangle given the measures of two sides.

9. 6 ft and 19 ft
10. 7 km and 29 km
11. 13 in. and 27 in.
12. 18 ft and 23 ft
13. 25 yd and 38 yd
14. 31 cm and 39 cm
15. 42 m and 6 m
16. 54 in. and 7 in.

17. **Given:** $H$ is the centroid of $\triangle EDF$.

   **Prove:** $EY + FY > DE$

   **Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $H$ is the centroid of $\triangle EDF$.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\overline{EY}$ is a median.</td>
<td>2.</td>
</tr>
<tr>
<td>3.</td>
<td>3. Definition of median</td>
</tr>
<tr>
<td>4.</td>
<td>4. Definition of midpoint</td>
</tr>
<tr>
<td>5. $EY + DY &gt; DE$</td>
<td>5.</td>
</tr>
</tbody>
</table>

18. **GARDENING** Ha Poong has 4 lengths of wood from which he plans to make a border for a triangular-shaped herb garden. The lengths of the wood borders are 8 inches, 10 inches, 12 inches, and 18 inches. How many different triangular borders can Ha Poong make?
1. **STICKS** Jamila has 5 sticks of lengths 2, 4, 6, 8, and 10 inches. Using three sticks at a time as the sides of triangles, how many triangles can she make?

Use the figure at the right for Exercises 2 and 3.

2. **PATHS** To get to the nearest supermarket, Tanya must walk over a railroad track. There are two places where she can cross the track (points A and B). Which path is longer? Explain.

3. **PATHS** While out walking one day Tanya finds a third place to cross the railroad tracks. Show that the path through point C is longer than the path through point B.

4. **CITIES** The distance between New York City and Boston is 187 miles and the distance between New York City and Hartford is 97 miles. Hartford, Boston, and New York City form a triangle on a map. What must the distance between Boston and Hartford be greater than?

5. **TRIANGLES** The figure shows an equilateral triangle ABC and a point P outside the triangle.

   a. Draw the figure that is the result of turning the original figure 60° counterclockwise about A. Denote by $P'$, the image of $P$ under this turn.

   b. Note that $PB$ is congruent to $PC$. It is also true that $PP'$ is congruent to $PA$. Why?

   c. Show that $PA$, $PB$, and $PC$ satisfy the triangle inequalities.
Hinge Theorem  The following theorem and its converse involve the relationship between the sides of two triangles and an angle in each triangle.

<table>
<thead>
<tr>
<th>Hinge Theorem</th>
<th>Converse of the Hinge Theorem</th>
</tr>
</thead>
<tbody>
<tr>
<td>If two sides of a triangle are congruent to two sides of another triangle and the included angle of the first is larger than the included angle of the second, then the third side of the first triangle is longer than the third side of the second triangle.</td>
<td>If two sides of a triangle are congruent to two sides of another triangle, and the third side in the first is longer than the third side in the second, then the included angle in the first triangle is greater than the included angle in the second triangle.</td>
</tr>
</tbody>
</table>

Example 1  Compare the measures of $GF$ and $FE$.

Two sides of $\triangle HGF$ are congruent to two sides of $\triangle HEF$, and $m\angle GHF > m\angle EHF$. By the Hinge Theorem, $GF > FE$.

Example 2  Compare the measures of $\angle ABD$ and $\angle CBD$.

Two sides of $\triangle ABD$ are congruent to two sides of $\triangle CBD$, and $AD > CD$. By the Converse of the Hinge Theorem, $m\angle ABD > m\angle CBD$.

Exercises

Compare the given measures.

1. $MR$ and $RP$

2. $AD$ and $CD$

3. $m\angle C$ and $m\angle Z$

4. $m\angle XYW$ and $m\angle WYZ$

Write an inequality for the range of values of $x$.

5. $(4x - 10)$

6. $(3x - 3)$
Inequalities Involving Two Triangles

PROVE RELATIONSHIPS IN TWO TRIANGLES  You can use the Hinge Theorem and its converse to prove relationships in two triangles.

Example

Given: \( RX = XS \)
\( m\angle SXT = 97 \)
Prove: \( ST > RT \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle SXT ) and ( \angle RXT ) are supplementary.</td>
<td>1. Def. of linear pair.</td>
</tr>
<tr>
<td>2. ( m \angle SXT + m \angle RXT = 180 )</td>
<td>2. Def. of supplementary.</td>
</tr>
<tr>
<td>3. ( m \angle SXT = 97 )</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( 97 + m \angle RXT = 180 )</td>
<td>4. Substitution</td>
</tr>
<tr>
<td>5. ( m \angle RXT = 83 )</td>
<td>5. Subtraction</td>
</tr>
<tr>
<td>6. ( 97 &gt; 83 )</td>
<td>6. Inequality</td>
</tr>
<tr>
<td>7. ( m \angle SXT &gt; m \angle RXT )</td>
<td>7. Substitution</td>
</tr>
<tr>
<td>8. ( RX = XS )</td>
<td>8. Given</td>
</tr>
<tr>
<td>9. ( TX = TX )</td>
<td>9. Reflexive Property</td>
</tr>
<tr>
<td>10. ( ST &gt; RT )</td>
<td>10. Hinge Theorem</td>
</tr>
</tbody>
</table>

Exercises

Complete the proof.

Given: rectangle \( AFBC \)
\( ED = DC \)
Prove: \( AE > FB \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. rectangle ( AFBC, ED = DC )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( AD = AD )</td>
<td>2. Reflexive Property</td>
</tr>
<tr>
<td>3. ( m\angle EDA &gt; m\angle ADC )</td>
<td>3. Exterior Angle Inequality</td>
</tr>
<tr>
<td>4.</td>
<td>4. Hinge Theorem</td>
</tr>
<tr>
<td>5.</td>
<td>5. Opp sides in rectangle are ( \equiv )</td>
</tr>
<tr>
<td>6. ( AE &gt; FB )</td>
<td>6. Substitution</td>
</tr>
</tbody>
</table>
5-6 Practice

Inequalities in Two Triangles

Compare the given measures.

1. $AB$ and $BK$

2. $ST$ and $SR$

3. $m\angle CDF$ and $m\angle EDF$

4. $m\angle R$ and $m\angle T$

5. PROOF Write a two-column proof.

Given: $G$ is the midpoint of $DF$.

$m\angle 1 > m\angle 2$

Prove: $ED > EF$

6. TOOLS Rebecca used a spring clamp to hold together a chair leg she repaired with wood glue. When she opened the clamp, she noticed that the angle between the handles of the clamp decreased as the distance between the handles of the clamp decreased. At the same time, the distance between the gripping ends of the clamp increased. When she released the handles, the distance between the gripping end of the clamp decreased and the distance between the handles increased. Is the clamp an example of the Hinge Theorem or its converse?
1. **CLOCKS** The minute hand of a grandfather clock is 3 feet long and the hour hand is 2 feet long. Is the distance between their ends greater at 3:00 or at 8:00?

2. **FERRIS WHEEL** A Ferris wheel has carriages located at the 10 vertices of a regular decagon.

   Which carriages are farther away from carriage number 1 than carriage number 4?

3. **WALKWAY** Tyree wants to make two slightly different triangles for his walkway. He has three pieces of wood to construct the frame of his triangles. After Tyree makes the first concrete triangle, he adjusts two sides of the triangle so that the angle they create is smaller than the angle in the first triangle. Explain how this changes the triangle.

4. **MOUNTAIN PEAKS** Emily lives the same distance from three mountain peaks: High Point, Topper, and Cloud Nine. For a photography class, Emily must take a photograph from her house that shows two of the mountain peaks. Which two peaks would she have the best chance of capturing in one image?

5. **RUNNERS** A photographer is taking pictures of three track stars, Amy, Noel, and Beth. The photographer stands on a track, which is shaped like a rectangle with semicircles on both ends.

   a. Based on the information in the figure, list the runners in order from nearest to farthest from the photographer.

   b. Explain how to locate the point along the semicircular curve that the runners are on that is farthest away from the photographer.
**Parallelograms**

A quadrilateral with both pairs of opposite sides parallel is a **parallelogram**. Here are four important properties of parallelograms.

- If a quadrilateral is a parallelogram, then its opposite sides are congruent.
  
  \[ \overline{PQ} \cong \overline{SR} \text{ and } \overline{PS} \cong \overline{QR} \]

- If a quadrilateral is a parallelogram, then its opposite angles are congruent.
  
  \[ \angle P \cong \angle R \text{ and } \angle S \cong \angle Q \]

- If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.
  
  \[ \angle P \text{ and } \angle S \text{ are supplementary; } \angle S \text{ and } \angle R \text{ are supplementary; } \angle R \text{ and } \angle Q \text{ are supplementary; } \angle Q \text{ and } \angle P \text{ are supplementary.} \]

- If a parallelogram has one right angle, then it has four right angles.
  
  \[ \text{If } m \angle P = 90, \text{ then } m \angle Q = 90, m \angle R = 90, \text{ and } m \angle S = 90. \]

**Example**

If a quadrilateral is a parallelogram, find the value of each variable.

\[ AB \text{ and } CD \text{ are opposite sides, so } \overline{AB} \cong \overline{CD}. \]

1. \[ 2a = 34 \]
   
   \[ a = 17 \]

2. \[ \angle A \text{ and } \angle C \text{ are opposite angles, so } \angle A \cong \angle C. \]
   
   \[ 8b = 112 \]
   
   \[ b = 14 \]

**Exercises**

Find the value of each variable.

1. \[ \begin{array}{c} 3x^\circ \\ 4y^\circ \end{array} \]

2. \[ \begin{array}{c} 8y \\ 6x^\circ \\ 88 \end{array} \]

3. \[ \begin{array}{c} 6x \\ 3y \\ 12 \end{array} \]

4. \[ \begin{array}{c} 3y^\circ \\ 6x^\circ \\ 12x^\circ \end{array} \]

5. \[ \begin{array}{c} 55^\circ \\ 5x^\circ \\ 60^\circ \\ 2y^\circ \end{array} \]

6. \[ \begin{array}{c} 2y \\ 30x \\ 72x \\ 150 \end{array} \]
Parallelograms

Two important properties of parallelograms deal with their diagonals.

<table>
<thead>
<tr>
<th>If a quadrilateral is a parallelogram, then</th>
<th>If $ABCD$ is a parallelogram, then</th>
</tr>
</thead>
<tbody>
<tr>
<td>its diagonals bisect each other.</td>
<td>$AP = PC$ and $DP = PB$</td>
</tr>
<tr>
<td>each diagonal separates the parallelogram</td>
<td>$\triangle ACD \cong \triangle CAB$ and $\triangle ADB \cong \triangle CBD$</td>
</tr>
<tr>
<td>into two congruent triangles.</td>
<td></td>
</tr>
</tbody>
</table>

**Example**

Find the value of $x$ and $y$ in parallelogram $ABCD$.

The diagonals bisect each other, so $AE = CE$ and $DE = BE$.

$6x = 24 \quad 4y = 18$

$x = 4 \quad y = 4.5$

**Exercises**

Find the value of each variable.

1. $3x \quad 4y \quad 8 \quad 12$

2. $28 \quad 2y \quad 4x$

3. $20 \quad x \quad 2y \quad 4y$

4. $30^\circ \quad 10 \quad 3x$

5. $12 \quad 3x \quad 2y$

6. $x \quad 4 \quad y \quad 17$

**COORDINATE GEOMETRY**

Find the coordinates of the intersection of the diagonals of $\square ABCD$ with the given vertices.

7. $A(3, 6), B(5, 8), C(3, -2),$ and $D(1, -4)$

8. $A(-4, 3), B(2, 3), C(-1, -2),$ and $D(-7, -2)$

9. **PROOF** Write a paragraph proof of the following.

   **Given:** $\square ABCD$

   **Prove:** $\triangle AED \cong \triangle BEC$
Parallelograms

ALGEBRA Find the value of each variable.

1. \[
\begin{align*}
x & = 3a - 4 \\
y & = b + 1 \\
z & = 2b \\
w & = a + 2
\end{align*}
\]

2. \[
\begin{align*}
x & = (2y - 40)^\circ \\
y & = (y + 10)^\circ \\
z & = (4x)^\circ \\
w & = (4y - 80)^\circ
\end{align*}
\]

3. \[
\begin{align*}
x & = y + 3 \\
y & = 15 \\
z & = 12
\end{align*}
\]

4. \[
\begin{align*}
x & = 3y + 1 \\
y & = 5y - 8 \\
z & = 4y + 2
\end{align*}
\]

ALGEBRA Use \(\text{RSTU}\) to find each measure or value.

5. \(m\angle RST = \ldots\)

6. \(m\angle STU = \ldots\)

7. \(m\angle TUR = \ldots\)

8. \(b = \ldots\)

COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of \(\text{PRYZ}\) with the given vertices.

9. \(P(2, 5), R(3, 3), Y(-2, -3), Z(-3, -1)\)

10. \(P(2, 3), R(1, -2), Y(-5, -7), Z(-4, -2)\)

11. PROOF Write a paragraph proof of the following.

   Given: \(\text{PRST}\) and \(\text{PQVU}\)
   Prove: \(\angle V \cong \angle S\)

12. CONSTRUCTION Mr. Rodriquez used the parallelogram at the right to design a herringbone pattern for a paving stone. He will use the paving stone for a sidewalk. If \(m\angle 1 = 130\), find \(m\angle 2, m\angle 3\), and \(m\angle 4\).
1. **WALKWAY** A walkway is made by adjoining four parallelograms as shown.

Are the end segments \( a \) and \( e \) parallel to each other? Explain.

2. **DISTANCE** Four friends live at the four corners of a block shaped like a parallelogram. Gracie lives 3 miles away from Kenny. How far apart do Teresa and Travis live from each other?

3. **SOCCER** Four soccer players are located at the corners of a parallelogram. Two of the players in opposite corners are the goalies. In order for goalie A to be able to see the three others, she must be able to see a certain angle \( x \) in her field of vision.

What angle does the other goalie have to be able to see in order to keep an eye on the other three players?

4. **VENN DIAGRAMS** Make a Venn diagram showing the relationship between squares, rectangles, and parallelograms.

5. **SKYSCRAPERS** On vacation, Tony’s family took a helicopter tour of the city. The pilot said the newest building in the city was the building with this top view. He told Tony that the exterior angle by the front entrance is 72°. Tony wanted to know more about the building, so he drew this diagram and used his geometry skills to learn a few more things. The front entrance is next to vertex \( B \).
   
   a. What are the measures of the four angles of the parallelogram?
   
   b. How many pairs of congruent triangles are there in the figure? What are they?
**6-3 Study Guide**

**Tests for Parallelograms**

**Conditions for Parallelograms** There are many ways to establish that a quadrilateral is a parallelogram.

<table>
<thead>
<tr>
<th>If:</th>
<th>If:</th>
</tr>
</thead>
<tbody>
<tr>
<td>both pairs of opposite sides are parallel,</td>
<td>$\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$,</td>
</tr>
<tr>
<td>both pairs of opposite sides are congruent,</td>
<td>$\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$,</td>
</tr>
<tr>
<td>both pairs of opposite angles are congruent,</td>
<td>$\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$,</td>
</tr>
<tr>
<td>the diagonals bisect each other,</td>
<td>$\overline{AB} \parallel \overline{CD}$ and $\overline{AB} \cong \overline{CD}$, or $\overline{AD} \parallel \overline{BC}$ and $\overline{AD} \cong \overline{BC}$,</td>
</tr>
<tr>
<td>one pair of opposite sides is congruent and parallel,</td>
<td>then: $\overline{ABCD}$ is a parallelogram.</td>
</tr>
</tbody>
</table>

**Example** Find $x$ and $y$ so that $FGHJ$ is a parallelogram.

$FGHJ$ is a parallelogram if the lengths of the opposite sides are equal.

$6x + 3 = 15 \quad 4x - 2y = 2$

$6x = 12 \quad 4(2) - 2y = 2$

$x = 2 \quad 8 - 2y = 2$

$-2y = -6$

$y = 3$

**Exercises**

Find $x$ and $y$ so that the quadrilateral is a parallelogram.

1. \[ \begin{array}{c}
 2x - 2 \\
 2y \\
 8 \\
 12 \\
\end{array} \]

2. \[ \begin{array}{c}
 5y^\circ \\
 11x^\circ \\
 55^\circ \\
 \end{array} \]

3. \[ \begin{array}{c}
 5y^\circ \\
 5x^\circ \\
 25^\circ \\
\end{array} \]

4. \[ \begin{array}{c}
 18 \\
 45^\circ \\
 9x^\circ \\
 6y \\
\end{array} \]

5. \[ \begin{array}{c}
 (x + y)^\circ \\
 2x^\circ \\
 30^\circ \\
 24^\circ \\
\end{array} \]

6. \[ \begin{array}{c}
 6y^\circ \\
 3x^\circ \\
\end{array} \]

---

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Tests for Parallelograms

Parallelograms on the Coordinate Plane  On the coordinate plane, the Distance, Slope, and Midpoint Formulas can be used to test if a quadrilateral is a parallelogram.

Example Determine whether ABCD is a parallelogram.

The vertices are A(−2, 3), B(3, 2), C(2, −1), and D(−3, 0).

**Method 1:** Use the Slope Formula, \( m = \frac{y_2 - y_1}{x_2 - x_1} \).

\[
\text{slope of } \overline{AD} = \frac{3 - 0}{-2 - (-3)} = \frac{3}{1} = 3 \\
\text{slope of } \overline{BC} = \frac{2 - (-1)}{3 - 2} = \frac{3}{1} = 3
\]

Since opposite sides have the same slope, \( \overline{AB} || \overline{CD} \) and \( \overline{AD} || \overline{BC} \). Therefore, \( ABCD \) is a parallelogram by definition.

**Method 2:** Use the Distance Formula, \( d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \).

\[
\begin{align*}
\text{AB} &= \sqrt{(-2 - 3)^2 + (3 - 2)^2} = \sqrt{25 + 1} \text{ or } \sqrt{26} \\
\text{CD} &= \sqrt{(2 - (-3))^2 + (1 - 0)^2} = \sqrt{25 + 1} \text{ or } \sqrt{26} \\
\text{AD} &= \sqrt{(-2 - (-3))^2 + (3 - 0)^2} = \sqrt{1 + 9} \text{ or } \sqrt{10} \\
\text{BC} &= \sqrt{(3 - 2)^2 + (2 - (-1))^2} = \sqrt{1 + 9} \text{ or } \sqrt{10}
\end{align*}
\]

Since both pairs of opposite sides have the same length, \( \overline{AB} \cong \overline{CD} \) and \( \overline{AD} \cong \overline{BC} \). Therefore, \( ABCD \) is a parallelogram by Theorem 6.9.

Exercises

Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

1. A(0, 0), B(1, 3), C(5, 3), D(4, 0);   Slope Formula

2. D(−1, 1), E(2, 4), F(6, 4), G(3, 1);  Slope Formula

3. R(−1, 0), S(3, 0), T(2, −3), U(−3, −2);   Distance Formula

4. A(−3, 2), B(−1, 4), C(2, 1), D(0, −1);   Distance and Slope Formulas

5. S(−2, 4), T(−1, −1), U(3, −4), V(2, 1);   Distance and Slope Formulas

6. F(3, 3), G(1, 2), H(−3, 1), I(−1, 4);   Midpoint Formula

7. A parallelogram has vertices R(−2, −1), S(2, 1), and T(0, −3). Find all possible coordinates for the fourth vertex.
**6-3 Practice**

**Tests for Parallelograms**

Determine whether each quadrilateral is a parallelogram. Justify your answer.

1.  

2.  

3.  

4.  

**COORDINATE GEOMETRY** Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

5.  $P(-5, 1), S(-2, 2), F(-1, -3), T(2, -2)$; Slope Formula

6.  $R(-2, 5), O(1, 3), M(-3, -4), Y(-6, -2)$; Distance and Slope Formulas

**ALGEBRA** Find $x$ and $y$ so that the quadrilateral is a parallelogram.

7.  

8.  

9.  

10.  

11. **TILE DESIGN** The pattern shown in the figure is to consist of congruent parallelograms. How can the designer be certain that the shapes are parallelograms?
1. **BALANCING** Nikia, Madison, Angela, and Shelby are balancing themselves on an “X”-shaped floating object. To balance themselves, they want to make themselves the vertices of a parallelogram.

   In order to achieve this, do all four of them have to be the same distance from the center of the object? Explain.

2. **COMPASSES** Two compass needles placed side by side on a table are both 2 inches long and point due north. Do they form the sides of a parallelogram?

3. **FORMATION** Four jets are flying in formation. Three of the jets are shown in the graph. If the four jets are located at the vertices of a parallelogram, what are the three possible locations of the missing jet?

4. **STREET LAMPS** When a coordinate plane is placed over the Harrisville town map, the four street lamps in the center are located as shown. Do the four lamps form the vertices of a parallelogram? Explain.

5. **PICTURE FRAME** Aaron is making a wooden picture frame in the shape of a parallelogram. He has two pieces of wood that are 3 feet long and two that are 4 feet long.

   a. If he connects the pieces of wood at their ends to each other, in what order must he connect them to make a parallelogram?

   b. How many different parallelograms could he make with these four lengths of wood?

   c. Explain something Aaron might do to specify precisely the shape of the parallelogram.
Properties of Rectangles  A rectangle is a quadrilateral with four right angles. Here are the properties of rectangles.

A rectangle has all the properties of a parallelogram.
- Opposite sides are parallel.
- Opposite angles are congruent.
- Opposite sides are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.

Also:
- All four angles are right angles. \( \angle UTS, \angle TSR, \angle SRU, \text{ and } \angle RUT \) are right angles.
- The diagonals are congruent. \( TR \cong US \)

Example 1  Quadrilateral \( RUTS \) above is a rectangle. If \( US = 6x + 3 \) and \( RT = 7x - 2 \), find \( x \).

The diagonals of a rectangle are congruent, so \( US = RT \).

\[
6x + 3 = 7x - 2 \\
3 = x - 2 \\
5 = x
\]

Example 2  Quadrilateral \( RUTS \) above is a rectangle. If \( m\angle STR = 8x + 3 \) and \( m\angle UTR = 16x - 9 \), find \( m\angle STR \).

\( \angle UTS \) is a right angle, so \( m\angle STR + m\angle UTR = 90 \).

\[
8x + 3 + 16x - 9 = 90 \\
24x - 6 = 90 \\
24x = 96 \\
x = 4
\]

\[ m\angle STR = 8x + 3 = 8(4) + 3 = 35 \]

Exercises

Quadrilateral \( ABCD \) is a rectangle.

1. If \( AE = 36 \) and \( CE = 2x - 4 \), find \( x \).

2. If \( BE = 6y + 2 \) and \( CE = 4y + 6 \), find \( y \).

3. If \( BC = 24 \) and \( AD = 5y - 1 \), find \( y \).

4. If \( m\angle BEA = 62 \), find \( m\angle BAC \).

5. If \( m\angle AED = 12x \) and \( m\angle BEC = 10x + 20 \), find \( m\angle AED \).

6. If \( BD = 8y - 4 \) and \( AC = 7y + 3 \), find \( BD \).

7. If \( m\angle DBC = 10x \) and \( m\angle ACB = 4x^2 - 6 \), find \( m\angle ACB \).

8. If \( AB = 6y \) and \( BC = 8y \), find \( BD \) in terms of \( y \).
Rectangles

Prove that Parallelograms Are Rectangles The diagonals of a rectangle are congruent, and the converse is also true.

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

In the coordinate plane you can use the Distance Formula, the Slope Formula, and properties of diagonals to show that a figure is a rectangle.

Example Quadrilateral $ABCD$ has vertices $A(-3, 0), B(-2, 3), C(4, 1),$ and $D(3, -2)$. Determine whether $ABCD$ is a rectangle.

**Method 1:** Use the Slope Formula.

$$\text{slope of } AB = \frac{3 - 0}{-2 - (-3)} = 1$$

$$\text{slope of } AD = \frac{-2 - 0}{3 - (-3)} = \frac{-2}{6}$$

$$\text{slope of } CD = \frac{-2 - 1}{3 - 4} = -3$$

$$\text{slope of } BC = \frac{1 - 3}{4 - (-2)} = \frac{2}{6}$$

Opposite sides are parallel, so the figure is a parallelogram. Consecutive sides are perpendicular, so $ABCD$ is a rectangle.

**Method 2:** Use the Distance Formula.

$$AB = \sqrt{(-3 - (-2))^2 + (0 - 3)^2} = \sqrt{10}$$

$$BC = \sqrt{(-2 - 4)^2 + (3 - 1)^2} = \sqrt{40}$$

$$CD = \sqrt{(4 - 3)^2 + (1 - (-2))^2} = \sqrt{10}$$

$$AD = \sqrt{(-3 - 3)^2 + (0 - (-2))^2} = \sqrt{40}$$

Opposite sides are congruent, thus $ABCD$ is a parallelogram.

$$AC = \sqrt{(-3 - 4)^2 + (0 - 1)^2} = \sqrt{50}$$

$$BD = \sqrt{(-2 - 3)^2 + (3 - (-2))^2} = \sqrt{50}$$

$ABCD$ is a parallelogram with congruent diagonals, so $ABCD$ is a rectangle.

Exercises

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

1. $A(-3, 1), B(-3, 3), C(3, 3), D(3, 1)$; Distance Formula

2. $A(-3, 0), B(-2, 3), C(4, 5), D(3, 2)$; Slope Formula

3. $A(-3, 0), B(-2, 2), C(3, 0), D(2, -2)$; Distance Formula

4. $A(-1, 0), B(0, 2), C(4, 0), D(3, -2)$; Distance Formula
6-4 Practice

Rectangles

ALGEBRA Quadrilateral RSTU is a rectangle.

1. If $UZ = x + 21$ and $ZS = 3x - 15$, find $US$.

2. If $RZ = 3x + 8$ and $ZS = 6x - 28$, find $UZ$.

3. If $RT = 5x + 8$ and $RZ = 4x + 1$, find $ZT$.

4. If $m\angle SUT = 3x + 6$ and $m\angle RUS = 5x - 4$, find $m\angle SUT$.

5. If $m\angle SRT = x + 9$ and $m\angle UTR = 2x - 44$, find $m\angle UTR$.

6. If $m\angle RSU = x + 41$ and $m\angle TUS = 3x + 9$, find $m\angle RSU$.

Quadrilateral GHJK is a rectangle. Find each measure if $m\angle 1 = 37$.

7. $m\angle 2$

8. $m\angle 3$

9. $m\angle 4$

10. $m\angle 5$

11. $m\angle 6$

12. $m\angle 7$

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

13. $B(-4, 3), G(-2, 4), H(1, -2), L(-1, -3)$; Slope Formula

14. $N(-4, 5), O(6, 0), P(3, -6), Q(-7, -1)$; Distance Formula

15. $C(0, 5), D(4, 7), E(5, 4), F(1, 2)$; Slope Formula

16. LANDSCAPING Huntington Park officials approved a rectangular plot of land for a Japanese Zen garden. Is it sufficient to know that opposite sides of the garden plot are congruent and parallel to determine that the garden plot is rectangular? Explain.
6-4 Word Problem Practice

Rectangles

1. **FRAMES** Jalen makes the rectangular frame shown.

   ![](frame.png)

   In order to make sure that it is a rectangle, Jalen measures the distances \(BD\) and \(AC\). How should these two distances compare if the frame is a rectangle?

2. **BOOKSHELVES** A bookshelf consists of two vertical planks with five horizontal shelves. Are each of the four sections for books rectangles? Explain.

3. **LANDSCAPING** A landscaper is marking off the corners of a rectangular plot of land. Three of the corners are in place as shown.

   ![](landscape.png)

   What are the coordinates of the fourth corner?

4. **SWIMMING POOLS** Antonio is designing a swimming pool on a coordinate grid. Is it a rectangle? Explain.

   ![](swimming_pool.png)

5. **PATTERNS** Veronica made the pattern shown out of 7 rectangles with four equal sides. The side length of each rectangle is written inside the rectangle.

   ![](patterns.png)

   a. How many rectangles can be formed using the lines in this figure?

   b. If Veronica wanted to extend her pattern by adding another rectangle with 4 equal sides to make a larger rectangle, what are the possible side lengths of rectangles that she can add?
Rhombi and Squares

Properties of Rhombi and Squares A rhombus is a quadrilateral with four congruent sides. Opposite sides are congruent, so a rhombus is also a parallelogram and has all of the properties of a parallelogram. Rhombi also have the following properties.

<table>
<thead>
<tr>
<th>The diagonals are perpendicular.</th>
<th>MH ⊥ RO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Each diagonal bisects a pair of opposite angles.</td>
<td>MH bisects ∠RMO and ∠RHO. RO bisects ∠MRH and ∠MOH.</td>
</tr>
</tbody>
</table>

A square is a parallelogram with four congruent sides and four congruent angles. A square is both a rectangle and a rhombus; therefore, all properties of parallelograms, rectangles, and rhombi apply to squares.

Example In rhombus ABCD, m∠BAC = 32. Find the measure of each numbered angle.

ABCD is a rhombus, so the diagonals are perpendicular and △ABE is a right triangle. Thus m∠4 = 90 and m∠1 = 90 − 32 or 58. The diagonals in a rhombus bisect the vertex angles, so m∠1 = m∠2. Thus, m∠2 = 58.

A rhombus is a parallelogram, so the opposite sides are parallel. ∠BAC and ∠3 are alternate interior angles for parallel lines, so m∠3 = 32.

Exercises Quadrilateral ABCD is a rhombus. Find each value or measure.

1. If m∠ABD = 60, find m∠BDC.
2. If AE = 8, find AC.
3. If AB = 26 and BD = 20, find AE.
4. Find m∠CEB.
5. If m∠CBD = 58, find m∠ACB.
6. If AE = 3x − 1 and AC = 16, find x.
7. If m∠CDB = 6y and m∠ACB = 2y + 10, find y.
8. If AD = 2x + 4 and CD = 4x − 4, find x.
Rhombi and Squares

Conditions for Rhombi and Squares  The theorems below can help you prove that a parallelogram is a rectangle, rhombus, or square.

- If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.
- If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus.
- If one pair of consecutive sides of a parallelogram are congruent, the parallelogram is a rhombus.
- If a quadrilateral is both a rectangle and a rhombus, then it is a square.

Determine whether parallelogram \(ABCD\) with vertices \(A(-3, -3), B(1, 1), C(5, -3), D(1, -7)\) is a rhombus, a rectangle, or a square.

\[AC = \sqrt{(-3 - 5)^2 + ((-3 - (-3))^2} = \sqrt{64} = 8\]
\[BD = \sqrt{(1-1)^2 + (-7 - 1)^2} = \sqrt{64} = 8\]

The diagonals are the same length; the figure is a rectangle.

Slope of \(\overline{AC}\) = \(-\frac{3 - (-3)}{-3 - 5} = \frac{0}{-8} = 0\) The line is horizontal.

Slope of \(\overline{BD}\) = \(\frac{1 - (-7)}{-1 - 1} = \frac{8}{0} = undefined\) The line is vertical.

Since a horizontal and vertical line are perpendicular, the diagonals are perpendicular. Parallelogram \(ABCD\) is a square which is also a rhombus and a rectangle.

Exercises

Given each set of vertices, determine whether \(\square ABCD\) is a rhombus, rectangle, or square. List all that apply. Explain.

1. \(A(0, 2), B(2, 4), C(4, 2), D(2, 0)\)
2. \(A(-2, 1), B(-1, 3), C(3, 1), D(2, -1)\)
3. \(A(-2, -1), B(0, 2), C(2, -1), D(0, -4)\)
4. \(A(-3, 0), B(-1, 3), C(5, -1), D(3, -4)\)

5. **PROOF** Write a two-column proof.
   
   **Given:** Parallelogram \(RSTU\). \(RS \cong ST\)
   
   **Prove:** \(RSTU\) is a rhombus.
Rhombi and Squares

PRYZ is a rhombus. If RK = 5, RY = 13 and m∠YRZ = 67, find each measure.

1. KY
2. PK
3. m∠YKZ
4. m∠PZR

MNPQ is a rhombus. If PQ = 3√2 and AP = 3, find each measure.

5. AQ
6. m∠APQ
7. m∠MNP
8. PM

COORDINATE GEOMETRY Given each set of vertices, determine whether □BEFG is a rhombus, a rectangle, or a square. List all that apply. Explain.

9. B(-9, 1), E(2, 3), F(12, -2), G(1, -4)

10. B(1, 3), E(7, -3), F(1, -9), G(-5, -3)

11. B(-4, -5), E(1, -5), F(-2, -1), G(-7, -1)

12. TESSELLATIONS The figure is an example of a tessellation. Use a ruler or protractor to measure the shapes and then name the quadrilaterals used to form the figure.
1. **TRAY RACKS** A tray rack looks like a parallelogram from the side. The levels for the trays are evenly spaced.

What two labeled points form a rhombus with base \( AA' \)?

2. **SLICING** Charles cuts a rhombus along both diagonals. He ends up with four congruent triangles. Classify these triangles as acute, obtuse, or right.

3. **WINDOWS** The edges of a window are drawn in the coordinate plane.

Determine whether the window is a square or a rhombus.

4. **SQUARES** Mackenzie cut a square along its diagonals to get four congruent right triangles. She then joined two of them along their long sides. Show that the resulting shape is a square.

5. **DESIGN** Tatianna made the design shown. She used 32 congruent rhombi to create the flower-like design at each corner.

   a. What are the angles of the corner rhombi?

   b. What kinds of quadrilaterals are the dotted and checkered figures?
**6-6 Study Guide**

**Trapezoids and Kites**

**Properties of Trapezoids** A trapezoid is a quadrilateral with exactly one pair of parallel sides. The midsegment or median of a trapezoid is the segment that connects the midpoints of the legs of the trapezoid. Its measure is equal to one-half the sum of the lengths of the bases. If the legs are congruent, the trapezoid is an isosceles trapezoid. In an isosceles trapezoid both pairs of base angles are congruent and the diagonals are congruent.

**Example** The vertices of \(ABCD\) are \(A(-3, -1), B(-1, 3), C(2, 3),\) and \(D(4, -1)\). Show that \(ABCD\) is a trapezoid and determine whether it is an isosceles trapezoid.

\[
\text{slope of } \overline{AB} = \frac{3 - (-1)}{-1 - (-3)} = \frac{4}{2} = 2
\]

\[
\text{slope of } \overline{AD} = \frac{-1 - (-1)}{4 - (-3)} = \frac{0}{7} = 0
\]

\[
\text{slope of } \overline{BC} = \frac{2 - (-1)}{3 - 3} = \frac{3}{0} = 0
\]

\[
\text{slope of } \overline{CD} = \frac{-1 - 3}{4 - 2} = \frac{-4}{2} = -2
\]

Exactly two sides are parallel, \(\overline{AD}\) and \(\overline{BC}\), so \(ABCD\) is a trapezoid. \(AB = CD\), so \(ABCD\) is an isosceles trapezoid.

**Exercises**

Find each measure.

1. \(m\angle D\)

\[
\begin{align*}
\angle D &= 125^\circ \\
\end{align*}
\]

2. \(m\angle L\)

\[
\begin{align*}
\angle L &= 40^\circ \\
\end{align*}
\]

**COORDINATE GEOMETRY** For each quadrilateral with the given vertices, verify that the quadrilateral is a trapezoid and determine whether the figure is an isosceles trapezoid.

3. \(A(-1, 1), B(3, 2), C(1, -2), D(-2, -1)\)

4. \(J(1, 3), K(3, 1), L(3, -2), M(-2, 3)\)

For trapezoid \(HJKL\), \(M\) and \(N\) are the midpoints of the legs.

5. If \(HJ = 32\) and \(LK = 60\), find \(MN\).

6. If \(HJ = 18\) and \(MN = 28\), find \(LK\).
Properties of Kites  A kite is a quadrilateral with exactly two pairs of consecutive congruent sides. Unlike a parallelogram, the opposite sides of a kite are not congruent or parallel.

The diagonals of a kite are perpendicular.
For kite $RMNP$, $MP \perp RN$.

In a kite, exactly one pair of opposite angles is congruent.
For kite $RMNP$, $\angle M \cong \angle P$.

**Example 1**  If $WXYZ$ is a kite, find $m \angle Z$.

The measures of $\angle Y$ and $\angle W$ are not congruent, so $\angle X \cong \angle Z$.

$m \angle X + m \angle Y + m \angle Z + m \angle W = 360$
$m \angle X + 60 + m \angle Z + 80 = 360$
$m \angle X + m \angle Z = 220$
$m \angle X = 110, m \angle Z = 110$

**Example 2**  If $ABCD$ is a kite, find $BC$.

The diagonals of a kite are perpendicular. Use the Pythagorean Theorem to find the missing length.

$BP^2 + PC^2 = BC^2$
$5^2 + 12^2 = BC^2$
$169 = BC^2$
$13 = BC$

**Exercises**

If $GHJK$ is a kite, find each measure.

1. Find $m \angle JRK$.

2. If $RJ = 3$ and $RK = 10$, find $JK$.

3. If $m \angle GHJ = 90$ and $m \angle GJK = 110$, find $m \angle HGK$.

4. If $HJ = 7$, find $HG$.

5. If $HG = 7$ and $GR = 5$, find $HR$.

6. If $m \angle GHJ = 52$ and $m \angle GJK = 95$, find $m \angle HGK$. 
Practice

**Trapezoids and Kites**

Find each measure.

1. \( m \angle T \)

2. \( m \angle Y \)

3. \( m \angle Q \)

4. \( BC \)

**ALGEBRA** For trapezoid \( FECD \), \( V \) and \( Y \) are midpoints of the legs.

5. If \( FE = 18 \) and \( VY = 28 \), find \( CD \).
6. If \( m \angle F = 140 \) and \( m \angle E = 125 \), find \( m \angle D \).

**COORDINATE GEOMETRY** \( RSTU \) is a quadrilateral with vertices \( R(-3, -3), S(5, 1), T(10, -2), U(-4, -9) \).

7. Verify that \( RSTU \) is a trapezoid.
8. Determine whether \( RSTU \) is an isosceles trapezoid. Explain.

**CONSTRUCTION** A set of stairs leading to the entrance of a building is designed in the shape of an isosceles trapezoid with the longer base at the bottom of the stairs and the shorter base at the top. If the bottom of the stairs is 21 feet wide and the top is 14 feet wide, find the width of the stairs halfway to the top.

**DESK TOPS** A carpenter needs to replace several trapezoid-shaped desktops in a classroom. The carpenter knows the lengths of both bases of the desktop. What other measurements, if any, does the carpenter need?
Word Problem Practice

Trapezoids and Kites

1. **PERSPECTIVE** Artists use different techniques to make things appear to be 3-dimensional when drawing in two dimensions. Kevin drew the walls of a room. In real life, all of the walls are rectangles. In what shape did he draw the side walls to make them appear 3-dimensional?

2. **PLAZA** In order to give the feeling of spaciousness, an architect decides to make a plaza in the shape of a kite. Three of the four corners of the plaza are shown on the coordinate plane. If the fourth corner is in the first quadrant, what are its coordinates?

3. **AIRPORTS** A simplified drawing of the reef runway complex at Honolulu International Airport is shown below.

   ![Diagram of a reef runway complex]

   How many trapezoids are there in this image?

4. **LIGHTING** A light outside a room shines through the door and illuminates a trapezoidal region $ABCD$ on the floor.

   ![Diagram of a trapezoid]

   Under what circumstances would trapezoid $ABCD$ be isosceles?

5. **RISERS** A riser is designed to elevate a speaker. The riser consists of 4 trapezoidal sections that can be stacked one on top of the other to produce trapezoids of varying heights.

   ![Diagram of a stack of risers]

   All of the stages have the same height. If all four stages are used, the width of the top of the riser is 10 feet.

   a. If only the bottom two stages are used, what is the width of the top of the resulting riser?

   b. What would be the width of the riser if the bottom three stages are used?
7-2 **Study Guide**

**Similar Polygons**

**Identify Similar Polygons**  Similar polygons have the same shape but not necessarily the same size.

**Example 1** If $\triangle ABC \sim \triangle XYZ$, list all pairs of congruent angles and write a proportion that relates the corresponding sides.

Use the similarity statement.

Congruent angles: $\angle A \cong \angle X$, $\angle B \cong \angle Y$, $\angle C \cong \angle Z$

Proportion: $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CA}{ZX}$

**Example 2** Determine whether the pair of figures is similar. If so, write the similarity statement and scale factor. Explain your reasoning.

**Step 1** Compare corresponding angles.

$\angle W \cong \angle P$, $\angle X \cong \angle Q$, $\angle Y \cong \angle R$, $\angle Z \cong \angle S$

Corresponding angles are congruent.

**Step 2** Compare corresponding sides.

$\frac{WX}{PQ} = \frac{12}{8} = \frac{3}{2}$, $\frac{XY}{QR} = \frac{18}{12} = \frac{3}{2}$, $\frac{YZ}{RS} = \frac{15}{10} = \frac{3}{2}$, and

$\frac{ZW}{SP} = \frac{9}{6} = \frac{3}{2}$. Since corresponding sides are proportional,

$WXYZ \sim PQRS$. The polygons are similar with a scale factor of $\frac{3}{2}$.

**Exercises**

List all pairs of congruent angles, and write a proportion that relates the corresponding sides for each pair of similar polygons.

1. $\triangle DEF \sim \triangle GHJ$

2. $PQRS \sim TUWX$

Determine whether each pair of figures is similar. If so, write the similarity statement and scale factor. If not, explain your reasoning.

3. 

4. 

---

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Chapter 7  129

North Carolina StudyText, Math BC, Volume 1
7-2 Study Guide (continued)  

**Similar Polygons**

**Use Similar Figures** You can use scale factors and proportions to find missing side lengths in similar polygons.

**Example 1**  

The two polygons are similar. Find \( x \) and \( y \).

\[
\begin{align*}
\triangle RST & \sim \triangle MNP \\
\text{Use the congruent angles to write the corresponding vertices in order.} \\
\triangle RST & \sim \triangle MNP
\end{align*}
\]

Write proportions to find \( x \) and \( y \).

\[
\begin{align*}
\frac{32}{16} &= \frac{x}{13} & \frac{38}{16} &= \frac{32}{16} \\
16x &= 32(13) & 32y &= 38(16) \\
x &= 26 & y &= 19
\end{align*}
\]

**Example 2**  

If \( \triangle DEF \sim \triangle GHJ \), find the scale factor of \( \triangle DEF \) to \( \triangle GHJ \) and the perimeter of each triangle.

The scale factor is

\[
\frac{EF}{HJ} = \frac{8}{12} = \frac{2}{3}.
\]

The perimeter of \( \triangle DEF \) is \( 10 + 8 + 12 = 30 \).

\[
\begin{align*}
\frac{2}{3} &= \frac{\text{Perimeter of } \triangle DEF}{\text{Perimeter of } \triangle GHJ} \\
\frac{2}{3} &= \frac{30}{x} \\
(3)(30) &= 2x \\
45 &= x
\end{align*}
\]

So, the perimeter of \( \triangle GHJ \) is 45.

**Exercises**

Each pair of polygons is similar. Find the value of \( x \).

1.

\[
\begin{align*}
\frac{8}{10} &= \frac{x}{12} \\
x &= \frac{10}{8} \times 12 = 15
\end{align*}
\]

2.

\[
\begin{align*}
\frac{4.5}{10} &= \frac{2.5}{x} \\
x &= \frac{10}{4.5} \times 2.5 = \frac{25}{9}
\end{align*}
\]

3.

\[
\begin{align*}
\frac{12}{18} &= \frac{x + 1}{24} \\
x &= \frac{18}{12} \times 24 - 1 = 28
\end{align*}
\]

4.

\[
\begin{align*}
\frac{x + 15}{40} &= \frac{36}{18} \\
x &= \frac{40}{36} \times 18 - 15 = 5
\end{align*}
\]

5. If \( \triangle ABCD \sim \triangle PQRS \), find the scale factor of \( \triangle ABCD \) to \( \triangle PQRS \) and the perimeter of each polygon.
7-2 Practice

Similar Polygons

Determine whether each pair of figures is similar. If so, write the similarity statement and scale factor. If not, explain your reasoning.

1. \[
\begin{align*}
\text{Triangle } LMJ &\sim \text{Triangle } KPS \\
\frac{LM}{KP} &= \frac{24}{14.4} = \frac{20}{15} \\
\text{scale factor} &= \frac{5}{6}
\end{align*}
\]

2. \[
\begin{align*}
\text{Triangle } CBA &\sim \text{Triangle } TUV \\
\frac{CA}{TU} &= \frac{12}{18} = \frac{15}{21} \\
\text{scale factor} &= \frac{2}{3}
\end{align*}
\]

Each pair of polygons is similar. Find the value of \(x\).

3. \[
\begin{align*}
\text{Pentagon } ABCD &\sim \text{Pentagon } MNCP \\
\frac{AD}{NC} &= \frac{14}{x + 6} = \frac{10}{x + 9} \\
\text{scale factor} &= \frac{x + 6}{x + 9} \\
x &= 3
\end{align*}
\]

4. \[
\begin{align*}
\text{Triangle } APB &\sim \text{Triangle } EDF \\
\frac{AB}{EF} &= \frac{14}{x + 1} = \frac{12}{40} \\
\text{scale factor} &= \frac{12}{40} \\
x &= 27
\end{align*}
\]

5. PENTAGONS If \(ABCDE \sim PQRST\), find the scale factor of \(ABCDE\) to \(PQRST\) and the perimeter of each polygon.

6. SWIMMING POOLS The Minnitte family and the neighboring Gaudet family both have in-ground swimming pools. The Minnitte family pool, \(PQRS\), measures 48 feet by 84 feet. The Gaudet family pool, \(WXYZ\), measures 40 feet by 70 feet. Are the two pools similar? If so, write the similarity statement and scale factor.
1. PANELS When closed, an entertainment center is made of four square panels. The three smaller panels are congruent squares.

What is the scale factor of the larger square to one of the smaller squares?

2. WIDESCREEN TELEVISIONS An electronics company manufactures widescreen television sets in several different sizes. The rectangular viewing area of each television size is similar to the viewing areas of the other sizes. The company’s 42-inch widescreen television has a viewing area perimeter of approximately 144.4 inches. What is the viewing area perimeter of the company’s 46-inch widescreen television?

3. ICE HOCKEY An official Olympic-sized ice hockey rink measures 30 meters by 60 meters. The ice hockey rink at the local community college measures 25.5 meters by 51 meters. Are the ice hockey rinks similar? Explain your reasoning.

4. ENLARGING Camille wants to make a pattern for a four-pointed star with dimensions twice as long as the one shown. Help her by drawing a star with dimensions twice as long on the grid below.

5. BIOLOGY A paramecium is a small single-cell organism. The paramecium magnified below is actually one tenth of a millimeter long.

   a. If you want to make a photograph of the original paramecium so that its image is 1 centimeter long, by what scale factor should you magnify it?

   b. If you want to make a photograph of the original paramecium so that its image is 15 centimeters long, by what scale factor should you magnify it?

   c. By approximately what scale factor has the paramecium been enlarged to make the image shown?
Identify Similar Triangles

Here are three ways to show that two triangles are similar.

<table>
<thead>
<tr>
<th>Similarity Type</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>AA Similarity</strong></td>
<td>Two angles of one triangle are congruent to two angles of another triangle.</td>
</tr>
<tr>
<td><strong>SSS Similarity</strong></td>
<td>The measures of the corresponding side lengths of two triangles are proportional.</td>
</tr>
<tr>
<td><strong>SAS Similarity</strong></td>
<td>The measures of two side lengths of one triangle are proportional to the measures of two corresponding side lengths of another triangle, and the included angles are congruent.</td>
</tr>
</tbody>
</table>

**Example 1**

Determine whether the triangles are similar.

\[
\begin{align*}
AC &= \frac{6}{9} = \frac{2}{3} \\
DF &= \frac{9}{12} = \frac{3}{4} \\
BC &= \frac{8}{12} = \frac{2}{3} \\
EF &= \frac{10}{15} = \frac{2}{3}
\end{align*}
\]

\[\triangle ABC \sim \triangle DEF \text{ by SSS Similarity.}\]

**Example 2**

Determine whether the triangles are similar.

\[
\begin{align*}
\frac{3}{4} &= \frac{6}{8} \quad \text{so} \quad \frac{MN}{QR} = \frac{NP}{RS} \\
m\angle N &= m\angle R \quad \text{so} \quad \angle N \cong \angle R. \\
\triangle NMP &\sim \triangle RQS \text{ by SAS Similarity.}
\end{align*}
\]

**Exercises**

Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning.

1. \[
\begin{align*}
\triangle ABC &\sim \triangle DEF \quad \text{by SSS Similarity.} \\
\triangle GHI &\sim \triangle JKL \quad \text{by SAS Similarity.}
\end{align*}
\]

2. \[
\begin{align*}
\triangle ABC &\sim \triangle DEF \quad \text{by SSS Similarity.} \\
\triangle GHI &\sim \triangle JKL \quad \text{by SAS Similarity.}
\end{align*}
\]

3. \[
\begin{align*}
\triangle ABC &\sim \triangle DEF \quad \text{by SSS Similarity.} \\
\triangle GHI &\sim \triangle JKL \quad \text{by SAS Similarity.}
\end{align*}
\]

4. \[
\begin{align*}
\triangle ABC &\sim \triangle DEF \quad \text{by SSS Similarity.} \\
\triangle GHI &\sim \triangle JKL \quad \text{by SAS Similarity.}
\end{align*}
\]

5. \[
\begin{align*}
\triangle ABC &\sim \triangle DEF \quad \text{by SSS Similarity.} \\
\triangle GHI &\sim \triangle JKL \quad \text{by SAS Similarity.}
\end{align*}
\]

6. \[
\begin{align*}
\triangle ABC &\sim \triangle DEF \quad \text{by SSS Similarity.} \\
\triangle GHI &\sim \triangle JKL \quad \text{by SAS Similarity.}
\end{align*}
\]
Use Similar Triangles Similar triangles can be used to find measurements.

**Example 1** \( \triangle ABC \sim \triangle DEF \). Find the values of \( x \) and \( y \).

\[
\begin{align*}
\frac{AC}{DF} &= \frac{BC}{EF} \\
\frac{18\sqrt{3}}{x} &= \frac{18}{9} \\
x &= \frac{18\sqrt{3}}{9} = 9\sqrt{3}
\end{align*}
\]

\[
\begin{align*}
\frac{AB}{DE} &= \frac{BC}{EF} \\
\frac{18\sqrt{3}}{y} &= \frac{18}{9} \\
y &= \frac{18\sqrt{3}}{9} = 9
\end{align*}
\]

\[
\begin{align*}
18x &= 9(18\sqrt{3}) \\
x &= \frac{9(18\sqrt{3})}{18} = 9\sqrt{3}
\end{align*}
\]

\[
\begin{align*}
9y &= 324 \\
y &= \frac{324}{9} = 36
\end{align*}
\]

**Example 2** A person 6 feet tall casts a 1.5-foot-long shadow at the same time that a flagpole casts a 7-foot-long shadow. How tall is the flagpole?

\[
\begin{align*}
\frac{6}{x} &= \frac{1.5}{7} \\
x &= \frac{6 \times 7}{1.5} = 28
\end{align*}
\]

The Sun’s rays form similar triangles. Using \( x \) for the height of the pole, \( \frac{6}{x} = \frac{1.5}{7} \), so \( 1.5x = 42 \) and \( x = 28 \).

The flagpole is 28 feet tall.

**Exercises**

**ALGEBRA** Identify the similar triangles. Then find each measure.

1. **JL**

2. **IU**

3. **QR**

4. **BC**

5. **LM**

6. **QP**

7. The heights of two vertical posts are 2 meters and 0.45 meter. When the shorter post casts a shadow that is 0.85 meter long, what is the length of the longer post’s shadow to the nearest hundredth?
Similar Triangles

Determine whether the triangles are similar. If so, write a similarity statement. If not, what would be sufficient to prove the triangles similar? Explain your reasoning.

1. \( \triangle JYS \) and \( \triangle KSW \) with sides 18, 24, 42 and 16, 12, 42 respectively.

2. \( \triangle LMQ \) and \( \triangle LNQ \) with sides 10, 8, 12 and 5, 8, 12 respectively.

ALGEBRA Identify the similar triangles. Then find each measure.

3. \( \triangle LM, \triangle QP \)

4. \( \triangle NL, \triangle ML \)

5. \( \triangle PS, \triangle PR \)

6. \( \triangle EG, \triangle HG \)

7. INDIRECT MEASUREMENT A lighthouse casts a 128-foot shadow. A nearby lamppost that measures 5 feet 3 inches casts an 8-foot shadow.
   
   a. Write a proportion that can be used to determine the height of the lighthouse.

   b. What is the height of the lighthouse?
7-3 Word Problem Practice

Similar Triangles

1. CHAIRS A local furniture store sells two versions of the same chair: one for adults, and one for children. Find the value of $x$ such that the chairs are similar.

![](image1)

2. BOATING The two sailboats shown are participating in a regatta. Find the value of $x$.

![](image2)

3. GEOMETRY Georgia draws a regular pentagon and starts connecting its vertices to make a 5-pointed star. After drawing three of the lines in the star, she becomes curious about two triangles that appear in the figure, $\triangle ABC$ and $\triangle CEB$. They look similar to her. Prove that this is the case.

![](image3)

4. SHADOWS A radio tower casts a shadow 8 feet long at the same time that a vertical yardstick casts a shadow half an inch long. How tall is the radio tower?

5. MOUNTAIN PEAKS Gavin and Brianna want to know how far a mountain peak is from their houses. They measure the angles between the line of site to the peak and to each other’s houses and carefully make the drawing shown.

![](image4)

The actual distance between Gavin and Brianna’s houses is $1 \frac{1}{2}$ miles.

a. What is the actual distance of the mountain peak from Gavin’s house? Round your answer to the nearest tenth of a mile.

b. What is the actual distance of the mountain peak from Brianna’s house? Round your answer to the nearest tenth of a mile.
Parallel Lines and Proportional Parts

Proportional Parts within Triangles In any triangle, a line parallel to one side of a triangle separates the other two sides proportionally. This is the Triangle Proportionality Theorem. The converse is also true.

If \( \overrightarrow{XY} \parallel \overrightarrow{RS} \), then \( \frac{RX}{XT} = \frac{SY}{YT} \). If \( \frac{RX}{XT} = \frac{SY}{YT} \), then \( \overrightarrow{XY} \parallel \overrightarrow{RS} \).

If \( X \) and \( Y \) are the midpoints of \( \overrightarrow{RT} \) and \( \overrightarrow{ST} \), then \( \overrightarrow{XY} \) is a midsegment of the triangle. The Triangle Midsegment Theorem states that a midsegment is parallel to the third side and is half its length.

If \( \overrightarrow{XY} \) is a midsegment, then \( \overrightarrow{XY} \parallel \overrightarrow{RS} \) and \( XY = \frac{1}{2} RS \).

Example 1 In \( \triangle ABC \), \( \overrightarrow{EF} \parallel \overrightarrow{CB} \). Find \( x \).

Since \( \overrightarrow{EF} \parallel \overrightarrow{CB} \), \( \frac{AF}{FB} = \frac{AE}{EC} \).

\[
\frac{x + 22}{x + 2} = \frac{18}{6}
\]

\[
6x + 132 = 18x + 36
\]

\[
96 = 12x
\]

\[
x = 8
\]

Example 2 In \( \triangle GHJ \), \( HK = 5 \), \( KG = 10 \), and \( JL \) is one-half the length of \( LG \). Is \( \overrightarrow{HK} \parallel \overrightarrow{KL} \)?

Using the converse of the Triangle Proportionality Theorem, show that \( \frac{HK}{KG} = \frac{JL}{LG} \).

Let \( JL = x \) and \( LG = 2x \).

\[
\frac{HK}{KG} = \frac{5}{10} = \frac{1}{2}
\]

\[
\frac{JL}{LG} = \frac{x}{2x} = \frac{1}{2}
\]

Since \( \frac{1}{2} = \frac{1}{2} \), the sides are proportional and \( \overrightarrow{HK} \parallel \overrightarrow{KL} \).

Exercises

ALGEBRA Find the value of \( x \).

1. 5 7
   \[
   \frac{5}{7} = \frac{x}{10}
   \]

2. 20 18
   \[
   \frac{20}{18} = \frac{x}{9}
   \]

3. \[
   \frac{x}{35} = \frac{3}{10}
   \]

4. \[
   \frac{24}{x} = \frac{30}{10}
   \]

5. \[
   \frac{x + 12}{x} = \frac{11}{33}
   \]

6. \[
   \frac{x}{x + 10} = \frac{30}{10}
   \]
Parallel Lines and Proportional Parts

Proportional Parts with Parallel Lines

When three or more parallel lines cut two transversals, they separate the transversals into proportional parts. If the ratio of the parts is 1, then the parallel lines separate the transversals into congruent parts.

\[
\frac{a}{b} = \frac{c}{d}, \quad \text{if } \ell_1 \parallel \ell_2 \parallel \ell_3
\]

\[
\frac{u}{v} = 1, \quad \text{if } \ell_4 \parallel \ell_5 \parallel \ell_6
\]

Example

Refer to lines \( \ell_1, \ell_2, \) and \( \ell_3 \) above. If \( a = 3, \ b = 8, \) and \( c = 5, \) find \( d. \)

\( \ell_1 \parallel \ell_2 \parallel \ell_3 \) so \( \frac{3}{8} = \frac{5}{d}. \) Then \( 3d = 40 \) and \( d = 13 \frac{1}{3}. \)

Exercises

ALGEBRA  Find \( x \) and \( y. \)

1. 

\[
\begin{align*}
5x & \quad 3x \\
x + 12 & \quad 12
\end{align*}
\]

2. 

\[
\begin{align*}
12 & \quad 2x - 6 \\
x + 3 & \quad \phantom{12}
\end{align*}
\]

3. 

\[
\begin{align*}
2x + 4 & \quad 3y \\
3x - 1 & \quad 2y + 2
\end{align*}
\]

4. 

\[
\begin{align*}
5 & \quad y \\
8 & \quad y + 2
\end{align*}
\]

5. 

\[
\begin{align*}
3 & \quad 4 \\
x & \quad x + 4
\end{align*}
\]

6. 

\[
\begin{align*}
16 & \quad 32 \\
y & \quad y + 3
\end{align*}
\]
1. If $AD = 24$, $DB = 27$, and $EB = 18$, find $CE$.

2. If $QT = x + 6$, $SR = 12$, $PS = 27$, and $TR = x - 4$, find $QT$ and $TR$.

Determine whether $\overline{JK} \parallel \overline{NM}$. Justify your answer.

3. $JN = 18$, $JL = 30$, $KM = 21$, and $ML = 35$

4. $KM = 24$, $KL = 44$, and $NL = \frac{5}{6} JN$

$\overline{JH}$ is a midsegment of $\triangle KLM$. Find the value of $x$.

5.

6.

7. Find $x$ and $y$.

8. Find $x$ and $y$.

9. **MAPS** On a map, Wilmington Street, Beech Drive, and Ash Grove Lane appear to all be parallel. The distance from Wilmington to Ash Grove along Kendall is 820 feet and along Magnolia, 660 feet. If the distance between Beech and Ash Grove along Magnolia is 280 feet, what is the distance between the two streets along Kendall?
1. **CARPENTRY** Jake is fixing an A-frame. He wants to add a horizontal support beam halfway up and parallel to the ground. What is the length of the beam?

2. **STREETS** In the diagram, Cay Street and Bay Street are parallel. Find \( x \).

3. **JUNGLE GYMS** Prassad is building a two-story jungle gym according to the plans shown. Find \( x \).

4. **FIREMEN** A cat is stuck in a tree and firefighters try to rescue it. Based on the figure, if a firefighter climbs to the top of the ladder, how far away is the cat?

5. **EQUAL PARTS** Nick has a stick that he would like to divide into 9 equal parts. He places it on a piece of grid paper as shown. The grid paper is ruled so that vertical and horizontal lines are equally spaced.

   a. Explain how he can use the grid paper to help him find where he needs to cut the stick.

   b. Suppose Nick wants to divide his stick into 5 equal parts utilizing the grid paper. What can he do?
7-5  Study Guide

Parts of Similar Triangles

Special Segments of Similar Triangles  When two triangles are similar, corresponding altitudes, angle bisectors, and medians are proportional to the corresponding sides.

Example  In the figure, $\triangle ABC \sim \triangle XYZ$, with angle bisectors as shown. Find $x$.

Since $\triangle ABC \sim \triangle XYZ$, the measures of the angle bisectors are proportional to the measures of a pair of corresponding sides.

\[
\frac{AB}{XY} = \frac{BD}{YW} \\
\frac{24}{x} = \frac{10}{8} \\
10x = 24 \times 8 \\
10x = 192 \\
x = 19.2
\]

Exercises

Find $x$.

1. \[\begin{array}{c}
\text{20} \\
\text{36} \\
\text{18} \\
\end{array}\]

2. \[\begin{array}{c}
\text{12} \\
\text{9} \\
\text{6} \\
\end{array}\]

3. \[\begin{array}{c}
\text{4} \\
\text{3} \\
\text{1} \\
\text{x} \\
\end{array}\]

4. \[\begin{array}{c}
\text{10} \\
\text{7} \\
\text{8} \\
\text{8} \\
\end{array}\]

5. \[\begin{array}{c}
\text{42} \\
\text{45} \\
\text{30} \\
\text{x} \\
\end{array}\]

6. \[\begin{array}{c}
\text{12} \\
\text{12} \\
\text{14} \\
\text{x} \\
\end{array}\]
7-5 Study Guide (continued)

Parts of Similar Triangles

Triangle Angle Bisector Theorem  An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

Example  Find $x$.

Since $SU$ is an angle bisector, $\frac{RU}{TU} = \frac{RS}{TS}$.

\[
\frac{x}{20} = \frac{15}{30} \\
30x = 20(15) \\
30x = 300 \\
x = 10
\]

Exercises  Find the value of each variable.

1. $\frac{25}{20} = \frac{x}{28}$

2. $\frac{15}{a} = \frac{9}{3}$

3. $\frac{46.8}{n} = \frac{42}{36}$

4. $\frac{x}{10} = \frac{13}{15}$

5. $\frac{110}{100} = \frac{r}{160}$

6. $\frac{x}{x+7} = \frac{11}{17}$
7-5 Practice

Parts of Similar Triangles

ALGEBRA Find $x$.

1. 
2. 
3. 
4. 

5. If $\triangle JKL \sim \triangle NPR$, $KM$ is an altitude of $\triangle JKL$, $PT$ is an altitude of $\triangle NPR$, $KL = 28$, $KM = 18$, and $PT = 15.75$, find $PR$.

6. If $\triangle STU \sim \triangle XYZ$, $UA$ is an altitude of $\triangle STU$, $ZB$ is an altitude of $\triangle XYZ$, $UT = 8.5$, $UA = 6$, and $ZB = 11.4$, find $ZY$.

7. PHOTOGRAPHY Francine has a camera in which the distance from the lens to the film is 24 millimeters.

   a. If Francine takes a full-length photograph of her friend from a distance of 3 meters and the height of her friend is 140 centimeters, what will be the height of the image on the film? (Hint: Convert to the same unit of measure.)

   b. Suppose the height of the image on the film of her friend is 15 millimeters. If Francine took a full-length shot, what was the distance between the camera and her friend?
1. **FLAGS** An oceanliner is flying two similar triangular flags on a flag pole. The altitude of the larger flag is three times the altitude of the smaller flag. If the measure of a leg on the larger flag is 45 inches, find the measure of the corresponding leg on the smaller flag.

2. **TENTS** Jana went camping and stayed in a tent shaped like a triangle. In a photo of the tent, the base of the tent is 6 inches and the altitude is 5 inches. The actual base was 12 feet long. What was the height of the actual tent?

3. **PLAYGROUND** The playground at Hank’s school has a large right triangle painted on the ground. Hank starts at the right angle corner and walks toward the opposite side along an angle bisector and stops when he gets to the hypotenuse.

4. **FLAG POLES** A flag pole attached to the side of a building is supported with a network of strings as shown in the figure.

   The rigging is done so that $AE = EF$, $AC = CD$, and $AB = BC$. What is the ratio of $CF$ to $BE$?

5. **COPIES** Gordon made a photocopy of a page from his geometry book to enlarge one of the figures. The actual figure that he copied is shown below.

   The photocopy came out poorly. Gordon could not read the numbers on the photocopy, although the triangle itself was clear. Gordon measured the base of the enlarged triangle and found it to be 200 millimeters.

   **a.** What is the length of the drawn altitude of the enlarged triangle? Round your answer to the nearest millimeter.

   **b.** What is the length of the drawn median of the enlarged triangle? Round your answer to the nearest millimeter.
Geometric Mean

The geometric mean between two numbers is the positive square root of their product. For two positive numbers \(a\) and \(b\), the geometric mean of \(a\) and \(b\) is the positive number \(x\) in the proportion \(\frac{a}{x} = \frac{x}{b}\). Cross multiplying gives \(x^2 = ab\), so \(x = \sqrt{ab}\).

Example

Find the geometric mean between each pair of numbers.

a. 12 and 3

\[
\begin{align*}
x &= \sqrt{ab} \\
&= \sqrt{12 \cdot 3} \\
&= \sqrt{(2 \cdot 2 \cdot 3) \cdot 3} \\
&= 6
definition of geometric mean

equal to \(a = 12\) and \(b = 3\)

equal to \(\sqrt{2 \cdot 2 \cdot 3 \cdot 3}\)

division by \(2\) and \(3\)

The geometric mean between 12 and 3 is 6.

b. 8 and 4

\[
\begin{align*}
x &= \sqrt{ab} \\
&= \sqrt{8 \cdot 4} \\
&= \sqrt{(2 \cdot 4) \cdot 4} \\
&= \sqrt{16 \cdot 2} \\
&= 4\sqrt{2}
definition of geometric mean

equal to \(a = 8\) and \(b = 4\)

equal to \(\sqrt{2 \cdot 4 \cdot 4}\)

equal to \(\sqrt{16 \cdot 2}\)

The geometric mean between 8 and 4 is \(4\sqrt{2}\) or about 5.7.

Exercises

Find the geometric mean between each pair of numbers.

1. 4 and 4
2. 4 and 6
3. 6 and 9
4. \(\frac{1}{2}\) and 2
5. 12 and 20
6. 4 and 25
7. 16 and 30
8. 10 and 100
9. \(\frac{1}{2}\) and \(\frac{1}{4}\)
10. 17 and 3
11. 4 and 16
12. 3 and 24
Geometric Mean

Geometric Means in Right Triangles  In the diagram, \( \triangle ABC \sim \triangle ADB \sim \triangle BDC \). An altitude to the hypotenuse of a right triangle forms two right triangles. The two triangles are similar and each is similar to the original triangle.

**Example 1**  Use right \( \triangle ABC \) with \( BD \perp AC \). Describe two geometric means.

a. \( \triangle ADB \sim \triangle BDC \) so \( \frac{AD}{BD} = \frac{BD}{CD} \).

In \( \triangle ABC \), the altitude is the geometric mean between the two segments of the hypotenuse.

b. \( \triangle ABC \sim \triangle ADB \) and \( \triangle ABC \sim \triangle BDC \), so \( \frac{AC}{AB} = \frac{AB}{AD} \) and \( \frac{AC}{BC} = \frac{BC}{DC} \).

In \( \triangle ABC \), each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse adjacent to that leg.

**Example 2**  Find \( x, y, \) and \( z \).

\[
15 = \sqrt{RP \cdot SP} \quad \text{Geometric Mean (Leg) Theorem}
\]

\[
15 = \sqrt{25x} \quad RP = 25 \text{ and } SP = x
\]

Square each side.

\[
225 = 25x \quad x = 9
\]

Divide each side by 25.

Then use the value of \( x \) to find \( y \) and \( z \).

\[
y = RP - SP
\]

\[
= 25 - 9
\]

\[
y = 16
\]

\[
z = \sqrt{RS \cdot RP} \quad \text{Geometric Mean (Leg) Theorem}
\]

\[
z = \sqrt{16 \cdot 25}
\]

\[
z = 400
\]

Multiply.

\[
z = 20
\]

Simplify.

**Exercises**

Find \( x, y, \) and \( z \) to the nearest tenth.

1. \[ \begin{array}{c}
\text{1} \\
\text{2} \\
\text{3}
\end{array} \]

2. \[ \begin{array}{c}
\text{1} \\
\text{2} \\
\text{3}
\end{array} \]

3. \[ \begin{array}{c}
\text{1} \\
\text{2} \\
\text{3}
\end{array} \]

4. \[ \begin{array}{c}
\text{1} \\
\text{2} \\
\text{3}
\end{array} \]

5. \[ \begin{array}{c}
\text{1} \\
\text{2} \\
\text{3}
\end{array} \]

6. \[ \begin{array}{c}
\text{1} \\
\text{2} \\
\text{3}
\end{array} \]
8-1 Practice

Geometric Mean

Find the geometric mean between each pair of numbers.

1. 8 and 12  
2. 3 and 15  
3. $\frac{4}{5}$ and 2

Write a similarity statement identifying the three similar triangles in the figure.

4.  
5.  

Find $x$, $y$, and $z$.

6.  
7.  

8.  
9.  

10. CIVIL An airport, a factory, and a shopping center are at the vertices of a right triangle formed by three highways. The airport and factory are 6.0 miles apart. Their distances from the shopping center are 3.6 miles and 4.8 miles, respectively. A service road will be constructed from the shopping center to the highway that connects the airport and factory. What is the shortest possible length for the service road? Round to the nearest hundredth.
1. SQUARES  Wilma has a rectangle of dimensions \( \ell \) by \( w \). She would like to replace it with a square that has the same area. What is the side length of the square with the same area as Wilma’s rectangle?

2. EQUALITY  Gretchen computed the geometric mean of two numbers. One of the numbers was 7 and the geometric mean turned out to be 7 as well. What was the other number?

3. VIEWING ANGLE  A photographer wants to take a picture of a beach front. His camera has a viewing angle of \( 90^\circ \) and he wants to make sure two palm trees located at points \( A \) and \( B \) in the figure are just inside the edges of the photograph.

He walks out on a walkway that goes over the ocean to get the shot. If his camera has a viewing angle of \( 90^\circ \), at what distance down the walkway should he stop to take his photograph?

4. EXHIBITIONS  A museum has a famous statue on display. The curator places the statue in the corner of a rectangular room and builds a 15-foot-long railing in front of the statue. Use the information below to find how close visitors will be able to get to the statue.

5. CLIFFS  A bridge connects to a tunnel as shown in the figure. The bridge is 180 feet above the ground. At a distance of 235 feet along the bridge out of the tunnel, the angle to the base and summit of the cliff is a right angle.

a. What is the height of the cliff? Round to the nearest whole number.

b. How high is the cliff from base to summit? Round to the nearest whole number.

c. What is the value of \( d \)? Round to the nearest whole number.
The Pythagorean Theorem
In a right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse. If the three whole numbers \(a\), \(b\), and \(c\) satisfy the equation \(a^2 + b^2 = c^2\), then the numbers \(a\), \(b\), and \(c\) form a Pythagorean triple.

### Example

**a. Find \(a\).**

\[
\begin{align*}
    a^2 + b^2 &= c^2 \\
    a^2 + 12^2 &= 13^2 \\
    a^2 + 144 &= 169 \\
    a^2 &= 25 \\
    a &= 5
\end{align*}
\]

**b. Find \(c\).**

\[
\begin{align*}
    a^2 + b^2 &= c^2 \\
    20^2 + 30^2 &= c^2 \\
    400 + 900 &= c^2 \\
    1300 &= c^2 \\
    \sqrt{1300} &= c \\
    36.1 &\approx c
\end{align*}
\]

### Exercises

**Find \(x\).**

1. \[3 \quad 3 \quad x\]
2. \[9 \quad 15 \quad x\]
3. \[25 \quad 65 \quad x\]
4. \[4 \quad 5 \quad 9\]
5. \[16 \quad x \quad 33\]
6. \[11 \quad 28 \quad x\]

**Use a Pythagorean Triple to find \(x\).**

7. \[17 \quad 28 \quad 8\]
8. \[45 \quad x \quad 24\]
9. \[28 \quad x \quad 96\]
The Pythagorean Theorem and Its Converse

Converse of the Pythagorean Theorem  If the sum of the squares of the lengths of the two shorter sides of a triangle equals the square of the lengths of the longest side, then the triangle is a right triangle.

You can also use the lengths of sides to classify a triangle.

- if \(a^2 + b^2 = c^2\) then \(\triangle ABC\) is a right triangle.
- if \(a^2 + b^2 > c^2\) then \(\triangle ABC\) is acute.
- if \(a^2 + b^2 < c^2\) then \(\triangle ABC\) is obtuse.

**Example**

Determine whether \(\triangle PQR\) is a right triangle.

\[
a^2 + b^2 \equiv c^2 \\
10^2 + (10\sqrt{3})^2 \equiv 20^2 \\
100 + 300 \equiv 400 \\
400 = 400 \checkmark
\]

Since \(c^2 = a^2 + b^2\), the triangle is a right triangle.

**Exercises**

Determine whether each set of measures can be the measures of the sides of a triangle. If so, classify the triangle as **acute**, **obtuse**, or **right**. Justify your answer.

1. 30, 40, 50
2. 20, 30, 40
3. 18, 24, 30
4. 6, 8, 9
5. 6, 12, 18
6. 10, 15, 20
7. \(\sqrt{5}, \sqrt{12}, \sqrt{13}\)
8. 2, \(\sqrt{8}, \sqrt{12}\)
9. 9, 40, 41
8-2 Practice

The Pythagorean Theorem and Its Converse

Find $x$.

1. \[ x \]

2. \[ 34 \]

3. \[ 26 \]

4. \[ 34 \]

5. \[ 16 \]

6. \[ 24 \]

Use a Pythagorean Triple to find $x$.

7. \[ 27 \]

8. \[ 136 \]

9. \[ 39 \]

10. \[ 42 \]

Determine whether each set of numbers can be the measures of the sides of a triangle. If so, classify the triangle as **acute**, **obtuse**, or **right**. Justify your answer.

11. 10, 11, 20
12. 12, 14, 49
13. $5\sqrt{2}$, 10, 11

14. 21.5, 24, 55.5
15. 30, 40, 50
16. 65, 72, 97

17. **CONSTRUCTION** The bottom end of a ramp at a warehouse is 10 feet from the base of the main dock and is 11 feet long. How high is the dock?
1. **SIDEWALKS** Construction workers are building a marble sidewalk around a park that is shaped like a right triangle. Each marble slab adds 2 feet to the length of the sidewalk. The workers find that exactly 1071 and 1840 slabs are required to make the sidewalks along the short sides of the park. How many slabs are required to make the sidewalk that runs along the long side of the park?

2. **RIGHT ANGLES** Clyde makes a triangle using three sticks of lengths 20 inches, 21 inches, and 28 inches. Is the triangle a right triangle? Explain.

3. **TETHERS** To help support a flag pole, a 50-foot-long tether is tied to the pole at a point 40 feet above the ground. The tether is pulled taut and tied to an anchor in the ground. How far away from the base of the pole is the anchor?

4. **FLIGHT** An airplane lands at an airport 60 miles east and 25 miles north of where it took off.

   ![Diagram of an airplane landing](image)

   How far apart are the two airports?

5. **PYTHAGOREAN TRIPLES** Ms. Jones assigned her fifth-period geometry class the following problem.

   Let \( m \) and \( n \) be two positive integers with \( m > n \). Let \( a = m^2 - n^2 \), \( b = 2mn \), and \( c = m^2 + n^2 \).

   a. Show that there is a right triangle with side lengths \( a \), \( b \), and \( c \).

   b. Complete the following table.

<table>
<thead>
<tr>
<th>( m )</th>
<th>( n )</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
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<td>4</td>
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</tr>
<tr>
<td>5</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   c. Find a Pythagorean triple that corresponds to a right triangle with a hypotenuse \( 25^2 = 625 \) units long. *(Hint: Use the table you completed for Exercise b to find two positive integers \( m \) and \( n \) with \( m > n \) and \( m^2 + n^2 = 625 \).)*
Properties of 45°-45°-90° Triangles

The sides of a 45°-45°-90° right triangle have a special relationship.

**Example 1**

If the leg of a 45°-45°-90° right triangle is \( x \) units, show that the hypotenuse is \( x\sqrt{2} \) units.

Using the Pythagorean Theorem with \( a = b = x \), then

\[
\begin{align*}
   c^2 &= a^2 + b^2 \\
   c^2 &= x^2 + x^2 \\
   c^2 &= 2x^2 \\
   c &= \sqrt{2x^2} \\
   c &= x\sqrt{2}
\end{align*}
\]

**Example 2**

In a 45°-45°-90° right triangle the hypotenuse is \( \sqrt{2} \) times the leg. If the hypotenuse is 6 units, find the length of each leg.

The hypotenuse is \( \sqrt{2} \) times the leg, so divide the length of the hypotenuse by \( \sqrt{2} \).

\[
\begin{align*}
   a &= \frac{6}{\sqrt{2}} \\
   &= \frac{6 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} \\
   &= \frac{6\sqrt{2}}{2} \\
   &= 3\sqrt{2} \text{ units}
\end{align*}
\]

**Exercises**

Find \( x \).

1. \[
\begin{array}{c}
   \text{45°} \\
   x \ \ \ \ \\
   8 \\
   \text{45°}
\end{array}
\]

2. \[
\begin{array}{c}
   \text{45°} \\
   x \ \ \ \ \\
   3\sqrt{2} \\
   \text{45°}
\end{array}
\]

3. \[
\begin{array}{c}
   \text{45°} \\
   4 \ \ \ \ \\
   x \\
   \text{45°}
\end{array}
\]

4. \[
\begin{array}{c}
   18 \\
   x \ \ \ \ \\
   x \\
   \text{45°}
\end{array}
\]

5. \[
\begin{array}{c}
   16 \\
   x \ \ \ \ \\
   x \\
   \text{45°}
\end{array}
\]

6. \[
\begin{array}{c}
   24\sqrt{2} \\
   x \ \ \ \ \\
   x \\
   \text{45°}
\end{array}
\]

7. If a 45°-45°-90° triangle has a hypotenuse length of 12, find the leg length.

8. Determine the length of the leg of a 45°-45°-90° triangle with a hypotenuse length of 25 inches.

9. Find the length of the hypotenuse of a 45°-45°-90° triangle with a leg length of 14 centimeters.
Properties of $30^\circ$-$60^\circ$-$90^\circ$ Triangles The sides of a $30^\circ$-$60^\circ$-$90^\circ$ right triangle also have a special relationship.

Example 1 In a $30^\circ$-$60^\circ$-$90^\circ$ right triangle the hypotenuse is twice the shorter leg. Show that the longer leg is $\sqrt{3}$ times the shorter leg.

$\triangle MNQ$ is a $30^\circ$-$60^\circ$-$90^\circ$ right triangle, and the length of the hypotenuse $MN$ is two times the length of the shorter side $NQ$.

Use the Pythagorean Theorem.

$a^2 = (2x)^2 - x^2$  
$a^2 = 4x^2 - x^2$ Multiply.

$a^2 = 3x^2$ Subtract.

$a = \sqrt{3x^2}$ Take the positive square root of each side.

$a = x\sqrt{3}$ Simplify.

Example 2 In a $30^\circ$-$60^\circ$-$90^\circ$ right triangle, the hypotenuse is 5 centimeters. Find the lengths of the other two sides of the triangle.

If the hypotenuse of a $30^\circ$-$60^\circ$-$90^\circ$ right triangle is 5 centimeters, then the length of the shorter leg is one-half of 5, or 2.5 centimeters. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg, or $(2.5)(\sqrt{3})$ centimeters.

Exercises

Find $x$ and $y$.

1.  

2.  

3.  

4.  

5.  

6.  

7. An equilateral triangle has an altitude length of 36 feet. Determine the length of a side of the triangle.

8. Find the length of the side of an equilateral triangle that has an altitude length of 45 centimeters.
8-3 Practice

Special Right Triangles

Find \( x \).

1. \[ \angle 45^\circ, \quad 14 \]

2. \[ 45^\circ, \quad 22 \]

3. \[ 45^\circ, \quad \]

4. \[ 210, \quad 45^\circ \]

5. \[ 88, \quad 45^\circ \]

6. \[ 5\sqrt{2}, \quad 45^\circ \]

Find \( x \) and \( y \).

7. \[ 30^\circ, \quad 9, \quad y \]

8. \[ 4\sqrt{3}, \quad 60^\circ, \quad x, \quad y \]

9. \[ 30^\circ, \quad 20, \quad x, \quad y \]

10. \[ 98, \quad 60^\circ, \quad x, \quad y \]

11. Determine the length of the leg of a 45°-45°-90° triangle with a hypotenuse length of 38.

12. Find the length of the hypotenuse of a 45°-45°-90° triangle with a leg length of 77 centimeters.

13. An equilateral triangle has an altitude length of 33 feet. Determine the length of a side of the triangle.

14. BOTANICAL GARDENS One of the displays at a botanical garden is an herb garden planted in the shape of a square. The square measures 6 yards on each side. Visitors can view the herbs from a diagonal pathway through the garden. How long is the pathway?
1. **ORIGAMI** A square piece of paper 150 millimeters on a side is folded in half along a diagonal. The result is a 45°-45°-90° triangle. What is the length of the hypotenuse of this triangle?

2. **ESCALATORS** A 40-foot-long escalator rises from the first floor to the second floor of a shopping mall. The escalator makes a 30° angle with the horizontal. How high above the first floor is the second floor?

3. **HEXAGONS** A box of chocolates shaped like a regular hexagon is placed snugly inside of a rectangular box as shown in the figure. If the side length of the hexagon is 3 inches, what are the dimensions of the rectangular box?

4. **WINDOWS** A large stained glass window is constructed from six 30°-60°-90° triangles as shown in the figure.

5. **MOVIES** Kim and Yolanda are watching a movie in a movie theater. Yolanda is sitting \( x \) feet from the screen and Kim is 15 feet behind Yolanda. The angle that Kim’s line of sight to the top of the screen makes with the horizontal is 30°. The angle that Yolanda’s line of sight to the top of the screen makes with the horizontal is 45°.

   a. How high is the top of the screen in terms of \( x \)?

   b. What is \( \frac{x + 15}{x} \)?

   c. How far is Yolanda from the screen? Round your answer to the nearest tenth.
Lesson 8-4

**Study Guide**

*Trigonometry*

**Trigonometric Ratios** The ratio of the lengths of two sides of a right triangle is called a **trigonometric ratio**. The three most common ratios are **sine**, **cosine**, and **tangent**, which are abbreviated *sin*, *cos*, and *tan*, respectively.

\[
\sin R = \frac{\text{leg opposite } \angle R}{\text{hypotenuse}} = \frac{r}{t} \\
\cos R = \frac{\text{leg adjacent to } \angle R}{\text{hypotenuse}} = \frac{s}{t} \\
\tan R = \frac{\text{leg opposite } \angle R}{\text{leg adjacent to } \angle R} = \frac{r}{s}
\]

**Example** Find sin *A*, cos *A*, and tan *A*. Express each ratio as a fraction and a decimal to the nearest hundredth.

\[
\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{BC}{BA} = \frac{5}{13} \approx 0.38
\]

\[
\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{12}{13} \approx 0.92
\]

\[
\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{BC}{AC} = \frac{5}{12} \approx 0.42
\]

**Exercises**

Find sin *J*, cos *J*, tan *J*, sin *L*, cos *L*, and tan *L*. Express each ratio as a fraction and as a decimal to the nearest hundredth if necessary.

1. Find sin *J*, cos *J*, tan *J*.
3. Find sin *K*, cos *K*, tan *K*.
Trigonometry

Use Inverse Trigonometric Ratios  You can use a calculator and the sine, cosine, or tangent to find the measure of the angle, called the inverse of the trigonometric ratio.

**Example**  Use a calculator to find the measure of $\angle T$ to the nearest tenth.

The measures given are those of the leg opposite $\angle T$ and the hypotenuse, so write an equation using the sine ratio.

$$\sin T = \frac{\text{opp}}{\text{hyp}} \quad \sin T = \frac{29}{34}$$

If $\sin T = \frac{29}{34}$, then $\sin^{-1} \frac{29}{34} = m\angle T$.

Use a calculator. So, $m\angle T \approx 58.5$.

**Exercises**

Use a calculator to find the measure of $\angle T$ to the nearest tenth.

1. 
   ![](image1)
2. 
   ![](image2)
3. 
   ![](image3)
4. 
   ![](image4)
5. 
   ![](image5)
6. 
   ![](image6)
8-4 Practice

Trigonometry

Find $\sin L$, $\cos L$, $\tan L$, $\sin M$, $\cos M$, and $\tan M$.
Express each ratio as a fraction and as a decimal to the nearest hundredth.

1. $\ell = 15$, $m = 36$, $n = 39$
2. $\ell = 12$, $m = 12\sqrt{3}$, $n = 24$

Find $x$. Round to the nearest hundredth.

3. 

4. 

5. 

Use a calculator to find the measure of $\angle B$ to the nearest tenth.

6. 

7. 

8. 

9. GEOGRAPHY Diego used a theodolite to map a region of land for his class in geomorphology. To determine the elevation of a vertical rock formation, he measured the distance from the base of the formation to his position and the angle between the ground and the line of sight to the top of the formation. The distance was 43 meters and the angle was $36^\circ$. What is the height of the formation to the nearest meter?
8-4 Word Problem Practice

Trigonometry

1. RADIO TOWERS Kay is standing near a 200-foot-high radio tower.

   Use the information in the figure to determine how far Kay is from the top of the tower. Express your answer as a trigonometric function.

2. RAMPS A 60-foot ramp rises from the first floor to the second floor of a parking garage. The ramp makes a 15° angle with the ground.

   How high above the first floor is the second floor? Express your answer as a trigonometric function.

3. TRIGONOMETRY Melinda and Walter were both solving the same trigonometry problem. However, after they finished their computations, Melinda said the answer was 52 sin 27° and Walter said the answer was 52 cos 63°. Could they both be correct? Explain.

4. LINES Jasmine draws line \( m \) on a coordinate plane.

   What angle does \( m \) make with the \( x \)-axis? Round your answer to the nearest degree.

5. NEIGHBORS Amy, Barry, and Chris live on the same block. Chris lives up the street and around the corner from Amy, and Barry lives at the corner between Amy and Chris. The three homes are the vertices of a right triangle.

   a. Give two trigonometric expressions for the ratio of Barry’s distance from Amy to Chris’ distance from Amy.

   b. Give two trigonometric expressions for the ratio of Barry’s distance from Chris to Amy’s distance from Chris.

   c. Give a trigonometric expression for the ratio of Amy’s distance from Barry to Chris’ distance from Barry.
Angles of Elevation and Depression

Many real-world problems that involve looking up to an object can be described in terms of an **angle of elevation**, which is the angle between an observer’s line of sight and a horizontal line.

When an observer is looking down, the **angle of depression** is the angle between the observer’s line of sight and a horizontal line.

**Example**

The angle of elevation from point A to the top of a cliff is 34°. If point A is 1000 feet from the base of the cliff, how high is the cliff?

Let $x$ = the height of the cliff.

\[
\tan 34° = \frac{x}{1000} \quad \text{tan} = \frac{\text{opposite}}{\text{adjacent}}
\]

\[
1000(\tan 34°) = x \quad \text{Multiply each side by 1000.}
\]

\[
674.5 \approx x \quad \text{Use a calculator.}
\]

The height of the cliff is about 674.5 feet.

**Exercises**

1. **HILL TOP** The angle of elevation from point A to the top of a hill is 49°. If point A is 400 feet from the base of the hill, how high is the hill?

2. **SUN** Find the angle of elevation of the Sun when a 12.5-meter-tall telephone pole casts an 18-meter-long shadow.

3. **SKIING** A ski run is 1000 yards long with a vertical drop of 208 yards. Find the angle of depression from the top of the ski run to the bottom.

4. **AIR TRAFFIC** From the top of a 120-foot-high tower, an air traffic controller observes an airplane on the runway at an angle of depression of 19°. How far from the base of the tower is the airplane?
Two Angles of Elevation or Depression

Angles of elevation or depression to two different objects can be used to estimate distance between those objects. The angles from two different positions of observation to the same object can be used to estimate the height of the object.

Example

To estimate the height of a garage, Jason sights the top of the garage at a 42° angle of elevation. He then steps back 20 feet and sites the top at a 10° angle. If Jason is 6 feet tall, how tall is the garage to the nearest foot?

△ABC and △ABD are right triangles. We can determine \( AB = x \) and \( CB = y \), and \( DB = y + 20 \).

Use △ABC. Use △ABD.

\[
\tan 42° = \frac{x}{y} \quad \text{or} \quad y \tan 42° = x \\
\tan 10° = \frac{x}{y + 20} \quad \text{or} \quad (y + 20) \tan 10° = x
\]

Substitute the value for \( x \) from △ABD in the equation for △ABC and solve for \( y \).

\[
y \tan 42° = (y + 20) \tan 10° \\
y \tan 42° = y \tan 10° + 20 \tan 10° \\
y \tan 42° - y \tan 10° = 20 \tan 10° \\
y (\tan 42° - \tan 10°) = 20 \tan 10° \\
y = \frac{20 \tan 10°}{\tan 42° - \tan 10°} \approx 4.87
\]

If \( y \approx 4.87 \), then \( x \approx 4.87 \tan 42° \) or about 4.4 feet. Add Jason's height, so the garage is about 4.4 + 6 or 10.4 feet tall.

Exercises

1. CLIFF

Sarah stands on the ground and sights the top of a steep cliff at a 60° angle of elevation. She then steps back 50 meters and sights the top of the steep cliff at a 30° angle. If Sarah is 1.8 meters tall, how tall is the steep cliff to the nearest meter?

2. BALLOON

The angle of depression from a hot air balloon in the air to a person on the ground is 36°. If the person steps back 10 feet, the new angle of depression is 25°. If the person is 6 feet tall, how far off the ground is the hot air balloon?
3. **WATER TOWERS** A student can see a water tower from the closest point of the soccer field at San Lobos High School. The edge of the soccer field is about 110 feet from the water tower and the water tower stands at a height of 32.5 feet. What is the angle of elevation if the eye level of the student viewing the tower from the edge of the soccer field is 6 feet above the ground? Round to the nearest tenth.

4. **CONSTRUCTION** A roofer props a ladder against a wall so that the top of the ladder reaches a 30-foot roof that needs repair. If the angle of elevation from the bottom of the ladder to the roof is 55°, how far is the ladder from the base of the wall? Round your answer to the nearest foot.

5. **TOWN ORDINANCES** The town of Belmont restricts the height of flagpoles to 25 feet on any property. Lindsay wants to determine whether her school is in compliance with the regulation. Her eye level is 5.5 feet from the ground and she stands 36 feet from the flagpole. If the angle of elevation is about 25°, what is the height of the flagpole to the nearest tenth?

6. **GEOGRAPHY** Stephan is standing on the ground by a mesa in the Painted Desert. Stephan is 1.8 meters tall and sights the top of the mesa at 29°. Stephan steps back 100 meters and sights the top at 25°. How tall is the mesa?

7. **INDIRECT MEASUREMENT** Mr. Dominguez is standing on a 40-foot ocean bluff near his home. He can see his two dogs on the beach below. If his line of sight is 6 feet above the ground and the angles of depression to his dogs are 34° and 48°, how far apart are the dogs to the nearest foot?
Word Problem Practice

Angles of Elevation and Depression

1. **LIGHTHOUSES** Sailors on a ship at sea spot the light from a lighthouse. The angle of elevation to the light is 25°.

   ![Diagram of a lighthouse with a 25° angle of elevation]

   The light of the lighthouse is 30 meters above sea level. How far from the shore is the ship? Round your answer to the nearest meter.

2. **RESCUE** A hiker dropped his backpack over one side of a canyon onto a ledge below. Because of the shape of the cliff, he could not see exactly where it landed.

   ![Diagram of a backpack with a 32° angle of depression]

   From the other side, the park ranger reports that the angle of depression to the backpack is 32°. If the width of the canyon is 115 feet, how far down did the backpack fall? Round your answer to the nearest foot.

3. **AIRPLANES** The angle of elevation to an airplane viewed from the control tower at an airport is 7°. The tower is 200 feet high and the pilot reports that the altitude is 5200 feet. How far away from the control tower is the airplane? Round your answer to the nearest foot.

4. **PEAK TRAM** The Peak Tram in Hong Kong connects two terminals, one at the base of a mountain, and the other at the summit. The angle of elevation of the upper terminal from the lower terminal is about 15.5°. The distance between the two terminals is about 1365 meters. About how much higher above sea level is the upper terminal compared to the lower terminal? Round your answer to the nearest meter.

5. **HELICOPTERS** Jermaine and John are watching a helicopter hover above the ground.

   ![Diagram of a helicopter with angles 48° and 55°]

   Jermaine and John are standing 10 meters apart.

   a. Find two different expressions that can be used to find the $h$, height of the helicopter.

   b. Equate the two expressions you found for Exercise a to solve for $x$. Round your answer to the nearest hundredth.

   c. How high above the ground is the helicopter? Round your answer to the nearest hundredth.
**Dilations**

**Draw Dilations** A dilation is a similarity transformation that enlarges or reduces a figure proportionally. Dilations are completed with respect to a center point and a scale factor.

**Example** Draw the dilation image of \( \triangle ABC \) with center \( O \) and \( r = 2 \).

Draw \( \overline{OA}, \overline{OB}, \) and \( \overline{OC}. \) Label points \( A', B', \) and \( C' \) so that \( OA' = 2(OA), OB' = 2(OB) \) and \( OC' = 2(OC). \) Connect the points to draw \( \triangle A'B'C'. \) \( \triangle A'B'C' \) is a dilation of \( \triangle ABC. \)

**Exercises**

Use a ruler to draw the image of the figure under a dilation with center \( S \) and the scale factor \( r \) indicated.

1. \( r = 2 \)  
   ![Image 1]

2. \( r = \frac{1}{2} \)  
   ![Image 2]

3. \( r = 1 \)  
   ![Image 3]

4. \( r = 3 \)  
   ![Image 4]

5. \( r = \frac{2}{3} \)  
   ![Image 5]

6. \( r = 1 \)  
   ![Image 6]
Dilations In The Coordinate Plane  To find the coordinates of an image after a
dilation centered at the origin, multiply the x- and y-coordinates of each point on the
preimage by the scale factor of the dilation, r.

\[(x, y) \rightarrow (rx, ry)\]

\[\triangle ABC \text{ has vertices } A(-2, -2), B(1, -1), \text{ and } C(2, 0). \text{ Find the image of } \triangle ABC \text{ after a dilation centered at the origin with a scale factor of 2.}\]

Multiply the x- and y-coordinates of each vertex by
the scale factor, 2.

\[(x, y) \rightarrow (2x, 2y)\]

\[A(-2, -2), \quad A'(4, 4)\]

\[B(1, -1), \quad B'(2, -2)\]

\[C(2, 0), \quad C'(4, 0)\]

Graph \(\triangle ABC\) and its image \(\triangle A'B'C'\).

Exercises

Graph the image of each polygon with the given vertices after a dilation centered
at the origin with the given scale factor.

1. \(E(-2, -2), F(-2, 4), G(2, 4), H(2, -2); \quad r = 0.5\)

2. \(A(0, 0), B(3, 3), C(6, 3), D(6, -3), E(3, -3); \quad r = \frac{1}{3}\)

3. \(A(-2, -2), B(-1, 2), C(2, 1); \quad r = 2\)

4. \(A(2, 2), B(3, 4), C(5, 2); \quad r = 2.5\)
9-6 Practice

Dilations

Use a ruler to draw the image of the figure under a dilation with center \( C \) and the indicated scale factor \( r \).

1. \( r = \frac{3}{2} \)

2. \( r = \frac{2}{3} \)

Determine whether the dilation from \( K \) to \( K' \) is an enlargement or a reduction. Then find the scale factor of the dilation and \( x \).

3.  

4.  

Graph the image of each polygon with the given vertices after a dilation centered at the origin with the given scale factor.

5. \( A(1, 1), C(2, 3), D(4, 2), E(3, 1); r = 0.5 \)

6. \( Q(-1, -1), R(0, 2), S(2, 1); r = \frac{3}{2} \)

7. PHOTOGRAPHY Estebe enlarged a 4-inch by 6-inch photograph by a factor of \( \frac{5}{2} \). What are the new dimensions of the photograph?
1. **CENTERS** Margot superimposed the image of the dilation of a figure on its original figure as shown. Identify the center of this dilation. Explain how you found it.

2. **SCALE FACTORS** Tyrone drew a shape together with one of its dilations on the same coordinate plane as shown.

   What is the scale factor of the dilation?

3. **DILATIONS** Cara is making images for a poster. She wants to thicken the five pointed star shown by dilating it, and then filling in the space between the original and its image. Sketch the dilated image with the indicated center and a scale factor of 1.5.

4. **COORDINATES** Leila drew a polygon with coordinates \((-1, 2), (1, 2), (1, -2),\) and \((-1, -2).\) She then dilated the image and obtained another polygon with coordinates \((-6, 12), (6, 12), (6, -12),\) and \((-6, -12).\) What was the scale factor and center of this dilation?

5. **PLANS** Fred drew the footprint of a stage he was planning to build for his band on a coordinate plane. He decided he wanted to make it smaller because he wanted to make sure it fit at every venue.

   a. Graph the image of Fred’s stage after a dilation centered at \((0, 0)\) with scale factor 0.5.

   b. The perimeter of the image is 26 units. What is the perimeter of the original figure?
10-1 Study Guide

Circles and Circumference

Segments in Circles  A circle consists of all points in a plane that are a given distance, called the radius, from a given point called the center.

A segment or line can intersect a circle in several ways.

- A segment with endpoints that are at the center and on the circle is a radius.
- A segment with endpoints on the circle is a chord.
- A chord that passes through the circle’s center and made up of collinear radii is a diameter.

For a circle that has radius \( r \) and diameter \( d \), the following are true

\[
 r = \frac{d}{2} \quad r = \frac{1}{2} d \quad d = 2r
\]

Example

a. Name the circle.
   The name of the circle is \( \odot O \).

b. Name radii of the circle.
   \( AO, BO, CO, \) and \( DO \) are radii.

c. Name chords of the circle.
   \( AB \) and \( CD \) are chords.

Exercises

For Exercises 1–7, refer to

1. Name the circle.

2. Name radii of the circle.

3. Name chords of the circle.

4. Name diameters of the circle.

5. If \( AB = 18 \) millimeters, find \( AR \).

6. If \( RY = 10 \) inches, find \( AR \) and \( AB \).

7. Is \( AB \cong XY \)? Explain.
Circles and Circumference

Circumference The circumference of a circle is the distance around the circle.

| Circumference | For a circumference of $C$ units and a diameter of $d$ units or a radius of $r$ units, $C = \pi d$ or $C = 2\pi r$ |

**Example**

Find the circumference of the circle to the nearest hundredth.

$C = 2\pi r$

Circumference formula

$= 2\pi(13)$

$r = 13$

$= 26\pi$

Simplify.

$\approx 81.68$

Use a calculator.

The circumference is $26\pi$ or about 81.68 centimeters.

**Exercises**

Find the diameter and radius of a circle with the given circumference. Round to the nearest hundredth.

1. $C = 40$ in.
2. $C = 256$ ft
3. $C = 15.62$ m
4. $C = 9$ cm
5. $C = 79.5$ yd
6. $C = 204.16$ m

Find the exact circumference of each circle using the given inscribed or circumscribed polygon.

7. ![Diagram 1](8 cm, 6 cm)
8. ![Diagram 2](9 in., 9 in.)
9. ![Diagram 3](3 mm, 7 mm)
10. ![Diagram 4](11 yd, 11 yd)
11. ![Diagram 5](5 cm, 12 cm)
12. ![Diagram 6](\sqrt{2}$ cm, \sqrt{2}$ cm)
Circles and Circumference

For Exercises 1–7, refer to $\bigcirc L$.

1. Name the circle.  
2. Name a radius.  
3. Name a chord.  
4. Name a diameter.  
5. Name a radius not drawn as part of a diameter.  
6. Suppose the radius of the circle is 3.5 yards. Find the diameter.  
7. If $RT = 19$ meters, find $LW$.

The diameters of $\bigcirc L$ and $\bigcirc M$ are 20 and 13 units, respectively, and $QR = 4$. Find each measure.

8. $LQ$  
9. $RM$

Find the diameter and radius of a circle with the given circumference. Round to the nearest hundredth.

10. $C = 21.2$ ft  
11. $C = 5.9$ m

Find the exact circumference of each circle using the given inscribed or circumscribed polygon.

12.  
13.  

14. **SUNDIALS** Herman purchased a sundial to use as the centerpiece for a garden. The diameter of the sundial is 9.5 inches.

   a. Find the radius of the sundial.
   
   b. Find the circumference of the sundial to the nearest hundredth.
1. **WHEELS**  Zack is designing wheels for a concept car. The diameter of the wheel is 18 inches. Zack wants to make spokes in the wheel that run from the center of the wheel to the rim. In other words, each spoke is a radius of the wheel. How long are these spokes?

2. **CAKE CUTTING**  Kathy slices through a circular cake. The cake has a diameter of 14 inches. The slice that Kathy made is straight and has a length of 11 inches. Did Kathy cut along a *radius*, a *diameter*, or a *chord* of the circle?

3. **COINS**  Three identical circular coins are lined up in a row as shown.

   The distance between the centers of the first and third coins is 3.2 centimeters. What is the radius of one of these coins?

4. **PLAZAS**  A rectangular plaza has a surrounding circular fence. The diagonals of the rectangle pass from one point on the fence through the center of the circle to another point on the fence.

   Based on the information in the figure, what is the diameter of the fence? Round your answer to the nearest tenth of a foot.

5. **EXERCISE HOOPS**  Taiga wants to make a circular loop that he can twirl around his body for exercise. He will use a tube that is 2.5 meters long.

   a. What will be the diameter of Taiga’s exercise hoop? Round your answer to the nearest thousandth of a meter.

   b. What will be the radius of Taiga’s exercise hoop? Round your answer to the nearest thousandth of a meter.
Measuring Angles and Arcs

Angles and Arcs  A central angle is an angle whose vertex is at the center of a circle and whose sides are radii. A central angle separates a circle into two arcs, a major arc and a minor arc.

Here are some properties of central angles and arcs.

• The sum of the measures of the central angles of a circle with no interior points in common is 360.
• The measure of a minor arc is less than 180 and equal to the measure of its central angle.
• The measure of a major arc is 360 minus the measure of the minor arc.
• The measure of a semicircle is 180.
• Two minor arcs are congruent if and only if their corresponding central angles are congruent.
• The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. (Arc Addition Postulate)

Example  AC is a diameter of \( \odot R \). Find \( m\overline{AB} \) and \( m\overline{ACB} \).

\( \angle ARB \) is a central angle and \( m\angle ARB = 42 \), so \( m\overline{AB} = 42 \). Thus \( m\overline{ACB} = 360 - 42 \) or 318.

Exercises

Find the value of \( x \).

1. \[ \begin{align*} x &\quad 105^\circ \quad 115^\circ \quad 60^\circ \end{align*} \]
2. \[ \begin{align*} x &\quad 120^\circ \end{align*} \]

\( \overline{BD} \) and \( \overline{AC} \) are diameters of \( \odot O \). Identify each arc as a major arc, minor arc, or semicircle of the circle. Then find its measure.

3. \( m\overline{BA} \)  
4. \( m\overline{BC} \)
5. \( m\overline{CD} \)  
6. \( m\overline{ACB} \)
7. \( m\overline{BCD} \)  
8. \( m\overline{AD} \)
Measuring Angles andArcs

Arc Length An arc is part of a circle and its length is a part of the circumference of the circle.

The length of arc $\ell$ can be found using the following equation:

$$\ell = \frac{x}{360} \cdot 2\pi r$$

**Example** Find the length of $\overparen{AB}$. Round to the nearest hundredth.

The length of arc $\overparen{AB}$, can be found using the following equation: $\overparen{AB} = \frac{x}{360} \cdot 2\pi r$

$$\overparen{AB} = \frac{x}{360} \cdot 2\pi r \quad \text{Arc Length Equation}$$

$$\overparen{AB} = \frac{135}{360} \cdot 2\pi(8) \quad \text{Substitution}$$

$$\overparen{AB} \approx 18.85 \text{ in.} \quad \text{Use a calculator.}$$

**Exercises**

Use $\odot O$ to find the length of each arc. Round to the nearest hundredth.

1. $\overparen{DE}$ if the radius is 2 meters
2. $\overparen{DEA}$ if the diameter is 7 inches
3. $\overparen{BC}$ if $BE = 24$ feet
4. $\overparen{CBA}$ if $DO = 3$ millimeters

Use $\odot P$ to find the length of each arc. Round to the nearest hundredth.

5. $\overparen{RT}$, if $MT = 7$ yards
6. $\overparen{MR}$, if $PR = 13$ feet
7. $\overparen{MST}$, if $MP = 2$ inches
8. $\overparen{MRS}$, if $PS = 10$ centimeters
Measuring Angles and Arcs

\( \overline{AC} \) and \( \overline{DB} \) are diameters of \( \bigcirc Q \). Identify each arc as a major arc, minor arc, or semicircle of the circle. Then find its measure.

1. \( m\widehat{AE} \)  
2. \( m\widehat{AB} \)
3. \( m\widehat{EDC} \)  
4. \( m\widehat{ADC} \)
5. \( m\widehat{ABC} \)  
6. \( m\widehat{BC} \)

\( \overline{FH} \) and \( \overline{EG} \) are diameters of \( \bigcirc P \). Find each measure.

7. \( m\widehat{EF} \)  
8. \( m\widehat{DE} \)
9. \( m\widehat{FG} \)  
10. \( m\widehat{DHG} \)
11. \( m\widehat{DFG} \)  
12. \( m\widehat{DGE} \)

Use \( \bigcirc Z \) to find each arc length. Round to the nearest hundredth.

13. \( \widehat{QPT} \), if \( OZ = 10 \) inches
14. \( \widehat{QR} \), if \( PZ = 12 \) feet
15. \( \widehat{PQR} \), if \( TR = 15 \) meters
16. \( \widehat{QPS} \), if \( ZQ = 7 \) centimeters

17. HOMEWORK Refer to the table, which shows the number of hours students at Leland High School say they spend on homework each night.

<table>
<thead>
<tr>
<th>Category</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 1 hour</td>
<td>8%</td>
</tr>
<tr>
<td>1–2 hours</td>
<td>29%</td>
</tr>
<tr>
<td>2–3 hours</td>
<td>58%</td>
</tr>
<tr>
<td>3–4 hours</td>
<td>3%</td>
</tr>
<tr>
<td>Over 4 hours</td>
<td>2%</td>
</tr>
</tbody>
</table>

a. If you were to construct a circle graph of the data, how many degrees would be allotted to each category?

b. Describe the arcs associated with each category.
1. **CONDIMENTS** A number of people in a park were asked to name their favorite condiment for hot dogs. The results are shown in the circle graph.

   - Ketchup: 198º
   - Mustard: 111.9º
   - Mayonnaise: 16.1º
   - Relish: 29.4º
   - Other: 4.6º

   What was the second most popular hot dog condiment?

2. **CLOCKS** Shiatsu is a Japanese massage technique. One of the beliefs is that various body functions are most active at various times during the day. To illustrate this, they use a Chinese clock that is based on a circle divided into 12 equal sections by radii.

   - PC: 7 PM - 9 PM
   - KID: 5 PM - 7 PM
   - ST: 7 AM - 9 AM
   - BL: 3 PM - 5 PM
   - SP: 9 AM - 11 AM
   - LU: 3 AM - 5 AM
   - GB: 11 PM - 1 AM
   - SI: 1 PM - 3 PM
   - HT: 11 AM - 1 PM
   - LIV: 1 AM - 3 AM
   - LI: 11 AM - 1 AM
   - TW: 9 PM - 11 PM

   What is the measure of any one of the 12 equal central angles?

3. **PIES** Yolanda has divided a circular apple pie into 4 slices by cutting the pie along 4 radii. The central angles of the 4 slices are $3x$, $6x - 10$, $4x + 10$, and $5x$ degrees. What exactly are the numerical measures of the central angles?

4. **RIBBONS** Cora is wrapping a ribbon around a cylinder-shaped gift box. The box has a diameter of 15 inches and the ribbon is 60 inches long. Cora is able to wrap the ribbon all the way around the box once, and then continue so that the second end of the ribbon passes the first end. What is the central angle formed between the ends of the ribbon? Round your answer to the nearest tenth of a degree.

5. **BIKE WHEELS** Lucy has to buy a new wheel for her bike. The bike wheel has a diameter of 20 inches.

   a. If Lucy rolls the wheel one complete rotation along the ground, how far will the wheel travel? Round your answer to the nearest hundredth of an inch.

   b. If the bike wheel is rolled along the ground so that it rotates 45º, how far will the wheel travel? Round your answer to the nearest hundredth of an inch.

   c. If the bike wheel is rolled along the ground for 10 inches, through what angle does the wheel rotate? Round your answer to the nearest tenth of a degree.
Arcs and Chords Points on a circle determine both chords and arcs. Several properties are related to points on a circle.

In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

Example In \( \odot K \), \( \overline{AB} \cong \overline{CD} \). Find \( AB \).

\[ \overline{AB} \] and \( \overline{CD} \) are congruent arcs, so the corresponding chords \( \overline{AB} \) and \( \overline{CD} \) are congruent.

\[ AB = CD \]

Definition of congruent segments

\[ 8x = 2x + 3 \]

Substitution

\[ x = \frac{1}{2} \]

Simplify.

So, \( AB = 8 \left( \frac{1}{2} \right) \) or 4.

Exercises

ALGEBRA Find the value of \( x \) in each circle.

1. \[ S \]
   \[ R \]
   \[ 64^\circ \]
   \[ T \]

2. \[ D \]
   \[ F \]
   \[ 116^\circ \]

3. \[ 82^\circ \]
   \[ K \]
   \[ M \]

4. \[ B \]
   \[ 90^\circ \]
   \[ A \]
   \[ 9 \]

5. \[ 2x + 4 \]
   \[ 18 \]

6. \[ J \]
   \[ 115^\circ \]
   \[ H \]
   \[ I \]
   \[ 115^\circ \]

7. \[ P \]
   \[ (2x + 4)^\circ \]
   \[ M \]
   \[ N \]
   \[ (3x + 2)^\circ \]

8. \( \odot M \equiv \odot P \)

9. \( \odot V \equiv \odot W \)
Diameters and Chords

- In a circle, if a diameter (or radius) is perpendicular to a chord, then it bisects the chord and its arc.
- In a circle, the perpendicular bisector of a chord is the diameter (or radius).
- In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

In $\odot O$, $CD \perp OE$, $OD = 15$, and $CD = 24$. Find $OE$.

A diameter or radius perpendicular to a chord bisects the chord, so $ED$ is half of $CD$.

$ED = \frac{1}{2}(24) = 12$

Use the Pythagorean Theorem to find $x$ in $\triangle OED$.

$(OE)^2 + (ED)^2 = (OD)^2$  \hspace{1cm} \text{Pythagorean Theorem}

$(OE)^2 + 12^2 = 15^2$  \hspace{1cm} \text{Substitution}

$(OE)^2 + 144 = 225$  \hspace{1cm} \text{Simplify}

$(OE)^2 = 81$  \hspace{1cm} \text{Subtract 144 from each side.}

$OE = 9$  \hspace{1cm} \text{Take the positive square root of each side.}

Exercises

In $\odot P$, the radius is 13 and $RS = 24$. Find each measure. Round to the nearest hundredth.
1. $RT$
2. $PT$
3. $TQ$

In $\odot A$, the diameter is 12, $CD = 8$, and $m\overline{CD} = 90$. Find each measure. Round to the nearest hundredth.
4. $m\overline{DE}$
5. $FD$
6. $AF$

7. In $\odot R$, $TS = 21$ and $UV = 3x$. What is $x$?
8. In $\odot Q$, $\overline{CD} \cong \overline{CB}$, $GQ = x + 5$ and $EQ = 3x - 6$. What is $x$?
10-3 Practice

Arrows and Chords

ALGEBRA Find the value of $x$ in each circle.

1. \[ \begin{align*}
    N & \quad 38 \\
    M & \quad 4x + 10 \\
    Q & \\
    P & \\
    \end{align*} \]

2. \[ \begin{align*}
    K & \quad x^2 \\
    J & \quad 70^\circ \\
    L & \\
    \end{align*} \]

3. \[ \begin{align*}
    A & \quad 109^\circ \\
    B & \quad \frac{3x+2}{5x-7} \\
    C & \quad 109^\circ \\
    \end{align*} \]

4. \[ \begin{align*}
    \odot R & \equiv \odot S \\
    R & \\
    S & \\
    \end{align*} \]

The radius of \( \odot N \) is 18, \( NK = 9 \), and \( m\overline{DE} = 120 \). Find each measure.

5. \( m\overline{GE} \)

6. \( m\angle HNE \)

7. \( m\angle HEN \)

8. \( HN \)

9. In \( \odot P \), \( QR = 7x - 20 \) and \( TS = 3x \). What is \( x \)?

10. In \( \odot K \), \( JL \cong LM \), \( KN = 3x - 2 \), and \( KP = 2x + 1 \). What is \( x \)?

11. GARDEN PATHS A circular garden has paths around its edge that are identified by the given arc measures. It also has four straight paths, identified by segments \( \overline{AC}, \overline{AD}, \overline{BE}, \) and \( \overline{DE} \), that cut through the garden’s interior. Which two straight paths have the same length?
1. **HEXAGON** A hexagon is constructed as shown in the figure.

How many different chord lengths occur as side lengths of the hexagon?

2. **WATERMARKS** For security purposes, a jewelry company prints a hidden watermark on the logo of all its official documents. The watermark is a chord located 0.7 cm from the center of a circular ring that has a 2.5 cm radius. To the nearest tenth, what is the length of the chord?

3. **ARCHAEOLOGY** Only one piece of a broken plate is found during an archaeological dig. Use the sketch of the pottery piece below to demonstrate how constructions with chords and perpendicular bisectors can be used to draw the plate’s original size.

4. **CENTERS** Neil wants to find the center of a large circle. He draws what he thinks is a diameter of the circle and then marks its midpoint and declares that he has found the center. His teacher asks Neil how he knows that the line he drew is the diameter of the circle and not a smaller chord. Neil realizes that he does not know for sure. What can Neil do to determine if it is an actual diameter?

5. **QUILTING** Miranda is following directions for a quilt pattern “In a 10-inch diameter circle, measure 3 inches from the center of the circle and mark a chord $AB$ perpendicular to the radius of the circle. Then cut along the chord.” Miranda is to repeat this for another chord, $CD$. Finally, she is to cut along chord $DB$ and $AC$. The result should be four curved pieces and one quadrilateral.

   **a.** If Miranda follows the directions, is she guaranteed that the resulting quadrilateral is a rectangle? Explain.

   **b.** Assume the resulting quadrilateral is a rectangle. One of the curved pieces has an arc measure of 74. What are the measures of the arcs on the other three curved pieces?
Inscribed Angles

**Inscribed Angles** An inscribed angle is an angle whose vertex is on a circle and whose sides contain chords of the circle. In \( \odot G \), minor arc \( 
\overarc{DF} \) is the **intercepted arc** for inscribed angle \( \angle DEF \).

| Inscribed Angle Theorem | If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc. |

If two inscribed angles intercept the same arc or congruent arcs, then the angles are congruent.

**Example** In \( \odot G \) above, \( m\overarc{DF} = 90 \). Find \( m\angle DEF \).

\( \angle DEF \) is an inscribed angle so its measure is half of the intercepted arc.

\[
m\angle DEF = \frac{1}{2} m\overarc{DF}
\]

\[
= \frac{1}{2} (90) \text{ or } 45
\]

**Exercises**

Find each measure.

1. \( m\overarc{AC} \)

![Diagram](image1)

2. \( m\angle N \)

![Diagram](image2)

3. \( m\angle QSR \)

![Diagram](image3)

**ALGEBRA** Find each measure.

4. \( m\angle U \)

![Diagram](image4)

5. \( m\angle T \)

![Diagram](image5)

6. \( m\angle A \)

![Diagram](image6)

7. \( m\angle C \)

![Diagram](image7)
### Inscribed Angles

**Angles of Inscribed Polygons** An **inscribed polygon** is one whose sides are chords of a circle and whose vertices are points on the circle. Inscribed polygons have several properties.

- An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle.
- If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

#### Example

*Find $m\angle K$.*

KL $\cong$ KM, so KL = KM. The triangle is an isosceles triangle, therefore $m\angle L = m\angle M = 3x + 5$.

$$m\angle L + m\angle M + m\angle K = 180$$

Substituting $3x + 5$ for both $m\angle L$ and $m\angle M$:

$$(3x + 5) + (3x + 5) + (5x + 5) = 180$$

Simplifying:

$$11x + 15 = 180$$

Subtract 15 from each side:

$$11x = 165$$

Divide each side by 11:

$$x = 15$$

So, $m\angle K = 5(15) + 5 = 80$.

### Exercises

**ALGEBRA** Find each measure.

1. $x$

2. $m\angle W$

3. $x$

4. $m\angle T$

5. $m\angle R$

6. $m\angle S$

7. $m\angle W$

8. $m\angle X$
Find each measure.

1. $m\widehat{AB}$

2. $m\angle X$

3. $m\widehat{JK}$

4. $m\angle Q$

ALGEBRA Find each measure.

5. $m\angle W$

6. $m\angle Y$

7. $m\angle A$

8. $m\angle D$

ALGEBRA Find each measure.

9. $m\angle A$

10. $m\angle C$

11. $m\angle G$

12. $m\angle H$

13. PROBABILITY In $\odot V$, point $C$ is randomly located so that it does not coincide with points $R$ or $S$. If $m\widehat{RS} = 140$, what is the probability that $m\angle RCS = 70$?
1. **ARENA** A circus arena is lit by five lights equally spaced around the perimeter.

What is \( m \angle 1 \)?

2. **FIELD OF VIEW** The figure shows a top view of two people in front of a very tall rectangular wall. The wall makes a chord of a circle that passes through both people.

Which person has more of their horizontal field of vision blocked by the wall?

3. **RHOMBI** Paul is interested in circumscribing a circle around a rhombus that is not a square. He is having great difficulty doing so. Can you help him? Explain.

4. **STREETS** Three kilometers separate the intersections of Cross and Upton and Cross and Hope.

What is the distance between the intersection of Upton and Hope and the point midway between the intersections of Upton and Cross and Cross and Hope?

5. **INSCRIBED HEXAGONS** You will prove that the sum of the measures of alternate interior angles in an inscribed hexagon is 360.

a. How are \( \angle A \) and \( \angle BCF \) related? Similarly, how are \( \angle E \) and \( \angle DCF \) related?

b. Show that \( m \angle A + m \angle BCD + m \angle E = 360 \).
**Equations of Circles**

**Equation of a Circle**  A circle is the locus of points in a plane equidistant from a given point. You can use this definition to write an equation of a circle.

| Standard Equation of a Circle | An equation for a circle with center at \((h, k)\) and a radius of \(r\) units is \((x - h)^2 + (y - k)^2 = r^2\). |

**Example**  Write an equation for a circle with center \((-1, 3)\) and radius 6.

Use the formula \((x - h)^2 + (y - k)^2 = r^2\) with \(h = -1, k = 3,\) and \(r = 6\).

\[
(x - (-1))^2 + (y - 3)^2 = 6^2
\]

Substitution

\[
(x + 1)^2 + (y - 3)^2 = 36
\]

Simplify.

**Exercises**

Write the equation of each circle.

1. center at \((0, 0)\), radius 8  
2. center at \((-2, 3)\), radius 5  
3. center at \((2, -4)\), radius 1  
4. center at \((-1, -4)\), radius 2  
5. center at \((-2, -6)\), diameter 8  
6. center at origin, diameter 4  
7. center at \((3, -4)\), passes through \((-1, -4)\)  
8. center at \((0, 3)\), passes through \((2, 0)\)  

9.  

10.  

---

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**Graph Circles** If you are given an equation of a circle, you can find information to help you graph the circle.

**Example** Graph \((x + 3)^2 + (y + 1)^2 = 9\).

Use the parts of the equation to find \((h, k)\) and \(r\).

Rewrite \((x + 3)^2 + (y + 1)^2 = 9\) to find the center and the radius.

\[
\begin{align*}
(x - h)^2 + (y - k)^2 &= r^2 \\
\end{align*}
\]

So \(h = -3, k = 1,\) and \(r = 3\). The center is at \((-3, 1)\) and the radius is 3.

**Exercises**

For each circle with the given equation, state the coordinates of the center and the measure of the radius. Then graph the equation.

1. \(x^2 + y^2 = 16\)

2. \((x - 2)^2 + (y - 1)^2 = 9\)

3. \((x + 2)^2 + y^2 = 16\)

4. \(x^2 + (y - 1)^2 = 9\)

Write an equation of a circle that contains each set of points. Then graph the circle.

5. \(F(-2, 2), G(-1, 1), H(-1, 3)\)

6. \(R(-2, 1), S(-4, -1), T(0, -1)\)
10-8 Practice

Equations of Circles

Write the equation of each circle.

1. center at origin, radius 7
2. center at (0, 0), diameter 18

3. center at (−7, 11), radius 8
4. center at (12, −9), diameter 22

5. center at (−1, 8), passes through (9, 3)
6. center at (−3, −3), passes through (−2, 3)

For each circle with the given equation, state the coordinates of the center and the measure of the radius. Then graph the equation.

7. \(x^2 + y^2 = 4\)

8. \((x + 3)^2 + (y − 3)^2 = 9\)

Write an equation of a circle that contains each set of points. Then graph the circle.

9. \(A(−2, 2), B(2, −2), C(6, 2)\)

10. \(R(5, 0), S(−5, 0), T(0, −5)\)

11. EARTHQUAKES When an earthquake strikes, it releases seismic waves that travel in concentric circles from the epicenter of the earthquake. Seismograph stations monitor seismic activity and record the intensity and duration of earthquakes. Suppose a station determines that the epicenter of an earthquake is located about 50 kilometers from the station. If the station is located at the origin, write an equation for the circle that represents one of the concentric circles of seismic waves of the earthquake.
10-8 Word Problem Practice

Equations of Circles

1. **DESIGN** Arthur wants to write the equation of a circle that is inscribed in the square shown in the graph.

![Graph of a square]

What is the equation of the desired circle?

2. **DRAFTING** The design for a park is drawn on a coordinate graph. The perimeter of the park is modeled by the equation \((x - 3)^2 + (x - 7)^2 = 225\). Each unit on the graph represents 10 feet. What is the radius of the actual park?

3. **WALLPAPER** The design of a piece of wallpaper consists of circles that can be modeled by the equation \((x - a)^2 + (y - b)^2 = 4\), for all even integers \(b\). Sketch part of the wallpaper on a grid.

4. **SECURITY RING** A circular safety ring surrounds a top-secret laboratory. On one map of the laboratory grounds, the safety ring is given by the equation \((x - 8)^2 + (y + 2)^2 = 324\). Each unit on the map represents 1 mile. What is the radius of the safety ring?

5. **DISTANCE** Cleo lives the same distance from the library, the post office, and her school. The table below gives the coordinates of these places on a map with a coordinate grid where one unit represents one yard.

<table>
<thead>
<tr>
<th>Location</th>
<th>Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Library</td>
<td>(–78, 202)</td>
</tr>
<tr>
<td>Post Office</td>
<td>(111, 193)</td>
</tr>
<tr>
<td>School</td>
<td>(202, –106)</td>
</tr>
</tbody>
</table>

a. What are the coordinates of Cleo’s home? Sketch the circle on a map locating all three places and Cleo’s home.

b. How far is Cleo’s house from the places mentioned?

c. Write an equation for the circle that passes through the library, post office, and school.
Lateral and Surface Areas of Prisms  In a solid figure, faces that are not bases are lateral faces. The lateral area is the sum of the area of the lateral faces. The surface area is the sum of the lateral area and the area of the bases.

| Lateral Area of a Prism | If a prism has a lateral area of $L$ square units, a height of $h$ units, and each base has a perimeter of $P$ units, then $L = Ph$. |
| Surface Area of a Prism | If a prism has a surface area of $S$ square units, a lateral area of $L$ square units, and each base has an area of $B$ square units, then $S = L + 2B$ or $S = Ph + 2B$. |

**Example**  Find the lateral and surface area of the regular pentagonal prism above if each base has a perimeter of 75 centimeters and the height is 10 centimeters.

$$L = Ph$$
$$= 75(10)$$
$$= 750$$

Multiply.

$$S = L + 2B$$
$$= 750 + 2 \left( \frac{1}{2} aP \right)$$
$$= 750 + \left( \frac{7.5}{\tan 36^\circ} \right)(75)$$
$$\approx 1524.2$$

The lateral area is 750 square centimeters and the surface area is about 1524.2 square centimeters.

**Exercises**

Find the lateral area and surface area of each prism. Round to the nearest tenth if necessary.

1. 

2. 

3. 

4. 

5. 

6. 
Lateral and Surface Areas of Cylinders  A \textit{cylinder} is a solid with bases that are congruent circles lying in parallel planes. The \textit{axis} of a cylinder is the segment with endpoints at the centers of these circles. For a \textit{right cylinder}, the axis is also the altitude of the cylinder.

\begin{tabular}{|l|l|}
\hline
Lateral Area of a Cylinder & If a cylinder has a lateral area of $L$ square units, a height of $h$ units, and a base has a radius of $r$ units, then $L = 2\pi rh$. \\
\hline
Surface Area of a Cylinder & If a cylinder has a surface area of $S$ square units, a height of $h$ units, and a base has a radius of $r$ units, then $S = L + 2B$ or $2\pi rh + 2\pi r^2$. \\
\hline
\end{tabular}

\textbf{Example}  Find the lateral and surface area of the cylinder. Round to the nearest tenth.

If $d = 12$ cm, then $r = 6$ cm.

\begin{align*}
L &= 2\pi rh \\
&= 2\pi(6)(14) \\
&= 527.8 \\
&\approx \text{Use a calculator.}
\end{align*}

\begin{align*}
S &= 2\pi rh + 2\pi r^2 \\
&\approx 527.8 + 2\pi(6)^2 \\
&\approx 754.0 \\
&\approx \text{Use a calculator.}
\end{align*}

The lateral area is about 527.8 square centimeters and the surface area is about 754.0 square centimeters.

\textbf{Exercises}  Find the lateral area and surface area of each cylinder. Round to the nearest tenth.

1. \hspace{2cm} 4 cm  \\
   \hspace{5cm} 12 cm

2. \hspace{2cm} 10 in.  \\
   \hspace{5cm} 6 in.

3. \hspace{2cm} 3 cm  \\
   \hspace{5cm} 3 cm  \\
   \hspace{5cm} 6 cm

4. \hspace{2cm} 8 cm  \\
   \hspace{5cm} 20 cm

5. \hspace{2cm} 12 m  \\
   \hspace{5cm} 4 m

6. \hspace{2cm} 2 m  \\
   \hspace{5cm} 1 m
12-2 Practice

Surface Areas of Prisms

Find the lateral and surface area of each prism. Round to the nearest tenth if necessary.

1. 

2. 

3. 

4. 

Find the lateral area and surface area of each cylinder. Round to the nearest tenth.

5. 

6. 

7. 

8.
1. LOGOS The Z company specializes in caring for zebras. They want to make a 3-dimensional “Z” to put in front of their company headquarters. The “Z” is 15 inches thick and the perimeter of the base is 390 inches.

What is the lateral surface area of this “Z”?

2. STAIRWELLS Management decides to enclose stairs connecting the first and second floors of a parking garage in a stairwell shaped like an oblique rectangular prism.

What is the lateral surface area of the stairwell?

3. CAKES A cake is a rectangular prism with height 4 inches and base 12 inches by 15 inches. Wallace wants to apply frosting to the sides and the top of the cake. What is the surface area of the part of the cake that will have frosting?

4. EXHAUST PIPES An exhaust pipe is shaped like a cylinder with a height of 50 inches and a radius of 2 inches. What is the lateral surface area of the exhaust pipe? Round your answer to the nearest hundredth.

5. TOWERS A circular tower is made by placing one cylinder on top of another. Both cylinders have a height of 18 inches. The top cylinder has a radius of 18 inches and the bottom cylinder has a radius of 36 inches.

a. What is the total surface area of the tower? Round your answer to the nearest hundredth.

b. Another tower is constructed by placing the original tower on top of another cylinder with a height of 18 inches and a radius of 54 inches. What is the total surface area of the new tower? Round your answer to the nearest hundredth.
Surface Areas of Pyramids and Cones

Lateral and Surface Areas of Pyramids

A **pyramid** is a solid with a polygon base. The lateral faces intersect in a common point known as the vertex. The altitude is the segment from the vertex that is perpendicular to the base. For a **regular pyramid**, the base is a regular polygon and the altitude has an endpoint at the center of the base. All the lateral edges are congruent and all the lateral faces are congruent isosceles triangles. The height of each lateral face is called the **slant height**.

| Lateral Area of a Regular Pyramid | The lateral area $L$ of a regular pyramid is $L = \frac{1}{2}P\ell$, where $\ell$ is the slant height and $P$ is the perimeter of the base. |
| Surface Area of a Regular Pyramid | The surface area $S$ of a regular pyramid is $S = \frac{1}{2}P\ell + B$, where $\ell$ is the slant height, $P$ is the perimeter of the base, and $B$ is the area of the base. |

**Example**

For the regular square pyramid above, find the lateral area and surface area if the length of a side of the base is 12 centimeters and the height is 8 centimeters. Round to the nearest tenth if necessary.

Find the slant height.

\[
\ell^2 = 6^2 + 8^2
\]
\[
\ell^2 = 100
\]
\[
\ell = 10
\]

\[
L = \frac{1}{2}P\ell = \frac{1}{2}(48)(10) = 240
\]

\[
S = \frac{1}{2}P\ell + B = 240 + 144 = 384
\]

The lateral area is 240 square centimeters, and the surface area is 384 square centimeters.

**Exercises**

Find the lateral area and surface area of each regular pyramid. Round to the nearest tenth if necessary.

1. \[ \text{20 cm} \]

2. \[ \text{8 ft} \]

3. \[ \text{10 cm} \]

4. \[ \text{8.7 in.} \]

---

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Surface Areas of Pyramids and Cones

Lateral and Surface Areas of Cones  A cone has a circular base and a vertex. The axis of the cone is the segment with endpoints at the vertex and the center of the base. If the axis is also the altitude, then the cone is a right cone. If the axis is not the altitude, then the cone is an oblique cone.

<table>
<thead>
<tr>
<th>Lateral Area of a Cone</th>
<th>The lateral area ( L ) of a right circular cone is ( L = \pi r \ell ), where ( r ) is the radius and ( \ell ) is the slant height.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface Area of a Cone</td>
<td>The surface area ( S ) of a right cone is ( S = \pi r \ell + \pi r^2 ), where ( r ) is the radius and ( \ell ) is the slant height.</td>
</tr>
</tbody>
</table>

**Example**  For the right cone above, find the lateral area and surface area if the radius is 6 centimeters and the height is 8 centimeters. Round to the nearest tenth if necessary.

Find the slant height.

\[
\ell^2 = 6^2 + 8^2
\]

Pythagorean Theorem

\[
\ell^2 = 100
\]

Simplify.

\[
\ell = 10
\]

Take the positive square root of each side.

Lateral area of a right cone

\[
L = \pi rl
\]

\[
= \pi(6)(10)
\]

\[
= 188.5
\]

Simplify.

Surface area of a right cone

\[
S = \pi rl + \pi r^2
\]

\[
= 188.5 + \pi(6^2)
\]

\[
\approx 301.6
\]

Simplify.

The lateral area is about 188.5 square centimeters and the surface area is about 301.6 square centimeters.

**Exercises**

Find the lateral area and surface area of each cone. Round to the nearest tenth if necessary.

1. 
   ![Diagram 1](image1.png)

2. 
   ![Diagram 2](image2.png)

3. 
   ![Diagram 3](image3.png)

4. 
   ![Diagram 4](image4.png)
12-3 Practice

Surface Areas of Pyramids and Cones

Find the lateral area and surface area of each regular pyramid. Round to the nearest tenth if necessary.

1. 

![Pyramid 1]

2. 

![Pyramid 2]

3. 

![Pyramid 3]

4. 

![Pyramid 4]

Find the lateral area and surface area of each cone. Round to the nearest tenth if necessary.

5. 

![Cone 1]

6. 

![Cone 2]

7. Find the surface area of a cone if the height is 14 centimeters and the slant height is 16.4 centimeters.

8. Find the surface area of a cone if the height is 12 inches and the diameter is 27 inches.

9. GAZEBOS The roof of a gazebo is a regular octagonal pyramid. If the base of the pyramid has sides of 0.5 meter and the slant height of the roof is 1.9 meters, find the area of the roof.

10. HATS Cuong bought a conical hat on a recent trip to central Vietnam. The basic frame of the hat is 16 hoops of bamboo that gradually diminish in size. The hat is covered in palm leaves. If the hat has a diameter of 50 centimeters and a slant height of 32 centimeters, what is the lateral area of the conical hat?
12-3 Word Problem Practice

Surface Areas of Pyramids and Cones

1. **PAPER MODELS** Patrick is making a paper model of a castle. Part of the model involves cutting out the net shown and folding it into a pyramid. The pyramid has a square base. What is the lateral surface area of the resulting pyramid?

2. **TETRAHEDRON** Sung Li builds a paper model of a regular tetrahedron, a pyramid with an equilateral triangle for the base and three equilateral triangles for the lateral faces. One of the faces of the tetrahedron has an area of 17 square inches. What is the total surface area of the tetrahedron?

3. **PAPERWEIGHTS** Daphne uses a paperweight shaped like a pyramid with a regular hexagon for a base. The side length of the regular hexagon is 1 inch. The altitude of the pyramid is 2 inches. What is the lateral surface area of this pyramid? Round your answers to the nearest hundredth.

4. **SPRAY PAINT** A can of spray paint shoots out paint in a cone shaped mist. The lateral surface area of the cone is $65\pi$ square inches when the can is held 12 inches from a canvas. What is the area of the part of the canvas that gets sprayed with paint? Round your answer to the nearest hundredth.

5. **MEGAPHONES** A megaphone is formed by taking a cone with a radius of 20 centimeters and an altitude of 60 centimeters and cutting off the tip. The cut is made along a plane that is perpendicular to the axis of the cone and intersects the axis 12 centimeters from the vertex. Round your answers to the nearest hundredth.

   a. What is the lateral surface area of the original cone?

   b. What is the lateral surface area of the tip that is removed?

   c. What is the lateral surface area of the megaphone?
Volumes of Prisms and Cylinders

Volumes of Prisms  The measure of the amount of space that a three-dimensional figure encloses is the volume of the figure. Volume is measured in units such as cubic feet, cubic yards, or cubic meters. One cubic unit is the volume of a cube that measures one unit on each edge.

<table>
<thead>
<tr>
<th>Volume of a Prism</th>
<th>If a prism has a volume of ( V ) cubic units, a height of ( h ) units, and each base has an area of ( B ) square units, then ( V = Bh ).</th>
</tr>
</thead>
</table>

Example 1  Find the volume of the prism.

\[ V = Bh \]
\[ = (7)(3)(4) \]
\[ = 84 \]

The volume of the prism is 84 cubic centimeters.

Example 2  Find the volume of the prism if the area of each base is 6.3 square feet.

\[ V = Bh \]
\[ = (6.3)(3.5) \]
\[ = 22.05 \]

The volume is 22.05 cubic feet.

Exercises  Find the volume of each prism.

1. 

2. 

3. 

4. 

5. 

6.
Volumes of Prisms and Cylinders

Volumes of Cylinders  The volume of a cylinder is the product of the height and the area of the base. When a solid is not a right solid, use Cavalieri’s Principle to find the volume. The principle states that if two solids have the same height and the same cross sectional area at every level, then they have the same volume.

| Volume of a Cylinder | If a cylinder has a volume of \( V \) cubic units, a height of \( h \) units, and the bases have a radius of \( r \) units, then \( V = \pi r^2 h \). |

**Example 1** Find the volume of the cylinder.

\[
V = \pi r^2 h \\
= \pi (3)^2 (4) \\
= \pi (9)(4) \\
= 113.1
\]

The volume is about 113.1 cubic centimeters.

**Example 2** Find the volume of the oblique cylinder.

Use the Pythagorean Theorem to find the height of the cylinder.

\[
h^2 + 5^2 = 13^2 \\
h^2 = 144 \\
h = 12
\]

Volume of a cylinder

\[
V = \pi r^2 h \\
= \pi (4)^2 (12) \\
= \pi (16)(12) \\
\approx 603.2
\]

The volume is about 603.2 cubic inches.

**Exercises**

Find the volume of each cylinder. Round to the nearest tenth.

1.  

![Cylinder 1](image1.png)

2.  

![Cylinder 2](image2.png)

3.  

![Cylinder 3](image3.png)

4.  

![Cylinder 4](image4.png)

5.  

![Cylinder 5](image5.png)

6.  

![Cylinder 6](image6.png)
Volumes of Prisms and Cylinders

Find the volume of each prism or cylinder. Round to the nearest tenth if necessary.

1. \( \text{26 m} \times 10 \text{ m} \times 17 \text{ m} \)

2. \( \text{5 in.} \times 9 \text{ in.} \times 5 \text{ in.} \)

3. \( \text{16 mm} \times 17.5 \text{ mm} \times \)

4. \( \text{7 ft} \times 25 \text{ ft} \times \)

5. \( \text{10 yd} \times 20 \text{ yd} \times 13 \text{ yd} \)

6. \( \text{8 cm} \times 30 \text{ cm} \times \)

7. **AQUARIUM** Mr. Gutierrez purchased a cylindrical aquarium for his office. The aquarium has a height of \( 25 \frac{1}{2} \) inches and a radius of 21 inches.

a. What is the volume of the aquarium in cubic feet?

b. If there are 7.48 gallons in a cubic foot, how many gallons of water does the aquarium hold?

c. If a cubic foot of water weighs about 62.4 pounds, what is the weight of the water in the aquarium to the nearest five pounds?
1. TRASH CANS The Meyer family uses a kitchen trash can shaped like a cylinder. 
   It has a height of 18 inches and a base diameter of 12 inches. 
   What is the volume of the trash can? Round your answer to the nearest tenth of a cubic inch.

2. BENCH Inside a lobby, there is a piece of furniture for sitting. The furniture is shaped like a simple block with a square base 6 feet on each side and a height of \(1\frac{3}{5}\) feet.

   What is the volume of the seat?

3. FRAMES Margaret makes a square frame out of four pieces of wood. Each piece of wood is a rectangular prism with a length of 40 centimeters, a height of 4 centimeters, and a depth of 6 centimeters. 
   What is the total volume of the wood used in the frame?

4. PENCIL GRIPS A pencil grip is shaped like a triangular prism with a cylinder removed from the middle. The base of the prism is a right isosceles triangle with leg lengths of 2 centimeters. The diameter of the base of the removed cylinder is 1 centimeter. The heights of the prism and the cylinder are the same, and equal to 4 centimeters.

   What is the exact volume of the pencil grip?

5. TUNNELS Construction workers are digging a tunnel through a mountain. 
   The space inside the tunnel is going to be shaped like a rectangular prism. The mouth of the tunnel will be a rectangle 20 feet high and 50 feet wide and the length of the tunnel will be 900 feet.

   a. What will the volume of the tunnel be?

   b. If instead of a rectangular shape, the tunnel had a semicircular shape with a 50-foot diameter, what would be its volume? Round your answer to the nearest cubic foot.
Volumes of Pyramids and Cones

Volumes of Pyramids This figure shows a prism and a pyramid that have the same base and the same height. It is clear that the volume of the pyramid is less than the volume of the prism. More specifically, the volume of the pyramid is one-third of the volume of the prism.

| Volume of a Pyramid | If a pyramid has a volume of $V$ cubic units, a height of $h$ units, and a base with an area of $B$ square units, then $V = \frac{1}{3}Bh$. |

Example

Find the volume of the square pyramid.

$V = \frac{1}{3}Bh$

$= \frac{1}{3}(8)(8)10$

$= 213.3$

The volume is about 213.3 cubic feet.

Exercises

Find the volume of each pyramid. Round to the nearest tenth if necessary.

1. 

2. 

3. 

4. 

5. 

6.
Volumes of Pyramids and Cones

Volumes of Cones For a cone, the volume is one-third the product of the height and the area of the base. The base of a cone is a circle, so the area of the base is \( \pi r^2 \).

<table>
<thead>
<tr>
<th>Volume of a Cone</th>
<th>If a cone has a volume of V cubic units, a height of h units, and the bases have a radius of r units, then ( V = \frac{1}{3} \pi r^2 h ).</th>
</tr>
</thead>
</table>

**Example** Find the volume of the cone.

\[
V = \frac{1}{3} \pi r^2 h \quad \text{Volume of a cone}
\]

\[
= \frac{1}{3} \pi (5)^2 12 \quad r = 5, h = 12
\]

\[
\approx 314.2 \quad \text{Simplify}
\]

The volume of the cone is about 314.2 cubic centimeters.

**Exercises**

Find the volume of each cone. Round to the nearest tenth.

1. \( \text{Radius} = 6 \text{ cm, Height} = 10 \text{ cm} \)

2. \( \text{Radius} = 4 \text{ ft, Height} = 8 \text{ ft} \)

3. \( \text{Radius} = 12 \text{ in, Height} = 30 \text{ in} \)

4. \( \text{Base angle} = 45^\circ, \text{Height} = 18 \text{ yd} \)

5. \( \text{Radius} = 20 \text{ ft, Height} = 26 \text{ ft} \)

6. \( \text{Radius} = 16 \text{ cm, Height} = 45^\circ \)
Volumes of Pyramids and Cones

Find the volume of each pyramid or cone. Round to the nearest tenth if necessary.

1. 

2. 

3. 

4. 

5. 

6. 

7. CONSTRUCTION Mr. Ganty built a conical storage shed. The base of the shed is 4 meters in diameter and the height of the shed is 3.8 meters. What is the volume of the shed?

8. HISTORY The start of the pyramid age began with King Zoser’s pyramid, erected in the 27th century B.C. In its original state, it stood 62 meters high with a rectangular base that measured 140 meters by 118 meters. Find the volume of the original pyramid.
1. **ICE CREAM DISHES** The part of a dish designed for ice cream is shaped like an upside-down cone. The base of the cone has a radius of 2 inches and the height is 1.2 inches.

What is the volume of the cone? Round your answer to the nearest hundredth.

2. **GREENHOUSES** A greenhouse has the shape of a square pyramid. The base has a side length of 30 yards. The height of the greenhouse is 18 yards.

What is the volume of the greenhouse?

3. **TEEPEE** Caitlyn made a teepee for a class project. Her teepee had a diameter of 6 feet. The angle the side of the teepee made with the ground was 65°.

What was the volume of the teepee? Round your answer to the nearest hundredth.

4. **SCULPTING** A sculptor wants to remove stone from a cylindrical block 3 feet high and turn it into a cone. The diameter of the base of the cone and cylinder is 2 feet.

What is the volume of the stone that the sculptor must remove? Round your answer to the nearest hundredth.

5. **STAGES** A stage has the form of a square pyramid with the top sliced off along a plane parallel to the base. The side length of the top square is 12 feet and the side length of the bottom square is 16 feet. The height of the stage is 3 feet.

a. What is the volume of the entire square pyramid, including the stage?

b. What is the volume of the top of the pyramid that is removed to get the stage?

c. What is the volume of the stage?
Surface Areas and Volumes of Spheres

Surface Areas of Spheres You can think of the surface area of a sphere as the total area of all of the nonoverlapping strips it would take to cover the sphere. If \( r \) is the radius of the sphere, then the area of a great circle of the sphere is \( \pi r^2 \). The total surface area of the sphere is four times the area of a great circle.

| Surface Area of a Sphere | If a sphere has a surface area of \( S \) square units and a radius of \( r \) units, then \( S = 4\pi r^2 \). |

Example Find the surface area of a sphere to the nearest tenth if the radius of the sphere is 6 centimeters.

\[
S = 4\pi r^2 \\
= 4\pi (6)^2 \\
\approx 452.4 \\
\approx 452.4 \\
\]

The surface area is 452.4 square centimeters.

Exercises

Find the surface area of each sphere or hemisphere. Round to the nearest tenth.

1. \( 5 \text{ m} \)
2. \( 7 \text{ in} \)
3. \( 3 \text{ ft} \)
4. \( 9 \text{ cm} \)
5. sphere: circumference of great circle = \( \pi \text{ cm} \)
6. hemisphere: area of great circle \( \approx 4\pi \text{ ft}^2 \)
Surface Areas and Volumes of Spheres

Volumes of Spheres  A sphere has one basic measurement, the length of its radius. If you know the length of the radius of a sphere, you can calculate its volume.

| Volume of a Sphere | If a sphere has a volume of \( V \) cubic units and a radius of \( r \) units, then \( V = \frac{4}{3} \pi r^3 \). |

**Example**  Find the volume of a sphere with radius 8 centimeters.

\[
V = \frac{4}{3} \pi r^3 \\
= \frac{4}{3} \pi (8)^3 \\
\approx 2144.7 \\
\]

The volume is about 2144.7 cubic centimeters.

**Exercises**

Find the volume of each sphere or hemisphere. Round to the nearest tenth.

1. [Diagram of a sphere with radius 5 ft]
2. [Diagram of a sphere with radius 6 in.]
3. [Diagram of a sphere with radius 16 in.]
4. hemisphere: radius 5 in.
5. sphere: circumference of great circle \( \approx 25 \) ft
6. hemisphere: area of great circle \( \approx 50 \) m²
Surface Areas and Volumes of Spheres

Find the surface area of each sphere or hemisphere. Round to the nearest tenth.

1. 6.5 cm
2. 89 ft

3. hemisphere: radius of great circle = 8.4 in.
4. sphere: area of great circle ≈ 29.8 m²

Find the volume of each sphere or hemisphere. Round to the nearest tenth.

5. 12.32 ft
6. 32 m

7. hemisphere: diameter = 18 mm
8. sphere: circumference ≈ 36 yd
9. sphere: radius = 12.4 in.
Word Problem Practice

Surface Areas and Volumes of Spheres

1. **ORANGES** Mandy cuts a spherical orange in half along a great circle. If the radius of the orange is 2 inches, what is the area of the cross section that Mandy cut? Round your answer to the nearest hundredth.

2. **BILLIARDS** A billiard ball set consists of 16 spheres, each \(2\frac{1}{4}\) inches in diameter. What is the total volume of a complete set of billiard balls? Round your answer to the nearest thousandth of a cubic inch.

3. **MOONS OF SATURN** The planet Saturn has several moons. These can be modeled accurately by spheres. Saturn’s largest moon Titan has a radius of about 2575 kilometers. What is the approximate surface area of Titan? Round your answer to the nearest tenth.

4. **THE ATMOSPHERE** About 99% of Earth’s atmosphere is contained in a 31-kilometer thick layer that enwraps the planet. The Earth itself is almost a sphere with radius 6378 kilometers. What is the ratio of the volume of the atmosphere to the volume of Earth? Round your answer to the nearest thousandth.

5. **CUBES** Marcus builds a sphere inside of a cube. The sphere fits snugly inside the cube so that the sphere touches the cube at one point on each side. The side length of the cube is 2 inches.

   a. What is the surface area of the cube?

   b. What is the surface area of the sphere? Round your answers to the nearest hundredth.

   c. What is the ratio of the surface area of the cube to the surface area of the sphere? Round your answer to the nearest hundredth.
**12-8 Study Guide**

**Congruent and Similar Solids**

**Identify Congruent or Similar Solids**  
*Similar solids* have exactly the same shape but not necessarily the same size. Two solids are similar if they are the same shape and the ratios of their corresponding linear measures are equal. All spheres are similar and all cubes are similar.  
*Congruent solids* have exactly the same shape and the same size. Congruent solids are similar solids with a scale factor of 1:1. Congruent solids have the following characteristics:

- Corresponding angles are congruent
- Corresponding edges are congruent
- Corresponding faces are congruent
- Volumes are equal

**Example**  
Determine whether the pair of solids is *similar*, *congruent*, or *neither*. If the solids are similar, state the scale factor.

**Exercise**

Determine whether the pair of solids is *similar*, *congruent*, or *neither*. If the solids are similar, state the scale factor.
Properties of Congruent or Similar Solids

When pairs of solids are congruent or similar, certain properties are known.

If two similar solids have a scale factor of \(a:b\) then,

- the ratio of their surface areas is \(a^2:b^2\).
- the ratio of their volumes is \(a^3:b^3\).

**Example**

Two spheres have radii of 2 feet and 6 feet. What is the ratio of the volume of the small sphere to the volume of the large sphere?

First, find the scale factor.

\[
\frac{\text{radius of the small sphere}}{\text{radius of the large sphere}} = \frac{2}{6} \quad \text{or} \quad \frac{1}{3}
\]

The scale factor is \(\frac{1}{3}\).

\[
\frac{a^3}{b^3} = \frac{(1)^3}{(3)^3} \quad \text{or} \quad \frac{1}{27}
\]

So, the ratio of the volumes is 1:27.

**Exercises**

1. Two cubes have side lengths of 3 inches and 8 inches. What is the ratio of the surface area of the small cube to the surface area of the large cube?

2. Two similar cones have heights of 3 feet and 12 feet. What is the ratio of the volume of the small cone to the volume of the large cone?

3. Two similar triangular prisms have volumes of 27 square meters and 64 square meters. What is the ratio of the surface area of the small prism to the surface area of the large prism?

4. **COMPUTERS** A small rectangular laptop has a width of 10 inches and an area of 80 square inches. A larger and similar laptop has a width of 15 inches. What is the length of the larger laptop?

5. **CONSTRUCTION** A building company uses two similar sizes of pipes. The smaller size has a radius of 1 inch and length of 8 inches. The larger size has a radius of 2.5 inches. What is the volume of the larger pipes?
12-8 Practice

Congruent and Similar Solids

Determine whether the pair of solids is similar, congruent, or neither. If the solids are similar, state the scale factor.

1. 

2. 

3. 

4. 

5. Two cubes have surface areas of 72 square feet and 98 square feet. What is the ratio of the volume of the small cube to the volume of the large cube?

6. Two similar ice cream cones are made of a half sphere on top and a cone on bottom. They have radii of 1 inch and 1.75 inches respectively. What is the ratio of the volume of the small ice cream cone to the volume of the large ice cream cone? Round to the nearest tenth.

7. ARClTHeCTURE Architects make scale models of buildings to present their ideas to clients. If an architect wants to make a 1:50 scale model of a 4000 square foot house, how many square feet will the model have?
Word Problem Practice

Congruent and Similar Solids

1. **COOKING** A cylindrical pot is 4.5 inches tall and has a radius of 4 inches. How tall would a similar pot be if its radius is 6 inches?

2. **MANUFACTURING** Boxes, Inc. wants to make the two boxes below. How long does the second box need to be so that they are similar?

3. **FARMING** A farmer has two similar cylindrical grain silos. The smaller silo is 25 feet tall and the larger silo is 40 feet tall. If the smaller silo can hold 1500 cubic feet of grain, how much can the larger silo hold?

4. **PLANETS** Earth has a surface area of about 196,937,500 square miles. Mars has a surface area of about 89,500,000 square miles. What is the ratio of the radius of Earth to the radius of Mars? Round to the nearest tenth.

   Source: NASA

5. **BASEBALL** Major League Baseball or MLB, rules state that baseballs must have a circumference of 9 inches. The National Softball Association, or NSA, rules state that softballs must have a circumference not exceeding 12 inches.

   Source: MLB, NSA

   a. Find the ratio of the circumference of MLB baseballs to the circumference of NSA softballs.

   b. Find the ratio of the volume of MLB baseballs to the volume of NSA softballs. Round to the nearest tenth.
Represent a Sample Space  The sample space of an experiment is the set of all possible outcomes. A sample space can be found using an organized list, table, or tree diagram.

Example  Maurice packs suits, shirts, and ties that can be mixed and matched. Using the packing list at the right, draw a tree diagram to represent the sample space for business suit combinations.

The sample space is the result of three stages:
- Suit color (G, B, or K)
- Shirt color (W or L)
- Tie (T or NT)

Draw a tree diagram with three stages.

Exercises
Represent the sample space for each experiment by making an organized list, a table, and a tree diagram.

1. The baseball team can wear blue or white shirts with blue or white pants.

2. The dance club is going to see either Sleeping Beauty or The Nutcracker at either Symphony Hall or The Center for the Arts.

3. Mikey’s baby sister can drink either apple juice or milk from a bottle or a toddler cup.

4. The first part of the test consisted of two true-or-false questions.
Fundamental Counting Principle The number of all possible outcomes for an experiment can be found by multiplying the number of possible outcomes from each stage or event.

Example The pattern for a certain license plate is 3 letters followed by 3 numbers. The letter “O” is not used as any of the letters and the number “0” is not used as any of the numbers. Any other letter or number can be used multiple times. How many license plates can be created with this pattern?

Use the Fundamental Counting Principle.

\[
\begin{array}{cccccc}
\text{1st Space} & \text{2nd Space} & \text{3rd Space} & \text{4th Space} & \text{5th Space} & \text{6th Space} \\
25 & 25 & 25 & 9 & 9 & 9 \\
\end{array}
\]

\[= 11,390,625\]

So 11,390,625 license plates can be created with this pattern.

Exercises

Find the number of possible outcomes for each situation.

1. A room is decorated with one choice from each category.

<table>
<thead>
<tr>
<th>Bedroom Décor</th>
<th>Number of Choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paint color</td>
<td>8</td>
</tr>
<tr>
<td>Comforter set</td>
<td>6</td>
</tr>
<tr>
<td>Sheet set</td>
<td>8</td>
</tr>
<tr>
<td>Throw rug</td>
<td>5</td>
</tr>
<tr>
<td>Lamp</td>
<td>3</td>
</tr>
<tr>
<td>Wall hanging</td>
<td>5</td>
</tr>
</tbody>
</table>

2. A lunch at Lincoln High School contains one choice from each category.

<table>
<thead>
<tr>
<th>Cafeteria Meal</th>
<th>Number of Choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main dish</td>
<td>3</td>
</tr>
<tr>
<td>Side dish</td>
<td>4</td>
</tr>
<tr>
<td>Vegetable</td>
<td>2</td>
</tr>
<tr>
<td>Salad</td>
<td>2</td>
</tr>
<tr>
<td>Salad Dressing</td>
<td>3</td>
</tr>
<tr>
<td>Dessert</td>
<td>2</td>
</tr>
<tr>
<td>Drink</td>
<td>3</td>
</tr>
</tbody>
</table>

3. In a catalog of outdoor patio plans, there are 4 types of stone, 3 types of edgers, 5 dining sets and 6 grills. Carl plans to order one item from each category.

4. The drama club held tryouts for 6 roles in a one-act play. Five people auditioned for lead female, 3 for lead male, 8 for the best friend, 4 for the mom, 2 for the dad, and 3 for the crazy aunt.
13-1 Practice

Representing Sample Spaces

Represent the sample space for each experiment by making an organized list, a table, and a tree diagram.

1. Tavya can spend the summer with her cousins or her grandparents at the lake or at the beach.

2. Jordan can write his final essay in class or at home on a scientific or an historical topic.

3. Julio can join the Air Force or the Army before or after college.

Find the number of possible outcomes for each situation.

4. Josh is making a stuffed animal.

<table>
<thead>
<tr>
<th>Animal Options</th>
<th>Number of Choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Animals</td>
<td>10</td>
</tr>
<tr>
<td>Type of stuffing</td>
<td>3</td>
</tr>
<tr>
<td>Sound effect</td>
<td>5</td>
</tr>
<tr>
<td>Eye color</td>
<td>3</td>
</tr>
<tr>
<td>Outfit</td>
<td>20</td>
</tr>
</tbody>
</table>

5. Kelley is buying an ice cream cone.

Assume one of each category is ordered.

<table>
<thead>
<tr>
<th>Ice Cream</th>
<th>Number of Choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of cone</td>
<td>3</td>
</tr>
<tr>
<td>Flavors</td>
<td>20</td>
</tr>
<tr>
<td>Cookie toppings</td>
<td>4</td>
</tr>
<tr>
<td>Candy toppings</td>
<td>8</td>
</tr>
</tbody>
</table>

6. Movie-themed gift baskets come with a choice of one of each of the following: 4 flavors of popcorn, 4 different DVDs, 4 types of drinks, and 8 different kinds of candy.

7. **INTERNSHIP** Jack is choosing an internship program that could take place in 3 different months, in 4 different departments of 3 different firms. Jack is only available to complete his internship in July. How many different outcomes are there for Jack’s internship?
13-1 Word Problem Practice

Representing Sample Spaces

1. SCHOOL SUPPLIES Eva is shopping for school supplies. She has a choice of one of each of the following: 6 backpacks, 8 notebooks, 3 pencil cases, 3 brands of pencils, 8 brands of pens, 4 types of calculator, and 4 colors of highlighter. How many different choices does she have for school supplies?

2. LAPTOPS Chloe is buying a laptop. She has a choice of 3 hard drive sizes, 3 processor speeds, 4 colors, 2 screen sizes, 2 warranty options, and 4 cases. She knows she wants a blue laptop with the longest warranty. How many choices does she have for laptops?

3. BOARD GAMES Below is a spinner used in a board game. If the spinner is spun 4 times, how many different possible outcomes are there?

4. BASKETBALL In the NBA there must be a minimum of 14 players on a team’s roster. A team has the minimum number of players where 3 are centers, 4 are power forwards, 2 are small forwards, 3 are shooting guards, and the rest are point guards. If all 5 positions must be filled, how many different choices does the coach have for the team’s lineup?

Source: NBA Players Association

5. VACATION RENTAL A brochure describes available vacation rentals in Colorado and Florida. In Colorado you can choose a 1 or 2 week stay in a 1- or 2-bedroom suite. In Florida you can choose a 1, 2 or 3 week stay in a 2- or 3-bedroom suite, along the beach or not.

a. How many outcomes are available in Colorado?

b. How many outcomes are available in Florida?

c. How many total outcomes are available?
Geometric Probability

Probability with Length  Probability that involves a geometric measure is called geometric probability. One type of measure is length.

Look at line segment $\overline{KL}$.
If a point, $M$, is chosen at random on the line segment, then

$$P(M \text{ is on } \overline{KL}) = \frac{KL}{RS}.$$ 

Example  Point $X$ is chosen at random on $\overline{AD}$. Find the probability that $X$ is on $\overline{AB}$.

$$P(X \text{ is on } \overline{AB}) = \frac{AB}{AD}$$

Length probability ratio

$$= \frac{8}{16}$$

$AB = 8$ and $AD = 8 + 2 + 6 = 16$

$$= \frac{1}{2}, 0.5, \text{ or } 50\%$$  Simplify.

Exercises

Point $M$ is chosen at random on $\overline{ZP}$. Find the probability of each event.

1. $P(M \text{ is on } \overline{ZQ})$

2. $P(M \text{ is on } \overline{QR})$

3. $P(M \text{ is on } \overline{RP})$

4. $P(M \text{ is on } \overline{QP})$

5. TRAFFIC LIGHT  In a 5-minute traffic cycle, a traffic light is green for 2 minutes 27 seconds, yellow for 6 seconds, and red for 2 minutes 27 seconds. What is the probability that when you get to the light it is green?

6. GASOLINE  Your mom’s mini van has a 24 gallon tank. What is the probability that, when the engine is turned on, the needle on the gas gauge is pointing between $\frac{1}{4}$ and $\frac{1}{2}$ full?
Geometric Probability

Probability with Area  Geometric probabilities can also involve area. When determining geometric probability with targets, assume that the object lands within the target area and that it is equally likely that the object will land anywhere in the region.

Example  Suppose a coin is flipped into a reflection pond designed with colored tiles that form 3 concentric circles on the bottom. The diameter of the center circle is 4 feet and the circles are spaced 2 feet apart. What is the probability the coin lands in the center?

\[
P(\text{coin lands in center}) = \frac{\text{area of center circle}}{\text{area of base of pond}}
\]

\[
= \frac{4\pi}{36\pi} = \frac{1}{9} \approx 0.11, \text{ about 0.11, or 11%}
\]

Exercises

1. LANDING  A parachutist needs to land in the center of a target on a rectangular field that is 120 yards by 30 yards. The target is a circular design with a 10 yard radius. What is the probability the parachutist lands somewhere in the target?

2. CLOCKS  Jonus watches the second hand on an analog clock as it moves past the numbers. What is the probability that at any given time the second hand on a clock is between the 2- and the 3-hour numbers?

Find the probability that a point chosen at random lies in the shaded region.

3.  

4.  

5.  

Use the spinner to find each probability. If the spinner lands on a line it is spun again.

6.  \( P(\text{pointer landing on red}) \)

7.  \( P(\text{pointer landing on blue}) \)

8.  \( P(\text{pointer landing on green}) \)
13-3 Practice

Geometric Probability

Point $L$ is chosen at random on $\overline{RS}$. Find the probability of each event.

1. $P(L$ is on $\overline{TV})$

2. $P(L$ is on $\overline{US})$

Find the probability that a point chosen at random lies in the shaded region.

3.

4.

5.

Use the spinner to find each probability. If the spinner lands on a line it is spun again.

6. $P$(pointer landing on purple)

7. $P$(pointer landing on red)

8. PIGS Four pigs are lined up at the feeding trough as shown in the picture. What is the probability that when a fifth pig comes to eat it lines up between the second and third pig?

9. MUSIC A certain company plays Mozart’s *Eine Kleine Nachtmusik* when its customers are on hold on the telephone. If the length of the complete recording is 2 hours long, what is the probability a customer put on hold will hear the Allegro movement which is 6 minutes, 31 seconds long?
1. **DARTS** A dart is thrown at the dartboard shown. Each sector has the same central angle. The dart has equal probability of hitting any point on the dartboard. What is the probability that the dart will land in a shaded sector?

2. **SPINNERS** Jamie, Joe, and Pat celebrate the end of each work week by ordering spring rolls from a Chinese restaurant. The order comes with 4 spring rolls so somebody gets an extra roll. Because Jamie works full time and Joe and Pat work half time, they decide who gets the extra roll by using a spinner that has a 50% chance of coming up Jamie, and 25% chances of coming up either Joe or Pat. Design such a spinner.

3. **RAIN** A container has a square top with a hole as shown. What is the probability that a raindrop that hits the container falls into the hole? Round your answer to the nearest thousandth.

4. **ELECTRON MICROSCOPES** Crystal places a 7 millimeter by 10 millimeter rectangular plate into the sample chamber of an electron microscope. A black and white checkerboard pattern of 1-millimeter squares was painted over the plate to identify different treatments of the material. When she turns on the monitor, she has no idea at what point on the plate she is looking because the white and black contrast does not show up on the screen. If there are 2 more black squares than white squares, what is the probability that she is looking at a white square?

5. **ENTERTAINMENT** A rectangular dance stage is lit by two lights that light up circular regions of the stage. The circles have radii of the same length and each circle passes through the center of the other. The stage perfectly circumscribes the two circles. A spectator throws a bouquet of flowers onto the stage. Assume the bouquet has an equal chance of landing anywhere on the stage. (Hint: Use inscribed equilateral triangles.)

   a. What is the probability that the flowers land on a lit part of the stage?

   b. What is the probability that the flowers land on the part of the stage where the spotlights overlap?
Probabilities of Independent and Dependent Events

Independent and Dependent Events

Compound events, or two or more simple events happening together, can be independent or dependent. Events are **independent events** if the probability of one event does not affect the probability of the other. Events are **dependent events** if one event in some way changes the probability that the other occurs.

The following are the **Multiplication Rules for Probability**.

<table>
<thead>
<tr>
<th>Probability of Two Independent Events</th>
<th>( P(A \text{ and } B) = P(A) \cdot P(B) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of Two Dependent Events</td>
<td>( P(A \text{ and } B) = P(A) \cdot P(B</td>
</tr>
</tbody>
</table>

\( P(B|A) \) is the **conditional probability** and is read the probability that event \( B \) occurs given that event \( A \) has already occurred.

**Example**

The P.E. teacher puts 10 red and 8 blue marbles in a bag. If a student draws a red marble, the student plays basketball. If a student draws a blue marble, the student practices long jump. Suppose Josh draws a marble, and not liking the outcome, he puts it back and draws a second time. What is the probability that on each draw his marble is blue?

Let \( B \) represent a blue marble.

\[
P(B \text{ and } B) = P(B) \cdot P(B) \quad \text{Probability of independent events}
\]

\[
= \frac{4}{9} \cdot \frac{4}{9} \text{ or } \frac{16}{81} \quad P(B) = \frac{4}{9}
\]

So, the probability of Josh drawing two blue marbles is \( \frac{16}{81} \) or about 20%.

**Exercises**

Determine whether the events are **independent** or **dependent**. Then find the probability.

1. A king is drawn from a deck of 52 cards, then a coin is tossed and lands heads up.

2. A spinner with 4 equally spaced sections numbered 1 through 4 is spun and lands on 1, then a die is tossed and rolls a 1.

3. A red marble is drawn from a bag of 2 blue and 5 red marbles and not replaced, then a second red marble is drawn.

4. A red marble is drawn from a bag of 2 blue and 5 red marbles and then replaced, then a red marble is drawn again.
Conditional Probabilities Conditional probability is used to find the probability of dependent events. It also can be used when additional information is known about an event.

The conditional probability of $B$ given $A$ is 

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

where $P(A) \neq 0$.

**Example** The Spanish Club is having a Cinco de Mayo fiesta. The 10 students randomly draw cards numbered with consecutive integers from 1 to 10. Students who draw odd numbers will bring main dishes. Students who draw even numbers will bring desserts. If Cynthia is bringing a dessert, what is the probability that she drew the number 10?

Since Cynthia is bringing dessert, she must have drawn an even number.

Let $A$ be the event that an even number is drawn.

Let $B$ be the event that the number 10 is drawn.

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$= \frac{0.5 \cdot 0.1}{0.5}$$

$$= 0.1$$

Simplify.

The probability Cynthia drew the number 10 is 0.1 or 10%.

**Exercises**

1. A blue marble is selected at random from a bag of 3 red and 9 blue marbles and not replaced. What is the probability that a second marble selected will be blue?

2. A die is rolled. If the number rolled is less than 5, what is the probability that it is the number 2?

3. A quadrilateral has a perimeter of 16 and all side lengths are even integers. What is the probability that the quadrilateral is a square?

4. A spinner with 8 evenly sized sections and numbered 1 through 8 is spun. Find the probability that the number spun is 6 given that it is an even number.
Probabilities of Independent and Dependent Events

Determine whether the events are independent or dependent. Then find the probability.

1. From a bag of 5 red and 6 green marbles, a red marble is drawn and not replaced. Then a green marble is drawn.

2. In a game, you roll an odd number on a die and then spin a spinner with 6 evenly sized spaces numbered 1 to 6 and get an even number.

3. A card is randomly chosen from a standard deck of 52 cards then replaced, and a second card is then chosen. What is the probability that the first card is the ace of hearts and the second card is the ace of diamonds?

Find each probability.

4. A die is tossed. If the number rolled is greater than 2, what is the probability that the number rolled is 3?

5. A black shoe is selected at random from a bin of 6 black shoes and 4 brown shoes and not replaced. What is the probability that a second shoe selected will be black?

6. A spinner with 12 evenly sized sections and numbered 1 to 12 is spun. What is the probability that the number spun is 12 given that the number is even?

7. GAME In a game, a spinner with 8 equally sized sections numbered 1 to 8 is spun and a die is tossed. What is the probability of landing on an odd number on the spinner and rolling an even number on the die?

8. APPROVAL A survey found that 8 out of 10 parents approved of the new principal’s performance. If 4 parents’ names are chosen, with replacement, what is the probability they all approve of the principal’s performance?
13-5 Word Problem Practice

Probabilities of Independent and Dependent Events

1. **DRIVING** The probability that a person has received a speeding ticket is 0.35. The probability of a person driving a red car is 0.15. What is the probability of randomly choosing a driver with a speeding ticket whose car is not red?

2. **GAMES** In a game, the spinner with 4 spaces numbered 1 to 4 is spun and a die is rolled.

   ![Spinner and Die]

What is the probability of spinning an even number on the spinner and rolling an even number on the die?

3. **CARDS** Three cards are drawn and not replaced from a standard deck. What is the probability that all three cards will be from different suits?

   ![Playing Cards]

4. **HEALTH** Jane conducted a survey at her school and found that the probability of a student contracting a version of the flu last year at her school was 5%. She also found the probability of a student contracting the stomach flu at her school last year was 1%. What is the probability that if a person develops the flu, it will be the stomach flu?

5. **BIRTHDAYS** Since there are 365 days in a year, the probability of a person’s birthday on any random day is about 0.00274.

   ![April Calendar]

- **a.** What is the probability that two people will have the same birthday?

- **b.** What is the probability that out of thirty people, two will have the same birthday?
Trees are connected vertex-edge graphs with the following properties.

<table>
<thead>
<tr>
<th>Trees</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. There are no cycles, paths that start and end at the same vertex.</td>
</tr>
<tr>
<td>2. There is exactly one path between any two vertices.</td>
</tr>
<tr>
<td>3. A tree with ( n ) vertices will always have ( n - 1 ) edges.</td>
</tr>
</tbody>
</table>

A **subgraph** is a graph that contains a subset of the edges and vertices of a larger graph. A tree that is a subgraph of a connected graph and includes all of the vertices of that graph is called a **spanning tree**. Since finding spanning trees in a large graph could be difficult, an algorithm exists to help you find them.

<table>
<thead>
<tr>
<th>Spanning Tree Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>To find a spanning tree in a graph, use the following process.</td>
</tr>
<tr>
<td><strong>Step 1</strong> Locate a cycle in the graph.</td>
</tr>
<tr>
<td><strong>Step 2</strong> Remove an edge from the cycle that will not disconnect the graph.</td>
</tr>
<tr>
<td><strong>Step 3</strong> If cycles remain, then repeat Step 2.</td>
</tr>
</tbody>
</table>

**Example** Find a spanning tree in the graph.

Remove \( b \) from cycle \( a, b, c \).  
Remove \( h \) from cycle \( c, h, i \).  
Remove \( i \) from cycle \( f, g, i \).  
Remove \( d \) from cycle \( c, d, e, f, g \).  

No cycles remain, so the result is a spanning tree.

**Exercises**

Find a spanning tree in each graph.

1.  
   ![Graph 1](image1)

2.  
   ![Graph 2](image2)
Minimal Spanning Trees

Trees can have weights. Since there are often several spanning trees for each graph, one of them must have a weight that is less than or equal to the weight of all the rest. This tree is called the **minimal spanning tree**. There may be more than one minimal spanning tree for a graph, but their weights will be the same.

### Kruskal's Algorithm

To find a minimal spanning tree in a weighted graph, use the following process.

**Step 1** Choose the edge with the least weight.

**Step 2** Choose the next edge with the least weight, as long as it does not create a cycle.

**Step 3** Continue as in Step 2 until all of the vertices are connected.

---

### Example

**Find a minimal spanning tree in the graph and state the weight of the tree.**

Since we can begin at any vertex, let’s start at **E**. Choose the edge of least weight connected to **E**, the edge between **E** and **D**.

Choose the edge of least weight connected to **D**, the edge between **D** and **A**.

Choose the edge of least weight connected to **A**, the edge between **A** and **B**.

Finally, the edge connecting **B** and **C** and the edge connecting **B** and **Z** both have the same weight 4. Choose them both; thus, connecting all vertices.

The weight of the tree is \(3 + 1 + 2 + 4 + 4\) or 14.

### Exercises

**Find a minimal spanning tree for each graph and state the weight of the tree.**

1. \[
\begin{array}{c}
B & 2 \\
C & 3 \\
D & 4 \\
E & 3
\end{array}
\]

2. \[
\begin{array}{c}
U & 4 \\
V & 4 \\
W & 8 \\
X & 5 \\
Y & 5 \\
Z & 3
\end{array}
\]
Minimal Spanning Trees

Find a spanning tree for each graph.

1. 

2. 

3. 

4. 

5. 

6. 

Find a minimal spanning tree for each graph and state the weight of the tree.

7. 

8. 

 Concepts and Skills Bank
1. **COMPUTERS** A small business plans to network fifteen computers to the same printer. How might this be done using the fewest communications lines possible?

3. **CABLE TELEVISION** Suppose the graph below represents the miles on a road map connecting 5 homes. A cable company wants to connect all of the homes on the map. Find the least number of miles connecting all of the locations.

3. **COMMUNICATIONS** A new fiber optics communication system is to be installed that will serve the cities in the map shown. The communications company has determined that the costs to connect each pair of cities are those that are shown on the map. Weights are in millions of dollars. Find the minimal cost to establish such a system and show its design.

4. **TRANSPORTATION** Suppose the graph below represents 7 local businesses where a mail carrier must make deliveries. Find a minimal spanning tree for the graph and state the weight of the tree.
Diagnostic Test

1. Which equation represents the circle shown on the graph below?

![Graph of a circle with center at (-13, 5) and radius of 13]

A. $(x - 13)^2 + (y + 5)^2 = 289$
B. $(x + 13)^2 + (y - 5)^2 = 289$
C. $(x - 13)^2 + (y - 5)^2 = 225$
D. $(x + 13)^2 + (y - 5)^2 = 225$

2. The capsule below is composed of a cylinder and two hemispheres. The volume of the cylinder is 195 cubic inches.

What is the total volume contained in two of these capsules?

A. 520 in$^3$
B. 585 in$^3$
C. 650 in$^3$
D. 975 in$^3$

3. The state of North Carolina elects 13 members to the United States House of Representatives. Suppose 2 privately funded grants will be given at random to North Carolina representatives. In which situation would the representative of the 8th district have the greatest chances of receiving a grant?

A. The first grant is chosen from all 13 representatives, and the second grant is also chosen from all 13 representatives.
B. The first grant is chosen from all 13 representatives, and the second grant is chosen from the remaining 12 representatives.
C. The first grant is chosen from districts 1 through 6, and the second grant is chosen from districts 7 through 13.
D. The first grant is chosen from districts 1 through 7, and the second grant is chosen from districts 8 through 13.

4. Which statement is true?

A. People generally agree about the meanings of undefined terms.
B. Theorems are accepted without proof.
C. A postulate can be used to prove an axiom.
D. A definition can be used to prove a postulate.
A skateboard ramp is $6\sqrt{3}$ feet high and makes a 30° angle with the ground, as shown in the figure below.

What is $x$, the length of the ramp?

- A 36 ft
- B $24\sqrt{3}$ ft
- C $12\sqrt{3}$ ft
- D 18 ft

What is the probability that a randomly selected point inside the square is also inside the right triangle?

- A 0.24
- B 0.48
- C 0.6
- D 0.8

Paras lives in Williamston, North Carolina. He is traveling to the Asheville Civic Center for an event this weekend. Possible routes with estimated distances in miles are shown in the diagram below.

According to the diagram, which route is the shortest from Williamston to the Asheville Civic Center?

- A through Rocky Mount and Raleigh
- B through Rocky Mount and Wilson
- C through Washington and Wilson
- D through Washington and Goldsboro
In a model of the solar system, Earth has a diameter of 15 inches, and Mars has a diameter of 8 inches.

How much greater is the surface area of Earth than Mars?

- A) $161\pi \text{ in}^2$
- B) $225\pi \text{ in}^2$
- C) $289\pi \text{ in}^2$
- D) $644\pi \text{ in}^2$

Given right triangle $JKL$ with right angle $K$, Stefan draws altitude $KM$ as a first step toward proving that $JL^2 = JK^2 + KL^2$.

Which reason allows him to state that $\frac{JL}{KL} = \frac{KL}{ML}$ and $\frac{JL}{JK} = \frac{JK}{JM}$?

- A) Concurrency of Altitudes
- B) Perpendicular Postulate
- C) definition of altitude
- D) Geometric Mean (Leg) Theorem

Alice used a scale of $3 \text{ cm} = 4 \text{ ft}$ to draw the side view of an underground pool.

What is the actual length of the hill that connects the shallow and deep ends of the pool?

- A) 5.625 ft
- B) 7.5 ft
- C) 10 ft
- D) 15 ft

Which statement could have been used to create the right column of the truth table below?

<table>
<thead>
<tr>
<th>New job in Raleigh</th>
<th>Move to new apartment</th>
<th>?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

- A) If Toni gets a new job in Raleigh, then she will move to a new apartment.
- B) Toni must take a new job in Raleigh and move to a new apartment to pay the bills.
- C) Toni must take a new job in Raleigh or move to a new apartment to pay the bills.
- D) If Toni does not get a new job in Raleigh, then she cannot pay the bills.
**Diagnostic Test** (continued)

**13** In the figure below, three squares are arranged so that they form right triangle FRH with right angle R. Maya is using the figure to prove the Pythagorean Theorem.

![Diagram of squares forming triangle FRH]

As a first step in her proof, Maya states that \( \triangle PFH \cong \triangle RFW \). Why is this true?

- **A** The triangles share a side and an angle.
- **B** \( FW = FH, PF = RF, \)
  \( m\angle PFH = 90^\circ + m\angle RFH, \)
  \( m\angle RFW = m\angle RFH + 90^\circ \)
- **C** \( RF = FH, PF = FW, \)
  \( m\angle PFH = 90^\circ + m\angle RFH, \)
  \( m\angle RFW = m\angle RFH + 90^\circ \)
- **D** \( \triangle RFW \) is the image of \( \triangle PFH \) after a rotation of 180° about point F.

**14.** On the tent shown below, a 16-foot cord is attached at a height of \( 8\sqrt{2} \) feet and secured to the ground.

![Diagram of tent with cord and height]

What angle does the cord form with the ground?

- **A** 30°
- **B** 45°
- **C** 60°
- **D** 90°

**15** Which statement is true about parallelogram HJKL shown below?

![Diagram of parallelogram HJKL]

- **A** Angles HJK and JKL are congruent corresponding angles.
- **B** Angles JKL and KLH are supplementary consecutive interior angles.
- **C** Angles LHK and JKH are congruent alternate exterior angles.
- **D** Angles LEH and KEJ are congruent alternate interior angles.
16 Steven and Corinne each have Global Positioning Systems for determining their precise locations. Starting at the same location, Steven travels northeast for 180 meters, and Corinne travels northwest for 350 meters. About how far apart are Steven and Corinne?

A 256.5 m  C 380.8 m
B 300.2 m  D 393.6 m

17 Elise is choosing an outfit. She selects a style of jeans. Then she chooses a top based on her choice of jeans. Finally, she chooses a pair of sandals. Her outfit choices are diagrammed below.

Jeans       Tops      Sandals
Red         Brown     Brown
            Black      Black
Boot Cut    Green     Brown
            Black      Black
            Blue       Brown
            Black      Black
Straight Leg Blue      Brown
            Purple     Black

If she makes each decision at random, what is the probability she will select straight leg jeans, a blue top, and black sandals?

A 6.25%  C 12.5%
B 10%     D 25%

18 Kirk drew the diagram below to represent a project that involves 7 tasks, labeled from T to Z. Each node is labeled with the number of hours it will take to complete the task.

What is the latest start time for task X?

A 9 hours  B 11 hours
C 14 hours  D 20 hours

Go on
19 In the figure below, $MN$ is a diameter of $\odot P$.

![Diagram of a circle with diameters and triangle MNO]

Which conclusion can be inferred?

A $\triangle MNO$ is an acute triangle.
B $\triangle MNO$ is an isosceles triangle.
C $\triangle MNO$ is an obtuse triangle.
D $\triangle MNO$ is a right triangle.

20 Triangle $VXW$ is isosceles with $VX = XW$, and $XZ$ is a median.

![Diagram of triangle VXW]

Which plan could be used to prove that $XZ$ is also an altitude of $\triangle VXW$?

A Show that $\angle VZX$ and $\angle WZX$ are congruent and form a linear pair.
B Show that $\angle VZX$ is a right angle because $VX^2 = VZ^2 + ZX^2$.
C Show that $\angle WZX$ is a right angle because $WX$ is a hypotenuse.
D Show that $XZ$ bisects the straight angle formed by $\overline{VW}$.

21 Quadrilateral $HJKL$ is graphed on a coordinate grid. The product of the slopes of any two adjacent sides of the quadrilateral equals $-1$. What additional information is needed to prove that quadrilateral $HJKL$ is similar to square $WXYZ$?

A $\frac{HJ}{WX} = \frac{KL}{YZ}$
B $HJ = JK = KL$
C The diagonals of quadrilateral $HJKL$ bisect each other.
D The diagonals of quadrilateral $HJKL$ are congruent.

22 Terry has four bills in his pocket: $1, $5, $10, and $20. He randomly selects two bills to pay the boy who cuts his yard. Which is a sample space of all the possible sums Terry could pull out?

A $6, 11, 16, 21, 30, 35$
B $6, 11, 15, 21, 25, 30$
C $6, 11, 15, 16, 30, 36$
D $6, 11, 15, 21, 30, 35$

23 Which two statements have equivalent truth values?

A conditional and inverse
B conditional and converse
C conditional and contrapositive
D converse and contrapositive
24 In right triangle $JKL$, $KM$ is an altitude.

Which statement can be inferred based on the given information?

- $JK = JM$
- $m \angle L = m \angle J$
- $\frac{JK}{KM} = \frac{KL}{ML}$
- $\frac{JK}{KM} = \frac{JL}{ML}$

25 A 20-foot ladder is placed against a house so that the bottom of the ladder is 12 feet from the edge of the house.

To the nearest degree, what is the measure of the angle that the ladder forms with the ground?

- $31^\circ$
- $37^\circ$
- $53^\circ$
- $59^\circ$

26 Which statement has a true converse?

- A If a figure is a square, it is a rectangle.
- B If a figure is a rectangle, it is a parallelogram.
- C If a figure is a parallelogram, it is a quadrilateral.
- D If a figure is a quadrilateral, it has four sides.

27 Mr. Hunan has a drawing of the North Carolina state flag in his classroom.

What is the probability that a point selected at random on the flag lies within the white rectangular portion of the flag?

- A 0.25
- B 0.375
- C 0.5
- D 0.75
28. Hedwin used a straightedge to draw line q shown below. He then used a compass and a straightedge to construct lines s and p.

Why must line p be parallel to line q?

A. Hedwin copied line q and named the new line p. Line s is a transversal.
B. Hedwin constructed a perpendicular bisector of lines p and q.
C. Hedwin used the same compass setting to mark the endpoints of lines p and q.
D. Hedwin copied angle 1 and named the new angle 2. Angles 1 and 2 are alternate interior angles.

30. Which reasoning could be used to show that \( m\angle F + m\angle G = 180^\circ \) in parallelogram \( EFGH \)?

A. \( EF \perp FG, FG \perp HG \) and \( 90^\circ + 90^\circ = 180^\circ \).
B. The sum of the interior angle measures of a quadrilateral is 360°, so \( m\angle F = 360^\circ \div 4 \) or 90°.
C. \( EG \) and \( HF \) bisect each other forming congruent triangles whose angle measures add up to 180°.
D. \( EF \parallel HG \) and \( FG \) is a transversal, so \( \angle F \) and \( \angle G \) are supplementary.

31. \( \angle QSR \) is inscribed in the circle below, intercepting arc \( QTR \).

If \( \overline{QTR} = 70^\circ \), then what is the measure of \( \angle QSR \)?

A. 35°  
B. 70°  
C. 105°  
D. 140°
Lynette is doing a research project for her social studies class. She wants to make a report on a North Carolina American Indian tribe. The list of available tribes is shown below.

<table>
<thead>
<tr>
<th>North Carolina American Indian Tribes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chowanoc</td>
</tr>
<tr>
<td>Coree</td>
</tr>
<tr>
<td>Hatteras</td>
</tr>
<tr>
<td>Machapunga</td>
</tr>
<tr>
<td>Meherrin</td>
</tr>
<tr>
<td>Tuscarora</td>
</tr>
</tbody>
</table>

Lynette chooses one of the tribes at random. Then she randomly chooses between giving a written report, an oral presentation, or a computerized slide show. What is the probability that she gives an oral presentation on the Tuscarora tribe?

A $\frac{1}{6}$  
B $\frac{1}{12}$  
C $\frac{1}{18}$  
D $\frac{1}{36}$

In a circle, central angle $\angle XYZ$ intercepts arc $XZ$ so that the measure of the arc is $74^\circ$. What is $m\angle XYZ$?

A $37^\circ$  
B $74^\circ$  
C $111^\circ$  
D $148^\circ$
35 The base of a wheelchair ramp is 72 inches away from the door. The height of the ramp is supposed to be $1/12$ the horizontal length of the ramp. A builder is planning to install a handrail along the length of the ramp.

How long will the handrail be?

- A 6 ft
- B 6 ft 1/50 in.
- C 6 ft 1/4 in.
- D 6 ft 2 in.

36 Two triangles have congruent corresponding exterior angles. What additional information could be used to prove that the triangles are congruent?

- A Both triangles are equilateral triangles.
- B Both triangles are isosceles triangles.
- C The triangles have a pair of congruent corresponding interior angles.
- D The triangles have a pair of congruent corresponding sides.

37 Which set of statements is an example of inductive reasoning?

- A Parallel lines never intersect. Line $a$ and line $b$ are parallel lines. Therefore, line $a$ and line $b$ never intersect.
- B In a random sample of line segments, line $a$ never intersects with line $b$. Therefore, line $a$ and line $b$ never intersect.
- C If line $a$ and line $b$ are parallel lines, then they never intersect. Line $a$ and line $b$ intersect. Therefore, line $a$ and line $b$ are not parallel lines.
- D If line $a$ and line $b$ are parallel lines, then they never intersect. Two lines never intersect, then they do not form an angle with each other. Therefore, if line $a$ and line $b$ are parallel lines, then they do not form an angle with each other.

38 What is the least amount of information needed to verify that two rectangles are similar?

- A Show that any 2 pairs of corresponding sides have the same ratio.
- B Show that the ratio shared by the 2 shorter sides is the same as the ratio shared by the 2 longer sides.
- C Show that 3 pairs of corresponding sides have the same ratio.
- D Show that 4 pairs of corresponding sides have the same ratio.
39 What must be the length of $\overline{XY}$ if $\triangle FGH$ is similar to $\triangle XYZ$?

- A 8
- B 10
- C 12.8
- D 20

40 The volume of the attic in the house shown below is 178 cubic feet.

What is the volume of the entire house?
- A $267 \text{ ft}^3$
- B $445 \text{ ft}^3$
- C $534 \text{ ft}^3$
- D $712 \text{ ft}^3$

41 Dolly has substituted different values for one of the dimensions of a cone into the formula for the volume of a cone. She has plotted the resulting volumes on a coordinate grid. If the data points she has plotted form a straight line, then which dimension of the cone has she changed?

- A diameter
- B height
- C radius
- D $\pi$

42 In the figure below, lines $m$ and $n$ are parallel. Dana used the angles formed by $\triangle WYX$ to determine that $\overline{WY} \perp \overline{YX}$.

Which angle relationships would be sufficient for Dana to arrive at this conclusion?

- A Alternate Interior Angles Theorem and Triangle Angle-Sum Theorem
- B Vertical Angles Theorem and Triangle Angle-Sum Theorem
- C definition of supplementary angles and Triangle Angle-Sum Theorem
- D Vertical Angles Theorem and definition of supplementary angles
43. Dipa created a scale drawing of the front view of a barn.

If the sum of the interior angle measures in her drawing is 900°, what is the measure of the angle formed by the top two sides of the actual barn?

A. 136°
B. 152°
C. 272°
D. 292°

44. Greg has 3 blue marbles, 3 red marbles, and 3 green marbles in a bag. He randomly selects two marbles from the bag. The first marble selected is red. Which statement is true about the second marble selected?

A. The chance of selecting a red marble next is greater if the first marble is put back into the bag.
B. The chance of selecting a red marble next is greater if the first marble is kept out of the bag.
C. The chance of selecting a red marble next is the same whether the first marble is put back into the bag or kept out of the bag.
D. The chance of selecting a red marble next is greater than that of selecting a blue marble next.

45. PQRT is a trapezoid with congruent diagonals PR and QT. The diagonals intersect at point F.

What must be true in order for △PQR and △QPT to be congruent?

A. PQ ≅ PR
B. PF ≅ TF
C. PR ≅ QR
D. PT ≅ QR

46. LP and LQ are secants of the circle below.

If \( m\widehat{MO} + m\widehat{PQ} = 112° \) and \( m\widehat{PQ} = 90° \), what is \( m\angle PLQ? \)

A. 11°
B. 22°
C. 34°
D. 56°
Diagnostic Test (continued)

47 Joseph draws two concentric circles in a coordinate plane, one with a radius twice that of the other. The equation that represents the inner circle is 

\[(x - 5)^2 + (y + 8)^2 = 49.\]

Which equation represents the outer circle?

A \((x + 5)^2 + (y - 8)^2 = 49\)
B \((x - 5)^2 + (y + 8)^2 = 98\)
C \((x - 5)^2 + (y + 8)^2 = 196\)
D \((x - 5)^2 + (y + 8)^2 = 2401\)

48 Mr. Brigg owns a rectangular piece of pastureland. He has a fence built in the corner of the land so that the rectangle formed by the fence is similar to the rectangle formed by the edges of the land.

What is the area of the entire property?

A 750 yd\(^2\)
B 1125 yd\(^2\)
C 1250 yd\(^2\)
D 1500 yd\(^2\)

49 Which of the following counters the assertion that two lines that are not parallel always intersect?

A skew lines
B parallel lines
C intersecting lines
D perpendicular lines

50 A trapezoid has congruent diagonals. Which statement must be true about the trapezoid?

A It has at least one pair of congruent sides.
B It has at least two pairs of congruent sides.
C Its diagonals bisect each other.
D Its diagonals are parallel to each other.

50 Marcela designed a dynamic triangular logo for a Web site using two side lengths that measure 4 inches and 6 inches. The third side is programmed to stretch and contract to create an animated visual effect on the Web site. If \(x\) represents the length of the third side, which statement defines all possible values of \(x\)?

A \(3 \leq x \leq 9\)
B \(3 < x < 9\)
C \(2 \leq x \leq 10\)
D \(2 < x < 10\)
52 Dynamic geometric software was used to drag vertex $E$ from its original position on the square to its new location shown below. The other three vertices move along with the figure so that the side lengths stay intact.

If the perimeter of the new figure is $y$ units, which statement must be true of the length of either diagonal?

A. The length must be less than $\frac{y}{8}$ units.
B. The length must be less than $\frac{y}{4}$ units.
C. The length must be less than $\frac{y}{2}$ units.
D. The length must be greater than $y$ units.

53 In order to prove the converse of the Pythagorean Theorem, Zulu assumes that the sides of a triangle are related by $r^2 = s^2 + t^2$ and that the angle opposite the side with length $r$ is either acute or obtuse. She draws each case and constructs a perpendicular segment with length $t$. Then she connects the endpoints to form side $v$.

What is Zulu’s next step?

A. \( \angle WZX \cong \angle WXZ \), so \( \triangle WZX \) is isosceles in each case.
B. \( \triangle WZX \) is isosceles, so \( \angle WZX \cong \angle WXZ \) in each case.
C. By the Pythagorean Theorem, \( v^2 = s^2 + t^2 \), so \( v = r \) in each case.
D. By the Angle Sum Theorem, \( m \angle WYZ + m \angle WXY = 180^\circ \) in each case.
Diagnostic Test (continued)

54 Marcus has a circular patio in his backyard. He is installing a border of bricks along the outer edge of the patio. So far, he has completed the section shown.

What is the measure of the arc that Marcus created?

A. 62°
B. 83°
C. 124°
D. 248°

56 Joy wants to prove by contradiction that the longest side of a right triangle is its hypotenuse. She begins by assuming that the hypotenuse of the right triangle measures 5 units and that one of the legs is 6 units. Which Theorem will Joy use to reach a contradiction?

A. If two sides of a triangle are congruent, then the angles opposite them are congruent.
B. If two complementary angles are congruent, each angle measures 45°.
C. If the measures of the interior angles of a triangle are added together, the sum equals 180°.
D. In a right triangle, the sum of the squares of the legs is equal to the square of the hypotenuse.

55 The Cape Lookout Lighthouse, in North Carolina, is 163 feet tall.

To the nearest foot, what is x, the distance from the top of the lighthouse to the boat?

A. 547 ft
B. 527 ft
C. 520 ft
D. 501 ft

57 What given information is needed to prove that \( \triangle GFK \) is similar to \( \triangle LMK \)?

A. \( GF \parallel LM \)
B. \( \angle KLM = \angle KML \)
C. \( \angle KGF = \angle KFG \)
D. \( \angle LKM = \angle KLM \)
58. Triangle $MKE$ and $\triangle GLF$ are isosceles right triangles.

Pedro writes the argument below to prove that trapezoid $MKGL$ is isosceles. What is the missing second step in Pedro’s argument?

i. $\overline{KG} \parallel \overline{EF}$ because $MKGL$ is a trapezoid.

ii. ?

iii. $\overline{KGFE}$ is a parallelogram because both pairs of opposite sides are parallel.

iv. $\overline{KE} \cong \overline{GF}$ because opposite sides of a parallelogram are congruent.

v. $\overline{ME} \cong \overline{KE}$, $\overline{GF} \cong \overline{LF}$, and $\angle MKE \cong \angle GFL$ because $\triangle MKE$ and $\triangle GLF$ are isosceles right triangles.

vi. $\triangle MKE \cong \triangle GLF$ by SAS. Therefore, $\overline{MK} \cong \overline{GL}$, and $MKGL$ is an isosceles trapezoid.

A. $\overline{EF} \cong \overline{FE}$ because all segments are congruent to themselves.

B. $\angle KEF \cong \angle GFE$ because all right angles are congruent.

C. $\overline{KG} \cong \overline{EF}$ because $\overline{KGFE}$ is a square.

D. $\overline{KE} \parallel \overline{GF}$ because $\overline{KE}$ and $\overline{GF}$ are both perpendicular to $\overline{ML}$.

59. Circle $F$ has a radius of 5.2 centimeters. Circle $P$ has a radius of 7.8 centimeters. Angle $GFH$ and $\angle QPR$ are congruent. If arc $GH$ is 6 centimeters, what is the length of arc $QR$?

A. 4 cm  
B. 8.6 cm  
C. 9 cm  
D. 15.6 cm

60. A DSL company needs to run wiring connections among several key points within a company. A diagram of possible paths with associated costs is shown below.

What is the minimum cost to make all the necessary connections?

A. $305  
B. $351  
C. $353  
D. $362
1. A city is planning a light-rail system. A graph that shows the possible links between neighborhoods and the costs in millions of dollars is shown below.

What is the minimum cost, in millions of dollars, of building a system that serves every neighborhood?

A. 173  B. 181  C. 186  D. 196

2. Sandeep wants to ride his bicycle to Dina’s house. Possible routes with the travel times, in minutes, are shown below in the vertex-edge graph.

What is the fastest way for Sandeep to bike to Dina’s house?

A. Pharmacy, School, Dina’s House
B. Library, Pool, Monument, Dina’s House
C. Pharmacy, School, Post Office, Dina’s House
D. Library, Pizza, Monument, Dina’s House

3. The vertex-edge graph below shows some of the routes and the approximate number of miles between several locations in North Carolina.

What is the distance of the shortest route from Chapel Hill to Gastonia?

A. 113 miles  B. 132 miles  C. 160 miles  D. 161 miles

4. A cable company is installing cable in all of the apartments in an apartment building. The vertex-edge graph below represents the possible paths of the cable and the associated costs. Each vertex represents an apartment.

What is the minimum cost of installing cable in every apartment?

A. $494  B. $479  C. $475  D. $473
1 High school music and dance director, Ms. Lopez, plans to have a spring concert. She makes a list of all the necessary tasks; approximately how many days each task will take to complete; and what tasks must precede other tasks.

<table>
<thead>
<tr>
<th>Task Code</th>
<th>Task</th>
<th>Time</th>
<th>Prior Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Choose music</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Rehearse choir</td>
<td>120</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Rehearse orchestra</td>
<td>30</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Rehearse dancers</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>Design ads, tickets, and programs</td>
<td>15</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>Print ads, tickets, and programs</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>Advertise</td>
<td>40</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>Sell tickets</td>
<td>40</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>Dress rehearsals</td>
<td>2</td>
<td>2, 3, 4</td>
</tr>
<tr>
<td>10</td>
<td>Concert</td>
<td>1</td>
<td>7, 8, 9</td>
</tr>
</tbody>
</table>

What is the critical path?

A 1, 3, 9, 10
B 1, 2, 9, 10
C 1, 5, 6, 7, 10
D 1, 2, 3, 4, 9, 10

2 Stella drew the weighted digraph below to represent a project that involves 15 tasks. The edges are labeled with the time, in weeks, it will take to complete each task.

What is the earliest finish time?

A 60 weeks
B 58 weeks
C 34 weeks
D 32 weeks
1 Which truth table shows the relationship between a statement $p$ and the negation of $p$?

A

<table>
<thead>
<tr>
<th>$p$</th>
<th>$\neg p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
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</table>

B

<table>
<thead>
<tr>
<th>$p$</th>
<th>$\neg p$</th>
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</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
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<tr>
<td>F</td>
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C

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<th>$p$</th>
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D

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<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

2 Which type of relation is represented in the right column of the truth table below?

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$?$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

A if $p$, then $q$
B not $q$
C $p$ and $q$
D $p$ or $q$

3 Which statement was used to create the truth table below?

<table>
<thead>
<tr>
<th>24 classes</th>
<th>3 months of training</th>
<th>Qualified?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

A Karate students must take 24 classes and complete 3 months of training to qualify for belt testing.
B Karate students must take 24 classes or complete 3 months of training to qualify for belt testing.
C If karate students complete 24 classes, then they have 3 months of training.
D If karate students take 24 classes in 3 months, they cannot qualify for belt testing.

4 Paul reads the following statement: The trainer will reward the dog if the dog sits down or rolls over. Then, he begins creating the truth table below.

<table>
<thead>
<tr>
<th>Dog sits down</th>
<th>Dog rolls over</th>
<th>Reward?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>No</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>No</td>
<td>No</td>
<td></td>
</tr>
</tbody>
</table>

How many times should Paul write “Yes” in the reward column?

A 1  B 2  C 3  D 4

Go on
Practice By Standard
Clarifying Objective MBC.G.1.2

1 In triangle $PQR$, $m\angle P + m\angle Q = 85^\circ$ and $m\angle R = 95^\circ$. What statement can be inferred based on the given information?

A Triangle $PQR$ is isosceles.

B $m\angle P = 95^\circ - m\angle Q$

C $m\angle P < m\angle R$

D $85^\circ < m\angle Q < 95^\circ$

2 Given:

I. 5, 6, and 7 are consecutive numbers.

II. 11, 12, and 13 are consecutive numbers.

III. 137, 138, and 139 are consecutive numbers.

IV. 419, 420, and 421 are consecutive numbers.

V. 5, 7, 11, 13, 137, 139, 419, and 421 are prime numbers.

Using the given statements, which conclusion can be inferred?

A Sets of three consecutive numbers generally contain two composite numbers and one prime number.

B There are more three-digit prime numbers than there are two-digit prime numbers.

C When a composite number occurs consecutively between two prime numbers, it is divisible by 6.

D When a composite number occurs consecutively between two prime numbers, the prime numbers mostly end in 7 or 9.

3 Given:

$\overrightarrow{EF}$ and $\overrightarrow{HG}$ intersect at point $J$.

Using the given statement, which conclusion can be inferred?

A $\overrightarrow{JE}$ and $\overrightarrow{JF}$ are opposite rays.

B Point $J$ is the midpoint of segment $HG$.

C $\angle EJH$ and $\angle GJF$ are supplementary.

D $HG$ bisects $\overrightarrow{EF}$.

4 Given:

I. Prisms with triangular bases have 3 lateral sides.

II. Prisms with quadrilateral bases have 4 lateral sides.

III. Prisms with pentagonal bases have 5 lateral sides.

IV. Prisms with hexagonal bases have 6 lateral sides.

What conclusion can be inferred based on the given information?

A When the base of a prism has $n$ sides, the prism has $2n$ lateral sides.

B When the base of a prism has $n$ sides, the prism has $n + 1$ lateral sides.

C When the base of a prism has $n$ sides, the prism has $n - 1$ lateral sides.

D When the base of a prism has $n$ sides, the prism has $n$ lateral sides.
1. The statement “if a quadrilateral is a square, then it is a rectangle” is true. Based on this statement, which conclusion is valid?
   - A. If a quadrilateral is not a square, then it is not a rectangle.
   - B. If a quadrilateral is not a rectangle, then it is not a square.
   - C. If a quadrilateral is a rectangle, then it is a square.
   - D. If a quadrilateral is a rectangle, then it is not a square.

2. Which are logically equivalent?
   - A. conditional and inverse
   - B. inverse and contrapositive
   - C. converse and contrapositive
   - D. converse and inverse

3. Suppose a given conditional statement is true. What can be concluded about its converse?
   - A. It is definitely true.
   - B. It is definitely false.
   - C. It is possibly true.
   - D. It is neither true nor false.

4. Given:
   An angle is obtuse if and only if its measure is between 90° and 180°.
   Using this true biconditional statement, which conclusion is valid?
   - A. If the measure of an angle is not between 90° and 180°, the angle is an obtuse angle.
   - B. If the measure of an angle is between 90° and 180°, the angle is an obtuse angle.
   - C. If the measure of an angle is not between 90° and 180°, the angle is not an obtuse angle.
   - D. If an angle is not an obtuse angle, then its measure is between 90° and 180°.

5. Which is a valid conclusion drawn from the true conditional statement in the box below?
   - If a polygon has eight sides, then it is an octagon.
   - A. If a polygon is not an octagon, then it does not have eight sides.
   - B. If a polygon is not an octagon, then it has eight sides.
   - C. If a polygon has eight sides, then it is not an octagon.
   - D. If a polygon does not have eight sides, then it is an octagon.
Practice By Standard
Clarifying Objective MBC.G.1.4

1 Which phrase best describes inductive reasoning?
   A accepting the meaning of a term without definition
   B using logic to draw conclusions based on accepted statements
   C inferring a general truth by examining a number of specific examples
   D defining mathematical terms to correspond with physical objects

2 Which statement is true?
   A A conjecture is a provable truth.
   B Before terms are defined, they are called undefined terms.
   C The converse of a theorem can be assumed to be true.
   D The converse of a definition can be assumed to be true.

3 Which type of statement below can be proven true with logical arguments?
   A axiom
   B common notion
   C postulate
   D theorem

4 Which statement best describes axioms and postulates?
   A Axioms and postulates are statements accepted to be true in order to prove theorems.
   B Axioms and postulates capture the essential qualities of objects.
   C Axioms can be used to prove postulates.
   D An axiom is a statement that includes previously defined terms, while a postulate is a statement that includes undefined terms.

5 Which set of statements illustrates the structure of deductive reasoning?
   A The Cape Hatteras Lighthouse had to be moved due to problems with erosion. Therefore, most lighthouses must be moved due to problems with erosion.
   B Charles Karault was born in Wilmington. Wilmington is located in southeastern North Carolina. Therefore, Charles Karault was born in southeastern North Carolina.
   C The Venus flytrap is native to Hampstead. Lou has a Venus flytrap. Therefore, Lou is native to Hampstead.
   D Many United States presidents have visited North Carolina. Therefore, all United States presidents have visited North Carolina.
Practice By Standard
Clarifying Objective MBC.G.1.5

**1** Keisha used a compass and a straightedge to create the figure below.

Why must $\triangle MKP$ and $\triangle NKP$ be congruent?

- **A** Keisha bisected $\overline{MN}$, so $PM = PN$. She bisected $\angle MKN$, so $m\angle MKP = m\angle NKP$. Both triangles share $\overline{KP}$.
- **B** Keisha constructed a perpendicular bisector of $\overline{MN}$, so $PM = PN$ and $m\angle KPM = m\angle KPN$. Both triangles share $\overline{KP}$.
- **C** Keisha used the same compass setting to mark points $M$ and $N$, so $MK = NK$. She bisected $\angle MKN$, so $m\angle MKP = m\angle NKP$. Both triangles share $\overline{KP}$.
- **D** Keisha used the same compass setting to mark points $M$ and $N$, so $MK = NK$. She constructed a perpendicular bisector of $\overline{MN}$, so $PM = PN$ and $m\angle KPM = m\angle KPN$.

**2** Julia draws a point and a line on a piece of paper. She folds the paper at the point so that her original line coincides with itself. She repeats this step using the crease she made with her previous fold. The process is shown in the diagram below.

Which explanation **best** justifies that Julia constructed parallel lines?

- **A** Each fold she made created a perpendicular line. If two lines are perpendicular to the same line, then those two lines are parallel.
- **B** Each fold created an intersection at a 75° angle. The lines are parallel because alternate interior angles are congruent.
- **C** She drew the first line very carefully so that her last fold would have an equal slope.
- **D** There is a third line that is parallel to both her original line and her last fold. If two lines are parallel to the same line, then those two lines are parallel.
3. Dynamic geometry software was used to create the figure below where $E$, $F$, $G$, and $H$ are points of tangency.

Why must $GE$ and $HF$ be equal in length?

A. The diameter of the larger circle passes through points $E$ and $F$, and the diameter of the smaller circle passes through points $G$ and $H$. When these two segments are drawn, isosceles trapezoid $EFHG$ is formed. Therefore, $GE = HF$.

B. Because points $E$, $G$, $F$, and $H$ are all points of tangency, the segments that occur between the circles are congruent. Therefore, $GE = HF$.

C. Because $JE$ and $JF$ are intersecting lines, the segments within the lines are congruent. Therefore, $GE = HF$.

D. Because $JE$ and $JF$ are tangent to both circles and intersect at a point outside both circles, $JE = JF$ and $JG = JH$. It follows that $JE - JG = JF - JH$. Since $GE = JE - JG$ and $HF = JF - JH$, $GE = HF$.

4. Latika folded a small piece of square paper in half to form a diagonal. Then she made the same fold to a larger piece of square paper, as shown.

Why must $\triangle 1$ and $\triangle 2$ be similar?

A. Each triangle has a $90^\circ$ angle. The diagonal of each square creates two $45^\circ$ angles within each triangle. Since the triangles have congruent corresponding angles, they are similar.

B. All squares are similar, so the two triangles formed by folding a square in half are also similar.

C. The side lengths and diagonal of the small square appears to be half the side lengths and diagonal of the large square, respectively. Since there appears to be a 1 to 2 ratio between each pair of corresponding sides, the triangles are similar.

D. The diagonals were folded so that their slopes decrease from left to right. Each triangle also has a vertical side and a horizontal side. Since the orientation is the same, the triangles are similar.
Suppose that $\overline{QR}$ is twice as long as $\overline{WX}$ and that $\overline{QS}$ is twice as long as $\overline{WY}$.

What must be true in order for $\triangle QRS$ and $\triangle WXY$ to be similar?

A. $RS$ is twice as long as $WY$.
B. $\angle RQS \equiv \angle XYW$
C. $RS$ is twice as long as $XY$.
D. $\angle QRS \equiv \angle XWY$

Quadrilateral $FGHJ$ is a parallelogram.

What must be true to prove that $\triangle FGL \equiv \triangle HGL$?

A. $FJ \equiv HJ$
B. $\angle HFJ \equiv \angle FHG$
C. $FL \equiv HL$
D. $\triangle FGL$ is an isosceles triangle.

Which type of triangle could be used to provide a counterexample to the conjecture below?

Each interior angle measure of a triangle is less than or equal to 90°.

A. right triangle
B. obtuse triangle
C. equilateral triangle
D. acute triangle

Which figure can be used to reject the conjecture “for any quadrilateral, the lengths of its diagonals are equal”?

A
B
C
D
Practice By Standard
Clarifying Objective MBC.G.2.2

1. Jose wants to prove that no quadrilateral can have exactly three right angles. He begins by drawing the quadrilateral below, assuming that $\angle 1$, $\angle 2$, and $\angle 3$ are right angles, and that $\angle 4$ is not a right angle.

What is the best way that Jose can finish his argument?

A. A square has four right angles. Therefore, $\angle 4$ must be a right angle.

B. The sum of the measures of the angles of a quadrilateral is 360°. So, $m\angle 4$ must be equal to $360° - (m\angle 1 + m\angle 2 + m\angle 3)$ or $360° - (90° + 90° + 90°)$, which is 90°. Therefore, $\angle 4$ must be a right angle.

C. In any quadrilateral, opposite angles are equal. So, $m\angle 1 = m\angle 4 = 90°$. Therefore, $\angle 4$ must be a right angle.

D. In any quadrilateral with three right angles, the fourth angle must also be a right angle, thus forming a rectangle or square. Therefore, $\angle 4$ must be a right angle.

2. In the figure below, $m\angle 1 + m\angle 4 = 175°$.

Sarah assumes that lines $\ell$ and $m$ are parallel. So, angles 2 and 4 are congruent. Since angles 1 and 2 are supplementary, then angles 1 and 4 are also supplementary. Why is Sarah’s assumption about $\ell$ and $m$ incorrect?

A. $m\angle 1 + m\angle 2 = 180°$ is inconsistent with $m\angle 1 + m\angle 4 = 180°$.

B. $m\angle 1 + m\angle 2 = 180°$ is inconsistent with $m\angle 2 + m\angle 4 = 180°$.

C. $m\angle 1 + m\angle 4 = 180°$ is inconsistent with $m\angle 1 + m\angle 4 = 175°$.

D. $m\angle 2 + m\angle 4 = 150°$ is inconsistent with $m\angle 1 + m\angle 4 = 175°$.

3. Which item can be used to support a statement in a direct argument?

A. conjecture

B. hypothesis

C. postulate

D. theory
Practice By Standard  (continued)
Clarifying Objective MBC.G.2.2

**4** Two congruent squares are placed side by side to form quadrilateral RSTV.

Which argument can be used to prove that quadrilateral RSTV is a rectangle?

A All squares are rhombuses, so the quadrilateral is formed by two rhombuses that are side by side. All rhombuses have four congruent sides, so quadrilateral RSTV has four congruent sides. Therefore, quadrilateral RSTV is a rectangle.

B The sum of the measures of the angles of a rectangle is 360°. So, \( m\angle R + m\angle S + m\angle T + m\angle V = 360° \). Therefore, quadrilateral RSTV is a rectangle.

C A square has four congruent sides. Since the squares are congruent and all the sides are congruent, \( TR \equiv SV \). Since RS is composed of two equal side lengths and TV is composed of two equal side lengths, \( RS \equiv TV \). Since opposite sides of the figure are congruent, quadrilateral RSTV is a rectangle.

D The quadrilateral is composed of two squares, so \( \angle R, \angle S, \angle T, \) and \( \angle V \) are right angles. Since quadrilateral RSTV has four right angles, it is a rectangle.

**5** In the figure below, \( \triangle MGF \cong \triangle LHJ \) and \( EF \equiv JK \).

Rurik writes the argument below to prove \( EH \equiv GK \). What is the missing step in Rurik’s argument?

i. \( \triangle MGF \cong \triangle LHJ \), so \( GF \equiv HJ \) and \( GF = HJ \).

ii. \( EF \equiv JK \), so \( EF = JK \).

iii. \( EF + GF + GH = JK + HJ + GH \).

iv. \( ? \)

v. Therefore, \( EH \equiv GK \).

A \( EF + GF + GH = EH \) and \( JK + HJ + GH = GK \), so \( EH = GK \).

B \( EF + GF = JK + HJ \), so \( EH = GK \).

C \( EF \equiv FJ \) and \( FJ \equiv GH \).

D \( FG + GH = FH \) and \( JK + HJ = HK \), so \( FH = HK \).
**Practice By Standard**

**Clarifying Objective MBC.G.2.3**

1. Parallel lines \( r \) and \( s \) are cut by a transversal in the figure below.

Which statement must be true?

A. \( \angle 1 \) and \( \angle 3 \) are congruent.
B. \( \angle 1 \) and \( \angle 3 \) are complementary.
C. \( \angle 2 \) and \( \angle 3 \) are congruent.
D. \( \angle 2 \) and \( \angle 3 \) are complementary.

2. Lines \( \ell \) and \( m \) are parallel in the figure below.

If \( \angle 1 \cong \angle 11 \), which relationship is true?

A. \( \angle 2 \cong \angle 14 \)
B. \( \angle 4 \cong \angle 5 \)
C. \( \angle 6 \cong \angle 15 \)
D. \( \angle 8 \cong \angle 10 \)

3. In the figure below, \( \overrightarrow{ML} \parallel \overrightarrow{PQ} \) and \( \overrightarrow{LP} \perp \overrightarrow{PQ} \).

Which statement is true?

A. \( \angle MLR \) is supplementary to \( \angle PQM \).
B. \( \angle TMS \) is congruent to \( \angle PQU \).
C. \( \angle MLR \) is a right angle.
D. \( \angle RLV \) is congruent to \( \angle PQM \).

4. If lines \( p \) and \( q \) are parallel in the figure below, which two angles are supplementary?

A. \( \angle 1 \) and \( \angle 5 \)
B. \( \angle 2 \) and \( \angle 3 \)
C. \( \angle 3 \) and \( \angle 6 \)
D. \( \angle 4 \) and \( \angle 7 \)
Practice By Standard
Clarifying Objective MBC.G.2.4

1. In the figure below, which additional piece of information would be sufficient to prove that $\triangle JKL \cong \triangle PRL$?

   - $JK \cong PR$
   - $\angle 2 \cong \angle 6$
   - $JK$ is parallel to $PR$.
   - $\angle 1 \cong \angle 2$

2. Which is a pair of similar triangles?

   - A
   - B
   - C
   - D

3. In the figure below, which given information could be used to prove that $\triangle WY \cong \triangle WXY$?

   - $WV \cong WX$ and $Y$ is the midpoint of $VX$.
   - $WV \cong WX$ and $\angle 2 \cong \angle 6$.
   - $Y$ is the midpoint of $VX$ and $\angle 1 \cong \angle 4$.
   - $Y$ is the midpoint of $VX$ and $\angle 3$ is supplementary to $\angle 5$.

4. In which case would triangles $MNP$ and $QST$ be both similar and congruent?

   - A $\angle M \cong \angle Q$ and $\frac{MN}{QS} = \frac{NP}{ST} = \frac{MP}{QT}$
   - B $\angle M \cong \angle Q$ and $\frac{MN}{QS} = \frac{MP}{QT} = 1$
   - C $\angle M \cong \angle Q$ and $\angle N \cong \angle S$
   - D $\angle M \cong \angle Q$, $MP \cong QT$, and $NP \cong ST$
Practice By Standard
Clarifying Objective MBC.G.3.1

1. In the figure below, $\triangle XYZ \sim \triangle EFG$.

What is the value of $x$?

- **A** 9
- **B** 10
- **C** 16
- **D** 25

2. In the figure below, $HJKL \sim PQRS$, $m\angle J = 84^\circ$, $m\angle K = 79^\circ$, and $m\angle P = 116^\circ$.

What is $m\angle S$?

- **A** 79°
- **B** 81°
- **C** 85°
- **D** 101°

3. $TUVWX \sim JKMNP$ with the following measures: $UV = 9$, $VW = 15$, $XT = 12$, $NP = 52$, $UT = 30$, and $PJ = 26$. What is the perimeter of $JKMNP$?

- **A** 54
- **B** 90
- **C** 195
- **D** 234

4. In the figure below, $PQTR \sim RTGF$.

What is the length of $FG$?

- **A** 4
- **B** 4.5
- **C** 5
- **D** 10

5. Given: $\triangle HIJ \sim \triangle SVW$

What is the value of $y$?

- **A** 8
- **B** 9
- **C** 11
- **D** 12
Practice By Standard
Clarifying Objective MBC.G.3.2

1 If quadrilaterals EFGH and KLMN each have interior angle measures of 40°, 70°, 120°, and 130°, which statement is true?

A The quadrilaterals are both similar and congruent regardless of the order that the angles occur.
B The quadrilaterals are similar regardless of the order that the angles occur, but they are not necessarily congruent.
C The quadrilaterals are both similar and congruent if and only if the angle measures occur in the same clockwise order within each quadrilateral.
D The quadrilaterals are similar if and only if the angle measures occur in the same clockwise order within each quadrilateral, but they are not necessarily congruent.

2 Which information would be sufficient to prove that GHKMN ~ PSTWX?

A \( \angle H \cong \angle S, \quad \frac{GH}{PS} = \frac{MN}{WX} \)
B \( \angle H \cong \angle S \)
C \( \angle M \cong \angle W, \quad \frac{GH}{PS} = \frac{HK}{ST} = \frac{KM}{TW} \)
D \( \frac{GH}{PS} = \frac{NG}{XP} \)

3 Which information would be sufficient to prove that two hexagons are similar?

A The hexagons are both regular.
B The hexagons have the same ratio between 6 pairs of sides.
C The hexagons have 6 pairs of congruent angles.
D The hexagons have the same ratio between 5 pairs of sides, and the hexagons have 4 pairs of congruent angles.

4 What is the least amount of information that would be sufficient to prove that \( EFJL \cong QRUV \)?

A \( FJ \cong RU, \quad JL \cong UV \)
B \( m\angle L = m\angle V \)
C \( m\angle L = m\angle V, \quad \overline{FJ} \cong \overline{RU} \)
D \( m\angle L = m\angle V, \quad \overline{FJ} \cong \overline{RU}, \quad JL \cong UV \)

5 Which statement is true?

A All parallelograms are similar.
B All rectangles are similar.
C All rhombuses are similar.
D All squares are similar.
Practice By Standard
Clarifying Objective MBC.G.3.3

1. The rectangular sign shown is a model of an actual billboard sign.

If the height of the actual sign is 18 feet, what scale was used for the model?

- A 2 in. = 9 ft
- B 3 in. = 8 ft
- C 4 in. = 9 ft
- D 6 in. = 9 ft

2. The figure shows the length of the northern border of North Carolina in an atlas of the United States.

If the map scale is 2 cm = 48 km, about how long is North Carolina’s northern border?

- A 1092 km
- B 1152 km
- C 2184 km
- D 24,570 km

3. Jana is creating two congruent hexagonal props for a show. A diagram of the two props is shown below.

What is the value of x?

- A 45
- B 115
- C 135
- D 270

4. The Carl Sandburg home is a historical site in western North Carolina. The original window frame on the second floor of the home is shown below.

Lang created a scale drawing of the original window frame. If the bottom edge of Lang’s drawing is 6 inches long, what ratio did Lang use for her drawing?

- A \( \frac{1}{8} \)
- B \( \frac{3}{8} \)
- C \( \frac{3}{4} \)
- D \( \frac{15}{16} \)
Practice By Standard
Clarifying Objective MBC.G.5.1

1. What is the standard equation of a circle with center (–8, 0) and radius 2.5?
   A. \(x^2 + y^2 = 2.5\)
   B. \((x - 8)^2 + y^2 = 2.5\)
   C. \((x + 8)^2 + y^2 = 6.25\)
   D. \(x^2 + (y + 8)^2 = 6.25\)

2. Which equation was used to create the circle in the graph below?
   A. \((x + 4)^2 + (y + 8)^2 = 144\)
   B. \((x - 4)^2 + (y - 8)^2 = 144\)
   C. \((x + 4)^2 + (y + 8)^2 = 169\)
   D. \((x - 4)^2 + (y - 8)^2 = 169\)

3. The standard equation of a circle is \(x^2 + (y - 36)^2 = 144\). What are the coordinates of the center of the circle?
   A. (0, 6)
   B. (6, 12)
   C. (0, 36)
   D. (36, 0)

4. Which is the graph of a circle whose equation is \((x - 3)^2 + (y + 5)^2 = 16\)?
   A. [Graph A]
   B. [Graph B]
   C. [Graph C]
   D. [Graph D]
1. Given: \( \overleftrightarrow{PO} \) is a perpendicular bisector of \( \overline{MN} \).

Which argument proves that point \( P \) is equidistant from points \( N \) and \( M \)?

\[ \begin{align*}
\text{A} & \quad \triangle PQM \sim \triangle PQN \text{ by SAS.} \\
& \quad \text{Therefore, } NP \cong MP. \\
\text{B} & \quad \triangle PQM \cong \triangle PQN \text{ by SAS.} \\
& \quad \text{Therefore, } NP \cong MP. \\
\text{C} & \quad \triangle PQM \cong \triangle PQN \text{ by SSS.} \\
& \quad \text{Therefore, } QP \cong MP. \\
\text{D} & \quad \triangle PQM \cong \triangle PQN \text{ by SAS.} \\
& \quad \text{Therefore, } QN \cong QM.
\end{align*} \]


2. Given: \( \triangle RST \) is an isosceles triangle, and \( \overrightarrow{SV} \) is an altitude from \( S \) to \( \overrightarrow{TR} \).

Which statement would be made in an argument that proves \( \overrightarrow{SV} \) bisects \( \angle S \)?

\[ \begin{align*}
\text{A} & \quad \angle VRT \cong \angle VSR \\
\text{B} & \quad \triangle SVR \cong \triangle SVT \text{ by SSS.} \\
\text{C} & \quad \triangle SVR \cong \triangle SVT \text{ by AAS.} \\
\text{D} & \quad S \text{ is equidistant from } R \text{ and } T.
\end{align*} \]

3. Given: The medians of \( \triangle EFG \) intersect at point \( K \).

Which plan of reasoning could be used to prove that point \( F \) is two thirds of the distance from each vertex to the midpoint of the opposite side?

\[ \begin{align*}
\text{A} & \quad \text{Graph } \triangle EFG \text{ and the medians on a coordinate plane. Write equations for } \overrightarrow{FJ}, \overrightarrow{EL}, \text{ and } \overrightarrow{GH}. \\
& \quad \text{Show that point } M \text{ occurs on each line.} \\
\text{B} & \quad \text{Graph } \triangle EFG \text{ and the medians on a coordinate plane. Use the distance formula to show that } \\
& \quad \triangle HMF \sim \triangle HME, \triangle JME \sim \triangle JMG, \text{ and } \triangle LMF \sim \triangle LMG. \\
\text{C} & \quad \text{Graph } \triangle EFG \text{ and the medians on a coordinate plane. Write equations for } \overrightarrow{FM}, \overrightarrow{EM}, \overrightarrow{GM}, \overrightarrow{FJ}, \overrightarrow{EL}, \text{ and } \overrightarrow{GH}. \text{ Use the equations to show that point } M \text{ is two thirds of the distance from each vertex to the midpoint of the opposite side.}
\end{align*} \]
4 In the figure below, $\overline{WZ}$ bisects $\angle XWY$.

Which statement would be made in an argument that **disproves** the conjecture that $ZX = ZY$?

A $\triangle ZWY \cong \triangle ZWX$ by AAS.
B $\angle XWZ \cong \angle YWZ$
C $\angle WXZ \cong \angle WZY$
D $\angle Y$ is not a right angle.

5 Which statement would prove that $\triangle FGH$ shown below is an isosceles triangle?

A $(\text{slope } \overline{FG}) \times (\text{slope } \overline{GH}) = 1$
B $(\text{slope } \overline{FG}) \times (\text{slope } \overline{GH}) = -1$
C $FG = GH$
D $m\angle GFH + m\angle GHF \leq 90^\circ$

6 In the figure below, $\triangle PQR$ is a right triangle, and $QS$ is an altitude from $Q$ to $PR$.

Which argument could be used to prove that $PQ$ is the geometric mean of $PS$ and $PR$?

A $\angle PQR \cong \angle PSQ$ because all right angles are congruent. $\angle P \cong \angle P$ because an angle is congruent to itself. So, $\triangle PQR \sim \triangle PSQ$ by AA. It follows that $\frac{PQ}{PS} = \frac{PR}{PQ}$ and $(PQ)^2 = (PS)(PR)$.
B $\angle PSQ \cong \angle QSR$ because all right angles are congruent. $\angle R \cong \angle R$ because an angle is congruent to itself. So, $\triangle PQR \sim \triangle PSQ$ by AA. It follows that $\frac{PQ}{PS} = \frac{PR}{PQ}$ and $(PQ)^2 = (PS)(PR)$.
C $\triangle PQR \sim \triangle PSQ$ because all right triangles are similar. Therefore, $\frac{PQ}{PS} = \frac{PR}{PQ}$ and $(PQ)^2 = (PS)(PR)$.
D $(PQ)^2 = (PR)^2 - (QR)^2$ and $(PQ)^2 = (PS)^2 + (QS)^2$. Therefore, $(PR)^2 - (QR)^2 = (PS)^2 + (QS)^2$. It follows that $\frac{PQ}{PS} = \frac{PR}{PQ}$ and $(PQ)^2 = (PS)(PR)$. 

Go on
Practice By Standard
Clarifying Objective MBC.G.6.2

1. Which statement could be used with the figure below in a proof of the Pythagorean Theorem?

   ![Figure of a square inside a larger square with areas marked]

   A. The area of the inner square is half the area of the outer square.
   B. The area of the outer square is $c^2$.
   C. The area of the outer square is equal to the sum of the areas of the inner square and two of the congruent triangles.
   D. The area of each triangle is $\frac{1}{2}ab$.

2. In $\triangle WXY$, $\angle WXY$ is a right angle, and $\overline{WZ} \perp \overline{WX}$.

   ![Diagram of a right triangle with a rectangle formed by extending one side]

   Which statement can be used in a proof of the Pythagorean Theorem?

   A. $\frac{WX}{VX} = \frac{VX}{ZX}$ and $\frac{WX}{WW} = \frac{WW}{WZ}$
   B. $(VZ)^2 = (WZ)(XZ)$
   C. $(VX)^2 = (VZ)^2 + (ZX)^2$
   D. $(WZ)^2 + (VZ)^2 = (VZ)^2 + (ZX)^2$

3. In the figure below, $\overline{GJ}$ is the longest side of $\triangle GJK$, and $z^2 = x^2 + y^2$.

   ![Diagram of a right triangle with a square added]

   Which plan of reasoning could be used to prove the converse of the Pythagorean Theorem?

   A. Draw a square formed by four triangles that are congruent to $\triangle GJK$ such that each side of the square is represented by $x + y$. Show that $x^2 = (2y)^2$ and $(2y)^2 = z^2 + y^2$.
   B. Draw a right triangle with side lengths $x$, $y$, and $n$ such that the right angle is formed between the sides with lengths $x$ and $y$. Prove that the two triangles are congruent by SSS, so $\angle JKG$ must be a right angle.
   C. Draw a quadrilateral formed by four triangles that are congruent to $\triangle GJK$ such that each side of the quadrilateral is represented by $x + y$. Show that $(x + y)^2 = z^2$.
   D. Draw a triangle that is congruent to $\triangle GJK$ by SAS. Show that the new triangle contains a right angle. Therefore, $\triangle GJK$ contains a right angle.
Practice By Standard
Clarifying Objective MBC.G.6.3

1. Which is the best plan of reasoning to prove the statement in the box below?

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

A. Draw a parallelogram with one diagonal to form two triangles. Prove that the triangles are congruent by ASA.
B. Draw a parallelogram with one diagonal to form two triangles. Prove that the triangles are congruent by SSS.
C. Draw a parallelogram with two diagonals to form four triangles. Prove that the top and bottom triangles are congruent by SAS.
D. Start with a rectangle and shift the vertices until it becomes a parallelogram.

2. A rectangle is graphed on a coordinate grid. One side has a slope of 2. Which is the best explanation for why the other three slopes are $-\frac{1}{2}$, $-\frac{1}{2}$, and 2?

A. Two sides must have the same slope. The other two sides must have opposite reciprocal slopes so they will be perpendicular to the first two sides.
B. Each pair of sides must have slopes that are the same.
C. Opposite sides must be parallel.
D. Adjacent sides must be parallel.

3. EFGH is a rhombus.

Which argument proves that $\overline{EG} \perp \overline{FH}$?

A. Because $EFGH$ is a parallelogram, $\overline{EJ} \cong \overline{GJ}$, $\angle EFG$ and $\angle FGH$ are bisected by the diagonals, and $\angle EFG$ is the supplement of $\angle FGH$. Therefore, $m\angle JFG + m\angle JGF = 90^\circ$. So, $\overline{EG} \perp \overline{FH}$.
B. Because $EFGH$ is a parallelogram, $\overline{EJ} \cong \overline{GJ}$, and $\overline{HJ} \cong \overline{FJ}$. Because $EFGH$ is a rhombus, $\overline{FE} \cong \overline{FG}$ and $\overline{GH} \cong \overline{EH}$. So, $\triangle EJF \cong \triangle GJF \cong \triangle GJH \cong \triangle EJH$. Because these four congruent triangles share a common vertex, $\overline{EG} \perp \overline{FH}$.
C. Because $EFGH$ is a parallelogram, $\overline{EF} \parallel \overline{HG}$. Because $\overline{EG}$ and $\overline{FH}$ are both transversals that cross a pair of parallel lines and $\overline{EG}$ intersects $\overline{FH}$, $\overline{EG} \perp \overline{FH}$.
D. Because $EFGH$ is a parallelogram, $\overline{EJ} \cong \overline{GJ}$. Because $EFGH$ is a rhombus, $\overline{FE} \cong \overline{FG}$. So, $\triangle EJF \cong \triangle GJF$ by SSS. Because $\angle EJF$ and $\angle GJF$ are congruent corresponding angles and a linear pair, they must each measure $90^\circ$. So, $\overline{EG} \perp \overline{FH}$. 

Go on
1. In the figure below, $ST$ is tangent to circle $R$ at point $S$, and $RS$ is a radius of the circle.

What is $m\angle QST$?

A. 23°  
B. 46°  
C. 67°  
D. 113°

2. Given: $WX$ is tangent to circle $O$ at point $W$, and $XY$ is tangent to circle $O$ at point $Y$.

What is $m\angle WXY$?

A. 40°  
B. 70°  
C. 80°  
D. 140°

3. In the circle below, $EF$ is a diameter, $FJ$ is tangent to the circle at point $F$, and $m\widehat{FGH} = 110°$.

What is $m\angle EFH$?

A. 27.5°  
B. 35°  
C. 55°  
D. 110°

4. Given: $MN$ is a radius of the circle, and $m\widehat{PLN} = 285°$.

What is $m\angle NMP$?

A. 37.5°  
B. 75°  
C. 142.5°  
D. 150°
1. In the figure below, \( K \) is the center of the circle, \( KL = 8 \) centimeters, and \( \widehat{HJL} = 16.8 \) centimeters.

What is the measure of \( \widehat{HJL} \) to the nearest degree?

\[ \begin{align*}
\text{A} & \quad 60^\circ \\
\text{B} & \quad 100^\circ \\
\text{C} & \quad 120^\circ \\
\text{D} & \quad 240^\circ
\end{align*} \]

2. A runner sprints 50 yards along the circular curve of a track. The diameter of the curve is 36 yards.

What is the arc measure to the nearest degree along the portion of the circle where the runner sprinted?

\[ \begin{align*}
\text{A} & \quad 80^\circ \\
\text{B} & \quad 95^\circ \\
\text{C} & \quad 143^\circ \\
\text{D} & \quad 159^\circ
\end{align*} \]

3. A group of singers is arranged on the school’s stage as shown below.

What is the approximate length of the portion of the stage’s circumference where the singers are standing?

\[ \begin{align*}
\text{A} & \quad 3.27 \text{ ft} \\
\text{B} & \quad 6.54 \text{ ft} \\
\text{C} & \quad 13.08 \text{ ft} \\
\text{D} & \quad 40.89 \text{ ft}
\end{align*} \]

4. A circular spinner with a radius of 3 inches is divided into 3 parts: Move Forward, Roll Again, and Lose a Turn.

What is the approximate length of the arc of the Lose a Turn section of the spinner?

\[ \begin{align*}
\text{A} & \quad 1.3 \text{ in.} \\
\text{B} & \quad 5 \text{ in.} \\
\text{C} & \quad 6.3 \text{ in.} \\
\text{D} & \quad 18.8 \text{ in.}
\end{align*} \]
Practice By Standard
Clarifying Objective MBC.G.8.1

1 A 275-foot transmission tower is stabilized by 4 wires that are each planted in the ground 450 feet from the tower, as shown below.

What is the approximate total length of the 4 wires?

A 1800.0 ft  C 2109.5 ft
B 1993.3 ft  D 2403.3 ft

2 A 40-foot ladder is set against a wall to form an isosceles right triangle.

Approximately how far is the base of the ladder from the wall?

A 28.3 ft  C 32.5 ft
B 31.2 ft  D 33.6 ft

3 A new highway is being built to route traffic around a residential area. A diagram of the old and new roads is shown below.

How many extra miles will cars now have to drive?

A 4 mi  C 13 mi
B 6 mi  D 17 mi

4 On a map of North Carolina, the cities of Asheboro, Graham, and Benson appear to form a right triangle. Using the map’s scale, Darcy estimates the distance from Asheboro to Graham as 34 miles and the distance from Asheboro to Benson as 75 miles.

What is the approximate distance from Benson to Graham?

A 16 mi  C 67 mi
B 33 mi  D 82 mi

Go on
Practice By Standard
Clarifying Objective MBC.G.8.2

1. In the figure below, the flagpole has height $h$, and $\tan x = 1.5$.

   ![Flagpole Diagram]

   How tall is the flagpole?
   - A. 16 ft
   - B. 25.5 ft
   - C. 36 ft
   - D. 48 ft

2. Mr. Delano is designing a walking path through the botanical gardens at a local park. The path is shown below.

   ![Garden Diagram]

   What is the approximate measure of the angle formed by the path at the Cottage Garden?
   - A. 37°
   - B. 39°
   - C. 53°
   - D. 72°

3. What is the approximate measure of $\angle E$ in the figure below?

   ![Triangle Diagram]

   - A. 31°
   - B. 37°
   - C. 40°
   - D. 74°

4. In the triangle below, which equation should be used to find the length of the hypotenuse?

   ![Triangle Diagram]

   - A. $n = 24 \sin 35°$
   - B. $n = 24 \cos 35°$
   - C. $n = \frac{24}{\sin 35°}$
   - D. $n = \frac{24}{\cos 35°}$

   Go on
1. Patrick cut out the triangular shape shown below to hang on a bulletin board in his room.

```
8 cm 8 cm
 x
```

How long is the hypotenuse of the triangle?

A. 4 cm  
B. 4√2 cm  
C. 8√2 cm  
D. 16 cm

2. Ofelia plans to use 4 congruent nylon triangles with the shapes shown below to construct a kite. The dowel that runs from L to N is 24 inches long.

```
L
60°

M

P

N
```

What is the perimeter of the kite?

A. 24 in.  
B. 24√3 in.  
C. 48√3 in.  
D. 96 in.

3. Maurice is collecting branches to outline a triangular welcome sign for soccer camp. He has a 3-foot branch and an 8-foot branch. He finds four more branches with lengths listed below. Which will allow him to construct the outline of a triangular sign?

A. the 2 ft branch  
B. the 4 ft branch  
C. the 7 ft branch  
D. the 12 ft branch

4. What is the greatest length, in meters, that can be the length of the base of an isosceles triangle with congruent sides each measuring 18.3 meters? Round to the nearest whole number.

A. 26 m  
B. 36 m  
C. 37 m  
D. 54 m

5. If \( y = 4\sqrt{3} \) in the right triangle below, then what is the value of \( z \)?

```
30°

y

z

x
```

A. 8  
B. 12  
C. 8\sqrt{3}  
D. 16
## Practice By Standard
Clarifying Objective MBC.G.9.1

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A baseball has a diameter of 3 inches.</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Baseball Diagram" /></td>
</tr>
<tr>
<td></td>
<td><strong>What is its approximate surface area?</strong></td>
</tr>
<tr>
<td>A</td>
<td>9 in²</td>
</tr>
<tr>
<td>B</td>
<td>10 in²</td>
</tr>
<tr>
<td>C</td>
<td>28.3 in²</td>
</tr>
<tr>
<td>D</td>
<td>113.1 in²</td>
</tr>
</tbody>
</table>

| 2 | Yianni uses cardstock paper to create the right circular cone shown below. |
|   | ![Cone Diagram](image) |
|   | **What is the approximate volume of the cone?** |
| A | 167.5 in³ |
| B | 251.2 in³ |
| C | 502.4 in³ |
| D | 670.2 in³ |

| 3 | Brigit constructed a tent by attaching a quarter of a sphere to half of a right circular cone. The diameter of the sphere is 4 meters, and the height of the cone is 12 meters. |
|   | ![Tent Diagram](image) |
|   | **What is the approximate volume of the tent?** |
| A | 21 m³ |
| B | 33.5 m³ |
| C | 41.9 m³ |
| D | 67.1 m³ |

| 4 | A manufacturer of watercolor markers is designing a balloon for a parade. In the figure below, the radius of the cone is 14 feet. |
|   | ![Balloon Diagram](image) |
|   | **What is the approximate surface area of the balloon?** |
| A | 6993.2 ft² |
| B | 7609 ft² |
| C | 8840.5 ft² |
| D | 9456.2 ft² |
Practice By Standard

Clarifying Objective MBC.G.9.2

1 A cone has a volume of 268 cubic millimeters. What is the volume of a circular cylinder with the same height and diameter as the cone?

A 134 mm³  
B 402 mm³  
C 536 mm³  
D 804 mm³

2 In the figure below, the base areas and heights of the solids are equal. The volume of the prism is 642 cubic yards.

What is the volume of the pyramid?

A 214 yd³  
B 428 yd³  
C 963 yd³  
D 1926 yd³

3 A cone with diameter x feet and height y feet has a volume of 45 cubic feet. What is the volume of a hemisphere with the same dimensions?

A 45 ft³  
B 67.5 ft³  
C 90 mm³  
D 135 mm³

4 A container shaped like a circular cylinder holds 171 cubic inches of water. How many cubic inches of water can a hemispherical container with the same dimensions hold?

A 57 in³  
B 85.5 in³  
C 114 in³  
D 256.5 in³

5 The volume of an octagonal pyramid is 18 cubic meters. What is the volume of an octagonal prism with the same base area and height?

A 6 m³  
B 12 m³  
C 27 m³  
D 54 m³

6 Which statement is true of the figures below?

A The volume of the cylinder is two-thirds the volume of the sphere.  
B The volume of the sphere is two-thirds the volume of the cylinder.  
C The volume of the cylinder is twice the volume of the sphere.  
D The volume of the sphere is three-fourths the volume of the cylinder.

Go on
1. Katrina is using a grid to recreate a drawing of the Grove Park Inn, located in Asheville.

What is the probability that a randomly selected point on the photo lies in an unfinished square?

A. 0.9  
B. 0.55  
C. 0.45  
D. 0.18

2. In the diagram below, $PR$ is the radius of circle $P$ and the diameter of circle $Q$.

What is the probability that a randomly chosen point in the interior of circle $P$ is also in the interior of circle $Q$?

A. $\frac{1}{2}$  
B. $\frac{\pi}{8}$  
C. $\frac{1}{4}$  
D. $\frac{\pi}{16}$

3. Ian drew the North Carolina State Seal on a poster board for a school project. The measurements of the poster board are 3 feet by 4 feet. The seal has a diameter of 2 feet.

If a point on the poster board were chosen at random, what is the probability that it would lie inside the seal?

A. $\frac{\pi}{12}$  
B. $\frac{4\pi}{12}$  
C. $1 - \frac{4\pi}{12}$  
D. $1 - \frac{\pi}{12}$

4. If a point is randomly chosen inside the square, what is the probability that it is outside of the gray triangle?

A. 0.1  
B. 0.25  
C. 0.5  
D. 0.75
1 For their vacation in North Carolina, Leah’s family has decided to visit one lighthouse and one museum. Their lighthouse choices are Bald Head, Cape Hatteras, and Oak Island. The museum choices are Lightfactory or Mint Museum of Art. Which is a diagram of the sample space for the family vacation?

A Bald Head
| Lightfactory
| Mint Museum of Art

Cape Hatteras
| Lightfactory
| Mint Museum of Art

Oak Island
| Lightfactory
| Mint Museum of Art

B Bald Head
| Cape Hatteras
| Mint Museum of Art

Lightfactory
| Bald Head

2 Wendy lives in Cary, North Carolina. When thinking of the first two digits of a password for her computer, she chooses one letter from the word CARY and pairs it with a number from 1 to 3. What is the sample space of Wendy’s choices?

A (C, 1), (A, 1), (R, 1), (Y, 1), (C, 2), (A, 2), (R, 2), (Y, 2)
B (C, A), (C, R), (C, Y), (A, C), (A, R), (A, Y), (R, O), (R, A), (R, Y), (Y, C), (Y, A), (Y, R), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2)
C (C, C), (A, A), (R, R), (Y, Y), (1, 1), (2, 2), (3, 3)
D (C, 1), (C, 2), (C, 3), (A, 1), (A, 2), (A, 3), (R, 1), (R, 2), (R, 3), (Y, 1), (Y, 2), (Y, 3)

3 Which experiment would yield the sample space shown in the table below?

<table>
<thead>
<tr>
<th>+</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
</tbody>
</table>

A sum when rolling two 14-sided dice
B sum when rolling a 6-sided die and an 8-sided die
C rolling a 14-sided die
D difference between lowest and highest roll of a 6-sided die and an 8-sided die
1. A spinner has 8 equal sections numbered 1 through 8. The faces of a cube are labeled A through F. Hannah will spin the spinner and roll the cube. What is the probability of spinning an even number and rolling a vowel?

   - A: \( \frac{1}{2} \)
   - B: \( \frac{1}{3} \)
   - C: \( \frac{1}{4} \)
   - D: \( \frac{1}{6} \)

2. Ricardo is picking out a scarf. His options of fabric and color are in the diagram below.

   - Cotton
     - Blue
     - Black
     - Red
   - Polyester
     - Green
     - Red
     - Yellow
   - Wool
     - Green
     - Red
     - Yellow

   If he chooses one option at random, what is the probability he will choose a red scarf?

   - A: \( \frac{3}{11} \)
   - B: \( \frac{2}{11} \)
   - C: \( \frac{1}{11} \)
   - D: \( \frac{1}{33} \)

3. The options of what Juan can do tonight are shown in the chart below.

   If all options are equally likely, what is the probability that Juan will do an activity at the Pavilion tonight?

   - A: \( \frac{5}{12} \)
   - B: \( \frac{2}{7} \)
   - C: \( \frac{1}{4} \)
   - D: \( \frac{1}{5} \)

4. Mr. Williams has a coin pouch with 12 quarters ranging from the year 1995 through 2006. There is one quarter from each year in the pouch. He has another container with 8 dimes ranging from the year 1998 through 2005. There is one dime from each year in the container. If he randomly selects one quarter and one dime, what is the probability that they will each be from the year 2000?

   - A: \( \frac{1}{16} \)
   - B: \( \frac{1}{20} \)
   - C: \( \frac{1}{48} \)
   - D: \( \frac{1}{96} \)
Practice By Standard
Clarifying Objective MBC.S.3.3

1 An artist has 7 new and 8 used paintbrushes in a container. He randomly selects two brushes from the container. Both brushes are new. Which statement about this situation is true?

A $P(\text{both brushes are new})$ without replacement $> P(\text{both brushes are new})$ with replacement

B $P(\text{both brushes are new})$ without replacement $< P(\text{both brushes are new})$ with replacement

C $P(\text{both brushes are new})$ without replacement $= P(\text{both brushes are new})$ with replacement

D $P(\text{both brushes are new})$ without replacement $+ P(\text{both brushes are new})$ with replacement $= 1$

2 Marla is one of 8 students whose names are in a bag for a drawing. Two names will be selected at random to win a book. The first student chosen is Jason. Which statement is true about Marla’s chances of winning the second book?

A Marla has a better chance if Jason’s name is kept out of the bag.

B Marla has a better chance if Jason’s name is returned to the bag.

C Marla has a better chance at winning the second book than she did at winning the first book, whether or not Jason’s name is returned to the bag.

D Marla has a better chance than Jason, even if his name is returned to the bag.

3 The table below lists several threatened and endangered animals of North Carolina. Lacey’s science teacher writes the name of each animal on a piece of paper and puts them all in a box. Then, each student is allowed to pick a name from the box. The first student selects a turtle and will either keep the name out or return it to the box.

<table>
<thead>
<tr>
<th>Sea Turtles</th>
<th>Whales</th>
<th>Wolves</th>
</tr>
</thead>
<tbody>
<tr>
<td>green</td>
<td>finback</td>
<td>gray</td>
</tr>
<tr>
<td>hawksbill</td>
<td>humpback</td>
<td>red</td>
</tr>
<tr>
<td>Kemp’s ridley</td>
<td>right</td>
<td></td>
</tr>
<tr>
<td>leatherback</td>
<td></td>
<td></td>
</tr>
<tr>
<td>loggerhead</td>
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<td></td>
</tr>
</tbody>
</table>

If Lacey is the second student who picks a name from the box, which statement is true?

A Lacey has the same chance of selecting a turtle whether the first name is returned to the box or not.

B Lacey will probably not select a turtle because the chances are too small.

C Lacey has a better chance of selecting a turtle if the first student does not put the name back in the box.

D Lacey has a better chance of selecting a turtle if the first student puts the name back in the box.
Practice Test

1 Which statement was used to create the truth table below?

<table>
<thead>
<tr>
<th>Writing scores from SAT</th>
<th>Writing scores from ACT</th>
<th>Qualified?</th>
</tr>
</thead>
<tbody>
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<td>Yes</td>
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<td>Yes</td>
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<tr>
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</tr>
<tr>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

A If an applicant submits writing scores from the SAT, then he or she must also submit writing scores from the ACT in order to qualify for admission to NC State.

B If an applicant submits writing scores from the ACT, then he or she must also submit writing scores from the SAT in order to qualify for admission to NC State.

C An applicant must submit writing scores from the SAT and from the ACT in order to qualify for admission to NC State.

D An applicant must submit writing scores from the SAT or from the ACT in order to qualify for admission to NC State.

2 $EFGH \sim MNPQ$ with the following measures: $EF = QM = 14$, $PQ = 3.2$, $FG = 21$, and $MN = 4$. What is the perimeter of $EFGH$?

A 27.2  C 102.7

B 95.2  D 147.7

3 A spherical water tank has a radius of 10 feet. If the tank is empty and water is pumped at a rate of 46.5 cubic feet per minute, about how long will it take to fill the water tank?

A 60 min  D 90 min

B 70 min  C 80 min

4 Given:

$\triangle QRS$ is equilateral.

$ST$ is an altitude of $\triangle QRS$.

Which set of statements can be used in an argument to prove that $ST$ is a perpendicular bisector of $QR$?

A $\triangle QRS \sim \triangle TRS$ and $m \angle STR + m \angle STQ = 180^\circ$.

B $QR \cong SR$, $ST \cong ST$, and $QS \cong SR$.

C $m \angle Q = m \angle R = 60^\circ$, $ST \cong ST$, and $m \angle STR = m \angle STQ = 90^\circ$.

D $SR \cong SR$, $SO \cong SO$, and $m \angle STR = m \angle STQ = 90^\circ$. 

Go on
5. Quinn is creating the truth table below.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>q</td>
<td>p and q</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

How should he complete the table?

A. Write “T”, “T”, “T”, and “F” in that order in the “p and q” column.
B. Write “T”, “T”, “F”, and “F” in that order in the “p and q” column.
C. Write “T”, “F”, “F”, and “F” in that order in the “p and q” column.
D. Write “T”, “F”, “T”, and “F” in that order in the “p and q” column.

6. Jamir goes for a hike using a global positioning system instead of following the local trails. He hikes 3.2 kilometers north and then 4.8 kilometers west. About how far is he from his starting point?

A. 8 km  B. 7.5 km  C. 6.4 km  D. 5.77 km

7. Which set of lengths could be used to create a triangle with string?

A. 19 in., 82 in., 120 in.
B. 7 in., 11 in., 20 in.
C. 5 in., 10 in., 12 in.
D. 3 in., 4 in., 8 in.

8. What is the length $x$ in the figure below?

A. 35 units  B. 37.5 units  C. 40 units  D. 42.5 units

9. Radha shares a room with her younger sister. For privacy, she hangs a cone of netting around and under the chair in her half of the room. The netting was priced at $3.40 per square meter.

Approximately how much did Radha spend on the canopy?

A. $11  B. $45  C. $59  D. $111
10. In circle $P$ below, the length of $\overline{STQ}$ is 24.2 yards, the length of $\overline{QR}$ is 13.4 yards, and $SP = 9$ yards.

To the nearest degree, what is the measure of $\overline{SQR}$?

A. $239^\circ$
B. $154^\circ$
C. $120^\circ$
D. $85^\circ$

11. Which set of conditions is sufficient to prove that two pentagons are congruent?

A. Four angles and the included sides of one pentagon are congruent to the corresponding four angles and included sides of the other pentagon.
B. Corresponding diagonals of the pentagons are congruent.
C. The five sides of one pentagon are congruent to the five sides of the other pentagon.
D. The five angles of one pentagon are congruent to the five angles of the other pentagon.

12. A circular ballroom with a diameter of 150 feet is having a square dance floor installed. The corners of the dance floor touch the circumference of the circular ballroom. What is the approximate area of the dance floor?

A. $7800 \text{ ft}^2$
B. $11,250 \text{ ft}^2$
C. $17,750 \text{ ft}^2$
D. $22,500 \text{ ft}^2$

13. Which of the following would have the same truth value as the statement in the box below?

If a line segment is a diameter of a circle, then it is also a chord.

A. inverse
B. contrapositive
C. inverse and converse
D. converse and contrapositive

14. Which type of statement can be proven true?

A. axiom
B. postulate
C. common notion
D. theorem
15. Which radius disproves the claim that the number representing the area of a circle is always greater than the number representing its circumference?

- A. $2\pi$
- B. 4
- C. $\pi$
- D. 1

16. Dynamic geometry software was used to construct congruent triangles using circle $P$, diameters $KM$ and $JL$, and parallel chords $JK$ and $ML$.

17. Which statement has an inverse that is always true?

- A. If a figure is a quadrilateral, then it has 4 sides.
- B. If a figure is a triangle, then it does not have 4 sides.
- C. If a figure is a square, then it is a rectangle.
- D. If a figure is a rhombus, then it is a parallelogram.

18. A builder is planning to install a water system in an area of new development. A graph that represents the possible connections between the neighborhoods and the costs in thousands of dollars is shown below.

What is the minimum cost, in thousands of dollars, of building a water system that reaches each neighborhood?

- A. 125
- B. 129
- C. 133
- D. 134
19. Jasmine wants to prove that $\triangle MNP \cong \triangle OPN$ in the parallelogram shown below.

Which statement supports Jasmine’s assertion that $\angle 1 \cong \angle 2$?

A. If two parallel lines are intersected by a transversal, then alternate interior angles are congruent.
B. If two parallel lines are intersected by a transversal, then corresponding angles are supplementary.
C. If two parallel lines are intersected by a transversal, then vertical angles are congruent.
D. If two parallel lines are intersected by a transversal, then linear pairs are supplementary.

20. Quadrilateral $ABCD$ is a parallelogram. Fredna wants to prove that $ABCD$ is also a rhombus. Which plan should she use?

A. Prove the diagonals bisect each other.
B. Prove the diagonals are congruent.
C. Prove both pairs of opposite sides are congruent.
D. Prove all four sides are congruent.

21. Katie works for a caterer on the weekends. The diagram shows a platter of different cheeses that she assembled for a party. The diameter of the platter is 14 inches.

About how long is the portion of the platter’s circumference along the edge of the Farmhouse Cheddar section?

A. 8.8 in.
B. 17.6 in.
C. 26.4 in.
D. 35.2 in.

22. In a right triangle, $\cos x = \frac{24}{a}$, and $\sin x = \frac{7}{a}$. What is $\tan x$?

A. $\frac{3}{7}$
B. $\frac{a}{24}$
C. $\frac{24}{7}$
D. $\frac{7}{24}$
Practice Test (continued)

23 Triangle $EGH$ is a right triangle.

Which statement would best support the argument that if $\angle E \cong \angle G$, then $m\angle E = 45^\circ$?

- **A** $e^2 + g^2 = h^2$
- **B** If two angles of a triangle are congruent, then the sides opposite those angles are congruent.
- **C** The smallest angle is opposite the shortest side in a triangle.
- **D** $\angle E$ and $\angle G$ are complementary.

24 $RT$ is a diameter in $\odot M$ below. What is $m\angle T$?

- **A** 12.5°
- **B** 25°
- **C** 50°
- **D** 65°

25 Which argument is an example of deductive reasoning?

- **A** The corresponding angles of two regular pentagons are congruent. So, the pentagons are congruent.
- **B** All sides of a regular pentagon are congruent. So, all sides of a regular hexagon are congruent.
- **C** The first three pairs of corresponding angles of a pentagon are congruent. So, the remaining two pairs of corresponding angles are congruent.
- **D** If two regular pentagons have the same perimeter, then they are congruent. Pentagons $ABCDE$ and $RSTUV$ have the same perimeter. So, they are congruent.

26 Which conclusion can be inferred from the given diagram of point $P$ and line $\ell$ below?

- **A** Point $P$ and line $\ell$ are collinear.
- **B** Point $P$ is equidistant to all points on line $\ell$.
- **C** There exists exactly one line through $P$ that is parallel to $\ell$.
- **D** There exist an infinite number of lines through $P$ that are parallel to $\ell$.
Practice Test (continued)

27 In the circle below, \( ST \) is a radius of circle \( S \), \( QR \) is a chord of circle \( S \), and \( \angle QXS \) is a right angle.

Larry wants to prove that \( QX \cong XR \). He makes the flow chart proof shown below.

\[ \angle QXS \text{ is a right angle.} \]
\[
\begin{align*}
\angle QXS \text{ is a right angle.} & \quad \text{Given} \quad ST \text{ is a radius of circle } S. \\
QR \perp ST & \quad ??? \quad \text{Given} \quad QX \cong XR \\
& \quad \text{If a radius is perpendicular to a chord, it bisects the chord.}
\end{align*}
\]

Which reason should Larry use to justify \( QR \perp ST \)?

- A given
- B definition of radius
- C All right angles are congruent.
- D definition of perpendicular lines

28 In the figure below, which given information could be used to prove that \( \triangle MNP \sim \triangle RST ?

\[ \begin{array}{c}
\angle N \cong \angle S \text{ and } PM = 6 \\
\angle P \cong \angle T \text{ and } RS = 6 \\
\angle N \cong \angle S \text{ and } RS = 6 \\
\angle M \cong \angle R \\
\end{array} \]

29 In the figure shown below, the volume of the sphere is 120 mm\(^3\).

What is the volume of the cone?

- A 60 mm\(^3\)
- B 80 mm\(^3\)
- C 180 mm\(^3\)
- D 240 mm\(^3\)
30. Lin folds a square piece of patty paper in half lengthwise. She draws a point on the crease and then folds the paper two more times so that lines are formed through the point and the bottom corners of the paper.

Which explanation best justifies that Lin constructed congruent triangles?

A. Each triangle contains a 90° angle, and the triangles share a common side. By SSS, the triangles are congruent.

B. She created an equilateral triangle that is cut in half. By SSS, the triangles are congruent.

C. All folds were made, so that two 30°–60°–90° triangles were created. The triangles are congruent by AAA.

D. The bottom is divided into two congruent segments. Each triangle contains a 90° angle, and the triangles share a common side. By SAS, the triangles are congruent.

31. Catalina is painting a front view of the Biltmore House, located in Asheville, North Carolina. She is working on the canvas below.

What is the area of the canvas?

A. 4320 in²
B. 4896 in²
C. 6528 in²
D. 9180 in²

32. In the right triangle below, which equation could be used to find x?

A. \( \sin x^\circ = \frac{4.6}{5} \)
B. \( \cos x^\circ = \frac{4.6}{5} \)
C. \( \sin x^\circ = \frac{6}{4.6} \)
D. \( \cos x^\circ = \frac{6}{4.6} \)
Practice Test (continued)

33 Given:
In \( \triangle EFG \), \( x^2 + y^2 = z^2 \), \( z > x \), and \( z > y \).

Nigel wants to prove that \( \triangle EFG \) is a right triangle. What is the best way that he could plan his proof?

A Draw three squares, one with side length \( x \), one with side length \( y \), and one with side length \( z \). Show algebraically that \( x^2 + y^2 = z^2 \).

B Draw a square with side length \( z \). Show that a corner of this square is supplementary to \( \angle F \).

C Draw right triangle \( HJK \) with side lengths \( x \), \( y \), and \( n \). Show algebraically that \( z = n \) and thus the triangles are congruent by SSS. Use corresponding parts to prove that \( \angle F \) is a right angle, and therefore, \( \triangle EFG \) is a right triangle.

D Draw right triangle \( HJK \) with side lengths \( x \), \( y \), and \( n \). Show geometrically that \( \angle G \cong \angle K \) and thus the triangles are congruent by SSA. Use corresponding parts to prove that \( \angle F \) is a right angle, and therefore, \( \triangle EFG \) is a right triangle.

34 What is the sample space for a two-stage experiment where the first stage is spinning a spinner labeled from \( Q \) to \( T \) in four equal sections and the second stage is spinning a spinner numbered 1 to 3 in three equal sections?

A \((Q, 1), (Q, 2), (Q, 3), (R, 1), (R, 2), (R, 3), (S, 1), (S, 2), (S, 3), (T, 1), (T, 2), (T, 3)\)

B \((1, 1), (1, 2), (1, 3), (1, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3)\)

C \((1, 1), (2, 2), (3, 3), (Q, Q), (R, R), (S, S), (T, T)\)

D \((Q, R), (Q, S), (Q, T), (R, Q), (R, S), (R, T), (S, Q), (S, R), (S, T), (T, Q), (T, R), (T, S), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2)\)

35 In \( \odot J \), \( m\angle EKL = 48^\circ \).

What is \( m\angle EJL \)?

A \( 48^\circ \)

B \( 72^\circ \)

C \( 96^\circ \)

D \( 144^\circ \)
Practice Test (continued)

36 The figure below shows parallelogram EFGH.

![Diagram of parallelogram EFGH]

Which statement proves that EFGH is a rectangle?

A  \( \text{slope } \overline{EF} = \text{slope } \overline{FG} \)

B  \( \text{slope } \overline{EF} = 2 \times (\text{slope } \overline{FG}) \)

C  \( (\text{slope } \overline{EF}) \times (\text{slope } \overline{FG}) = -1 \)

D  \( (\text{slope } \overline{EF}) \times (\text{slope } \overline{FG}) = 1 \)

37 In the figure below, lines \( \ell \) and \( m \) are parallel. Which pair of angles is supplementary?

![Diagram of parallel lines \( \ell \) and \( m \)]

A  \( \angle 1 \) and \( \angle 6 \)

B  \( \angle 2 \) and \( \angle 3 \)

C  \( \angle 3 \) and \( \angle 7 \)

D  \( \angle 4 \) and \( \angle 5 \)

38 Quadrilaterals JKLM and WXYZ are graphed on the coordinate plane below.

![Diagram of quadrilaterals JKLM and WXYZ]

Which method could be used to prove that JKLM \( \sim \) WXYZ?

A  Show that when each coordinate of points \( W, X, Y, \) and \( Z \) is multiplied by 2, the resulting coordinates are those of points \( J, K, L, \) and \( M. \)

B  Use the distance formula to show that \( \frac{JK}{WX} = \frac{KL}{XY} = \frac{LM}{YZ}. \)

C  Use a protractor to show that \( m\angle J = m\angle W, m\angle K = m\angle X, \) and \( m\angle L = m\angle Y. \)

D  Use a protractor and the distance formula to show that \( m\angle K = m\angle X, m\angle L = m\angle Y, \) and \( \frac{JK}{WX} = \frac{KL}{XY}. \)
 Practice Test  (continued)

39 The right triangle below has vertices Q(0, 0) and R(x, 0).

Which conclusion can be inferred?

A. The coordinates of point S are (x, 2x).

B. The coordinates of point S are (x, x√2).

C. The coordinates of point S are (x, x√3).

D. The coordinates of point S are \( \left( x, \frac{x\sqrt{3}}{3} \right) \).

40 A circle is drawn on a coordinate plane. It has a center at (−8, 12), and it is tangent to the y-axis. Which equation represents the circle?

A. \((x + 8)^2 + (y - 12)^2 = 64\)

B. \((x - 8)^2 + (y + 12)^2 = 64\)

C. \((x + 8)^2 + (y - 12)^2 = 144\)

D. \((x - 8)^2 + (y + 12)^2 = 144\)

41 An octagonal prism has a volume of 495 cubic feet. What is the volume of a pyramid that has the same base area and height as the prism?

A. 165 ft³

B. 247.5 ft³

C. 990 ft³

D. 1485 ft³

42 Given:

\( \angle 1 \) and \( \angle 2 \) are supplementary.

\( \angle 1 \) and \( \angle 3 \) are supplementary.

Oscar wants to use an indirect argument to prove that \( \angle 2 \cong \angle 3 \). How should he begin his argument?

A. Assume that \( \angle 2 \) is not congruent to \( \angle 3 \).

B. \( m\angle 1 + m\angle 2 = 180° \) and \( m\angle 1 + m\angle 3 = 180° \)

C. Assume that \( \angle 1 \) and \( \angle 2 \) are not supplementary and that \( \angle 1 \) and \( \angle 3 \) are not supplementary.

D. Assume that supplementary angles do not exist.
Practice Test (continued)

43 Casey selects a jack at random from a standard deck of 52 playing cards and then replaces it. After shuffling the deck, she hopes the second card she selects will be an ace. Which statement about this situation is true?

A Casey has a better chance of selecting the ace if the jack is replaced.

B Casey has a better chance of selecting the ace if the jack is not replaced.

C It does not matter if the jack is replaced or kept out of the deck; either way, it is equally likely that Casey will select the ace.

D If the jack is replaced, the probability of selecting an ace as a second card is higher than the probability of selecting another jack as a second card.

44 At 871 feet tall, the Bank of America Corporate Center is the tallest building in North Carolina. Allie is using a scale of 3 in. = 150 ft to draw a model of the building. How many inches tall is the building in her drawing?

A 5.81 in.

B 11.61 in.

C 17.42 in.

D 43.55 in.

45 Josh is remodeling his kitchen. He makes a list of all the necessary tasks, the number of hours required for each task, and any prior tasks that must be completed in advance of each task when appropriate.

<table>
<thead>
<tr>
<th>Task Code</th>
<th>Task</th>
<th>Time (in hours)</th>
<th>Prior Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Remove cabinets.</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>Remove counter-tops.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>Remove sink.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>Prepare walls.</td>
<td>3</td>
<td>E, F</td>
</tr>
<tr>
<td>I</td>
<td>Prepare floor.</td>
<td>5</td>
<td>H</td>
</tr>
<tr>
<td>J</td>
<td>Paint walls.</td>
<td>6</td>
<td>I</td>
</tr>
<tr>
<td>K</td>
<td>Install floor.</td>
<td>8</td>
<td>I</td>
</tr>
<tr>
<td>L</td>
<td>Install cabinets.</td>
<td>10</td>
<td>J, K</td>
</tr>
<tr>
<td>M</td>
<td>Install sink.</td>
<td>3</td>
<td>G, L</td>
</tr>
<tr>
<td>N</td>
<td>Install counter-tops.</td>
<td>2</td>
<td>N</td>
</tr>
</tbody>
</table>

What is the earliest possible finish time?

A 21 hours

B 30 hours

C 34 hours

D 47 hours
Practice Test (continued)

46 The school colors of Charles D. Owen High School in Black Mountain, North Carolina, are maroon and gray. A member of the school’s basketball team has a shirt in each color, shorts in each color, and white, black, and gray sneakers. What is the probability that a practice outfit chosen at random will have a maroon shirt and gray sneakers?

\[
\begin{align*}
A & : \quad \frac{1}{12} \\
B & : \quad \frac{1}{6} \\
C & : \quad \frac{1}{4} \\
D & : \quad \frac{1}{3}
\end{align*}
\]

47 Colleen is an interior designer. Her drawing of the layout of the Brown family’s new living room furniture is shown on the grid below.

If Colleen uses a laser pointer to present her design to the family, what is the probability that the first point she randomly selects is within the area labeled “sofa”?

\[
\begin{align*}
A & : \quad 0.4 \\
B & : \quad 0.1 \\
C & : \quad 0.25 \\
D & : \quad 0.05
\end{align*}
\]

48 Given:

I. \( \triangle JKM \) and \( \triangle LKM \) share side \( \overline{KM} \)

II. \( JM \cong LM \)

Which additional piece of information is sufficient to prove that \( \triangle JKM \) and \( \triangle LKM \) are congruent?

A \( \overline{KM} \cong \overline{JM} \)

B \( \angle JKM \cong \angle LKM \)

C \( \angle JMK \cong \angle LMK \)

D \( \angle MJK \cong \angle KML \)

49 Belk Theatre, located in the North Carolina Blumenthal Performing Arts Center, has 2200 seats. Suppose Henri is attending a show at this theatre, and the announcer says that two seats will be chosen at random to win a prize. The first person chosen is sitting several rows back from Henri. Which statement is true about Henri’s chances of winning?

A Henri had a better chance of being chosen first than he has of being chosen second.

B Henri has a better chance of being chosen second if the first person cannot win twice.

C Henri has the same chance of being chosen second whether the first person chosen can win twice or not.

D Henri has a better chance of being chosen second if the first person can win twice.
Practice Test  (continued)

50 The sample space from a two-stage experiment is shown below.

yellow, green
yellow, orange
red, green
red, orange
blue, green
blue, orange
purple, green
purple, orange

What are the stages of the experiment?

A Randomly select one of four fill colors and then randomly select one of two stroke colors.
B Randomly select one of two fill colors and then randomly select one of four stroke colors.
C Randomly select one of eight fill colors and then randomly select one of four stroke colors.
D Randomly select one of three fill colors and then randomly select one of four stroke colors.

51 A hemisphere with diameter $d$ meters and radius $r$ meters has a volume of 54 cubic meters. What is the volume of a cylinder with diameter $d$ meters and height $r$ meters?

A $18 \text{ m}^3$  C $81 \text{ m}^3$
B $36 \text{ m}^3$  D $162 \text{ m}^3$

52 What is the approximate length of the ladder in the figure below?

A $8 \text{ ft}$  C $15 \text{ ft}$
B $10 \text{ ft}$  D $25 \text{ ft}$

53 Don is leading a tour group through the museum. Possible tours with walking distances in yards are shown in the vertex-edge graph below. Tours must begin and end at the entrance. The tour guide must pass each exhibit on a tour exactly once.

What is the shortest tour that includes exhibit $T$?

A $R, S, T, Q$  C $M, P, Q, T, S, R$
B $M, P, U, T, Q$  D $M, N, O, U, T, S, R$
Practice Test (continued)

54 What is \( m\angle T \) in the figure below?

\[ T \]
\[ 110^\circ \]
\[ 148^\circ \]
\[ 75^\circ \]

\[ \begin{array}{ll}
A & 19^\circ \\
B & 24^\circ \\
C & 27^\circ \\
D & 37.5^\circ \\
\end{array} \]

55 Laine's choices for a lake activity with her friends are shown in the diagram below.

Kerr Lake
- Fishing
- Waterskiing
- Pontoon Ride

Lake James
- Fishing
- Waterskiing

Falls Lake
- Fishing
- Waterskiing
- Tubing
- Pontoon Ride

How could Milena use her drawing to prove the Pythagorean Theorem?

A Show that \( z^2 = (x + y)(x - y) \).
B Show that \( y^2 = (z - x)^2 \).
C Show that \( x^2 = (z + y)^2 \).
D Show that \( x^2 = (z + y)(z - y) \).

56 Milena draws a circle with radius \( z \). She also constructs a right triangle with legs \( x \) and \( y \) and hypotenuse \( z \). She extends a leg of the triangle to create a chord that intersects the diameter of the circle.

57 In the figure below, \( \overline{RT} \) bisects \( \angle QRS \).

José is going to write a proof to show that \( \triangle QRS \) is an isosceles triangle. Which additional given information does he need?

A \( \overline{RT} \perp \overline{QS} \)
B \( RT = \frac{1}{2} QS \)
C \( \overline{RT} \cong \overline{QT} \)
D \( \angle Q \cong \angle SRT \)
58. Dipa drew a scale model of her backyard. The side that measures 20 inches in her drawing corresponds to an actual property line that is 8 meters long.

![Diagram of scale model]

What is the perimeter of Dipa’s actual backyard in meters?

A. 27.2 m  
B. 30.4 m  
C. 32.8 m  
D. 45.6 m

59. In the figure below, the diameter of the circle is equal to one-half the side length of the square.

![Diagram of square and circle]

What is the probability that a randomly selected point inside the square is also inside the circle?

A. \( \frac{\pi}{2} \)  
B. \( \frac{\pi}{4} \)  
C. \( \frac{\pi}{8} \)  
D. \( \frac{\pi}{16} \)

60. Which graph shows a circle with the equation \((x + 4)^2 + (y - 1)^2 = 36\)?

A.  
B.  
C.  
D.  

STOP