

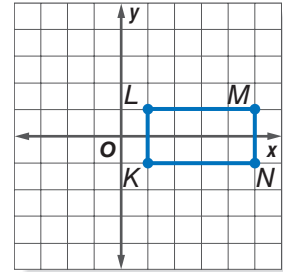
3 Multiple Transformations

You have already learned how to find the coordinates of a geometric figure after dilations, translations, reflections, and rotations. In addition, you can find the coordinates of a geometric figure after the result of more than one transformation.

EXAMPLE Find Coordinates After Multiple Transformations

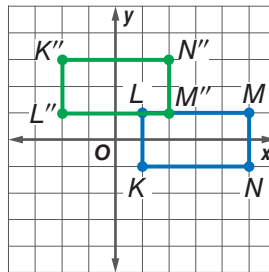
- 1** Quadrilateral $LMNK$ is translated 3 units left and 2 units down. Then the image is reflected across the x -axis. Find the coordinates after both transformations. Then graph the final image.

The vertices after the first transformation can be found by adding -3 units to the x -coordinates and -2 to the y -coordinates. The vertices after the second transformation can be found by multiplying the y -coordinates by -1 .



Vertices of Quadrilateral $LMNK$	$(x + (-3), y + (-2))$	Vertices After First Transformation	$(x, y \cdot -1)$	Vertices After Second Transformation
$L(1, 1)$	$(1 + (-3), 1 + (-2))$	$L'(-2, -1)$	$(-2, -1 \cdot -1)$	$L''(-2, 1)$
$M(5, 1)$	$(5 + (-3), 1 + (-2))$	$M'(2, -1)$	$(2, -1 \cdot -1)$	$M''(2, 1)$
$N(5, -1)$	$(5 + (-3), -1 + (-2))$	$N'(2, -3)$	$(2, -3 \cdot -1)$	$N''(2, 3)$
$K(1, -1)$	$(1 + (-3), -1 + (-2))$	$K'(-2, -3)$	$(-2, -3 \cdot -1)$	$K''(-2, 3)$

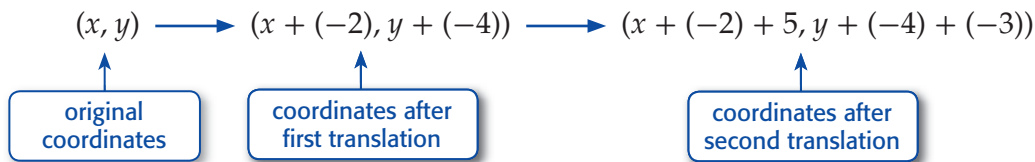
Use the vertices of quadrilateral L'' , M'' , N'' , and K'' to graph the final image.



You can often describe the result of multiple transformations without graphing. Instead, you can use algebra to represent the coordinates of each vertex after each transformation. Begin by representing the coordinates of each vertex of the original figure as (x, y) . A translation of 3 units left and 2 units down can be represented as $(x + (-3), y + (-2))$, or $(x - 3, y - 2)$. After this image is reflected across the x -axis, the coordinates become $(x - 3, -1(y - 2))$, or $(x - 3, -y + 2)$.

EXAMPLE**Describe the Result of Multiple Transformations**

- 2 Triangle PQR is translated 2 units left and 4 units down. Then the image is translated 5 units right and 3 units down. Describe the result after both transformations have been performed.



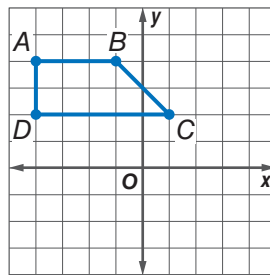
$$x + (-2) + 5 \text{ simplifies to } x + 3$$

$$y + (-4) + (-3) \text{ simplifies to } y + (-7)$$

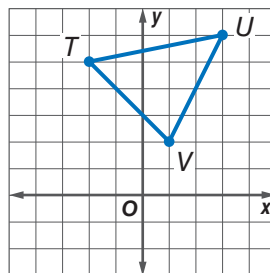
So, the result after both transformations have been performed is the same as a single translation of 3 units right and 7 units down.

Exercises

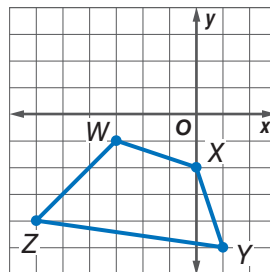
1. Quadrilateral $ABCD$ is translated 1 unit right and 3 units up. Then the image is reflected across the y -axis. Find the coordinates after both transformations. Then graph the final image.



2. Triangle TUV is translated 4 units up. Then the image is rotated 90° counterclockwise about the origin. Find the coordinates after both transformations. Then graph the final image.



3. Quadrilateral $WXYZ$ is reflected across the x -axis. Then the image is reflected across the y -axis. Find the coordinates after both transformations. Then graph the final image.



For Exercises 4–6, describe the final result after each set of transformations below without graphing.

4. Triangle XYZ is reflected across the x -axis and then across the y -axis.
5. Quadrilateral $GHJK$ is translated 3 units left and 2 units down. Then the image is reflected 4 units right and 5 units up.
6. Triangle CDF is rotated 90° counterclockwise about the origin. Then the image is rotated 180° counterclockwise about the origin.