10-2

Simplifying Radical Expressions

Then
You simplified radicals. (Lesson 0-2)

Now
- Simplify radical expressions by using the Product Property of Square roots.
- Simplify radical expressions by using the Quotient Property of Square roots.

New Vocabulary
radical expression
rationalizing the denominator
conjugate

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- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Why?

The Sunshine Skyway Bridge across Tampa Bay in Florida, is supported by 21 steel cables, each 9 inches in diameter.

To find the diameter a steel cable should have to support a given weight, you can use the equation $d = \sqrt{\frac{w}{8}}$, where d is the diameter of the cable in inches and w is the weight in tons.

Product Property of Square Roots A **radical expression** contains a radical, such as a square root. Recall the expression under the radical sign is called the radicand. A radicand is in simplest form if the following three conditions are true.

- No radicands have perfect square factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

The following property can be used to simplify square roots.

Key Concept — **Product Property of Square Roots** — For Your FOLDABLE

Words For any nonnegative real numbers a and b, the square root of ab is equal to the square root of a times the square root of b.

Symbols $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, if $a \geq 0$ and $b \geq 0$

Examples $\sqrt{4 \cdot 9} = \sqrt{36}$ or 6 $\qquad$ $\sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9} = 2 \cdot 3$ or 6

EXAMPLE 1 **Simplify Square Roots**

Simplify $\sqrt{80}$.

$\sqrt{80} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5}$ Prime factorization of 80

$= \sqrt{2^2} \cdot \sqrt{2^2} \cdot \sqrt{5}$ Product Property of Square Roots

$= 2 \cdot 2 \cdot \sqrt{5}$ or $4\sqrt{5}$ Simplify.

Check Your Progress

1A. $\sqrt{54}$ $\qquad$ **1B.** $\sqrt{180}$

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EXAMPLE 2 Multiply Square Roots

Simplify $\sqrt{2} \cdot \sqrt{14}$.

$\sqrt{2} \cdot \sqrt{14} = \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{7}$ Product Property of Square Roots

$= \sqrt{2^2} \cdot \sqrt{7}$ or $2\sqrt{7}$ Product Property of Square Roots

Check Your Progress

2A. $\sqrt{5} \cdot \sqrt{10}$ **2B.** $\sqrt{6} \cdot \sqrt{8}$

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Consider the expression $\sqrt{x^2}$. It may seem that $x = \sqrt{x^2}$, but when finding the principal square root of an expression containing variables, you have to be sure that the result is not negative. Consider $x = -3$.

$$\sqrt{x^2} \stackrel{?}{=} x$$

$$\sqrt{(-3)^2} \stackrel{?}{=} -3 \quad \text{Replace } x \text{ with } -3.$$

$$\sqrt{9} \stackrel{?}{=} -3 \quad (-3)^2 = 9$$

$$3 \neq -3 \quad \sqrt{9} = 3$$

Notice in this case, if the right hand side of the equation were $|x|$, the equation would be true. For expressions where the exponent of the variable inside a radical is even and the simplified exponent is odd, you must use absolute value.

$$\sqrt{x^2} = |x| \qquad \sqrt{x^3} = |x|\sqrt{x} \qquad \sqrt{x^4} = x^2 \qquad \sqrt{x^6} = |x^3|$$

EXAMPLE 3 Simplify a Square Root with Variables

Simplify $\sqrt{90x^3y^4z^5}$.

$\sqrt{90x^3y^4z^5} = \sqrt{2 \cdot 3^2 \cdot 5 \cdot x^3 \cdot y^4 \cdot z^5}$ Prime factorization

$= \sqrt{2} \cdot \sqrt{3^2} \cdot \sqrt{5} \cdot \sqrt{x^2} \cdot \sqrt{x} \cdot \sqrt{y^4} \cdot \sqrt{z^4} \cdot \sqrt{z}$ Product Property

$= \sqrt{2} \cdot 3 \cdot \sqrt{5} \cdot |x| \cdot \sqrt{x} \cdot y^2 \cdot z^2 \cdot \sqrt{z}$ Simplify.

$= 3y^2z^2|x|\sqrt{10xz}$ Simplify.

Check Your Progress

3A. $\sqrt{32r^2k^4t^5}$ **3B.** $\sqrt{56xy^{10}z^5}$

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Quotient Property of Square Roots To divide square roots and simplify radical expressions, you can use the Quotient Property of Square Roots.

ReadingMath

Fractions in the Radicand The expression $\sqrt{\frac{a}{b}}$ is read *the square root of a over b*, or *the square root of the quantity of a over b*.

Key Concept — Quotient Property of Square Roots

For Your FOLDABLE

Words For any real numbers a and b, where $a \geq 0$ and $b > 0$, the square root of $\frac{a}{b}$ is equal to the square root of a divided by the square root of b.

Symbols $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

You can use the properties of square roots to **rationalize the denominator** of a fraction with a radical. This involves multiplying the numerator and denominator by a factor that eliminates radicals in the denominator.

Test-TakingTip

Simplify Look at the radicand to see if it can be simplified first. This may make your computations simpler.

STANDARDIZED TEST EXAMPLE 4

Which expression is equivalent to $\sqrt{\frac{35}{15}}$?

A $\frac{5\sqrt{21}}{15}$ **B** $\frac{\sqrt{21}}{3}$ **C** $\frac{\sqrt{525}}{15}$ **D** $\frac{\sqrt{35}}{15}$

Read the Test Item

The radical expression needs to be simplified.

Solve the Test Item

$$\sqrt{\frac{35}{15}} = \frac{\sqrt{35}}{\sqrt{15}}$$ Quotient Property of Square Roots

$$= \frac{\sqrt{35}}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}}$$ Multiply by $\frac{\sqrt{15}}{\sqrt{15}}$.

$$= \frac{\sqrt{525}}{15}$$ Product Property of Square Roots

$$= \frac{\sqrt{3 \cdot 5 \cdot 5 \cdot 7}}{15}$$ Prime factorization

$$= \frac{5\sqrt{21}}{15} \text{ or } \frac{\sqrt{21}}{3}$$ The correct choice is B.

Check Your Progress

4. Simplify $\frac{\sqrt{6y}}{\sqrt{12}}$.

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Binomials of the form $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$, where a, b, c, and d are rational numbers, are called **conjugates**. For example, $2 + \sqrt{7}$ and $2 - \sqrt{7}$ are conjugates. The product of two conjugates is a rational number and can be found using the pattern for the difference of squares.

EXAMPLE 5 Use Conjugates to Rationalize a Denominator

Simplify $\frac{3}{5 + \sqrt{2}}$.

$$\frac{3}{5 + \sqrt{2}} = \frac{3}{5 + \sqrt{2}} \cdot \frac{5 - \sqrt{2}}{5 - \sqrt{2}}$$ The conjugate of $5 + \sqrt{2}$ is $5 - \sqrt{2}$.

$$= \frac{3(5 - \sqrt{2})}{5^2 - (\sqrt{2})^2}$$ $(a - b)(a + b) = a^2 - b^2$

$$= \frac{15 - 3\sqrt{2}}{25 - 2} \text{ or } \frac{15 - 3\sqrt{2}}{23}$$ $(\sqrt{2})^2 = 2$

Check Your Progress

Simplify each expression.

5A. $\frac{3}{2 + \sqrt{2}}$ **5B.** $\frac{7}{3 - \sqrt{7}}$

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Check Your Understanding

Examples 1–3 pp. 612–613

Simplify each expression.

1. $\sqrt{24}$
2. $3\sqrt{16}$
3. $2\sqrt{25}$
4. $\sqrt{10} \cdot \sqrt{14}$
5. $\sqrt{3} \cdot \sqrt{18}$
6. $3\sqrt{10} \cdot 4\sqrt{10}$
7. $\sqrt{60x^4y^7}$
8. $\sqrt{88m^3p^2r^5}$
9. $\sqrt{99ab^5c^2}$

Example 4 p. 614

10. **MULTIPLE CHOICE** Which expression is equivalent to $\sqrt{\frac{45}{10}}$?

A $\frac{5\sqrt{2}}{10}$ B $\frac{\sqrt{450}}{10}$ C $\frac{\sqrt{50}}{10}$ D $\frac{3\sqrt{2}}{2}$

Example 5 p. 614

Simplify each expression.

11. $\frac{3}{3+\sqrt{5}}$
12. $\frac{5}{2-\sqrt{6}}$
13. $\frac{2}{1-\sqrt{10}}$
14. $\frac{1}{4+\sqrt{12}}$
15. $\frac{4}{6-\sqrt{7}}$
16. $\frac{6}{5+\sqrt{11}}$

Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Examples 1 and 3 pp. 612–613

Simplify each expression.

17. $\sqrt{52}$
18. $\sqrt{56}$
19. $\sqrt{72}$
20. $3\sqrt{18}$
21. $\sqrt{243}$
22. $\sqrt{245}$
23. $\sqrt{5} \cdot \sqrt{10}$
24. $\sqrt{10} \cdot \sqrt{20}$
25. $3\sqrt{8} \cdot 2\sqrt{7}$
26. $4\sqrt{2} \cdot 5\sqrt{8}$
27. $3\sqrt{25t^2}$
28. $5\sqrt{81q^5}$
29. $\sqrt{28a^2b^3}$
30. $\sqrt{75qr^3}$
31. $7\sqrt{63m^3p}$
32. $4\sqrt{66g^2h^4}$
33. $\sqrt{2ab^2} \cdot \sqrt{10a^5b}$
34. $\sqrt{4c^3d^3} \cdot \sqrt{8c^3d}$

Real-World Link

In 1736, Benjamin Franklin founded the first volunteer fire organization, the Union Fire Company, in Philadelphia.

Source: *Firehouse Magazine*

35. **ROLLER COASTER** The velocity v of a roller coaster in feet per second at the bottom of a hill can be approximated by $v = \sqrt{64h}$, where h is the height of the hill in feet.

a. Simplify the equation.

b. Determine the velocity of a roller coaster at the bottom of a 134-foot hill.

36. **FIREFIGHTING** When fighting a fire, the velocity v of water being pumped into the air is modeled by the function $v = \sqrt{2hg}$, where h represents the maximum height of the water and g represents the acceleration due to gravity (32 ft/s^2).

a. Solve the function for h.

b. The Hollowville Fire Department needs a pump that will propel water 80 feet into the air. Will a pump advertised to project water with a velocity of 70 feet per second meet their needs? Explain.

c. The Jackson Fire Department must purchase a pump that will propel water 90 feet into the air. Will a pump that is advertised to project water with a velocity of 77 feet per second meet the fire department's need? Explain.

Examples 4 and 5 p. 614

Simplify each expression.

37. $\sqrt{\frac{32}{t^4}}$

38. $\sqrt{\frac{27}{m^5}}$

39. $\frac{\sqrt{68ac^3}}{\sqrt{27a^2}}$

40. $\frac{\sqrt{h^3}}{\sqrt{8}}$

41. $\sqrt{\frac{3}{16}} \cdot \sqrt{\frac{9}{5}}$

42. $\sqrt{\frac{7}{2}} \cdot \sqrt{\frac{5}{3}}$

43. $\frac{7}{5+\sqrt{3}}$

44. $\frac{9}{6-\sqrt{8}}$

45. $\frac{3\sqrt{3}}{-2+\sqrt{6}}$

46. $\frac{3}{\sqrt{7}-\sqrt{2}}$

47. $\frac{5}{\sqrt{6}+\sqrt{3}}$

48. $\frac{2\sqrt{5}}{2\sqrt{7}+3\sqrt{3}}$

Real-World Link

The first hand-held hair dryer was sold in 1925 and dried hair with 100 watts of heat. Modern hair dryers may have 2000 watts.

Source: *Enotes Encyclopedia*

49. **ELECTRICITY** The amount of current in amperes I that an appliance uses can be calculated using the formula $I = \sqrt{\frac{P}{R}}$, where P is the power in watts and R is the resistance in ohms.

   a. Simplify the formula.

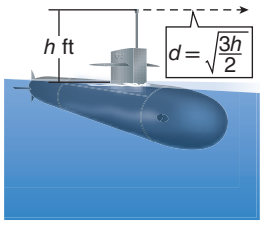
   b. How much current does an appliance use if the power used is 75 watts and the resistance is 5 ohms?

50. **KINETIC ENERGY** The speed v of a ball can be determined by the equation $v = \sqrt{\frac{2k}{m}}$, where k is the kinetic energy and m is the mass of the ball.

   a. Simplify the formula if the mass of the ball is 3 kilograms.

   b. If the ball is traveling 7 meters per second, what is the kinetic energy of the ball in Joules?

51. **SUBMARINES** The greatest distance d in miles that a lookout can see on a clear day is modeled by the formula shown. Determine how high the submarine would have to raise its periscope to see a ship, if the submarine is the given distances away from the ship.



Distance	3	6	9	12	15
Height					

H.O.T. Problems

Use Higher-Order Thinking Skills

52. **REASONING** Explain how to solve $(3x - 2)^2 = (2x + 6)^2$.

53. **CHALLENGE** Solve $|y^3| = \frac{1}{3\sqrt{3}}$ for y.

54. **REASONING** Marge takes a number, subtracts 4, multiplies by 4, takes the square root, and takes the reciprocal to get $\frac{1}{2}$. What number did she start with? Write a formula to describe the process.

55. **OPEN ENDED** Write two binomials of the form $a\sqrt{b} + c\sqrt{f}$ and $a\sqrt{b} - c\sqrt{f}$. Then find their product.

56. **CHALLENGE** Use the Quotient Property of Square Roots to derive the Quadratic Formula by solving the quadratic equation $ax^2 + bx + c = 0$. (*Hint*: Begin by completing the square.)

57. **WRITING IN MATH** Summarize how to write a radical expression in simplest form.

Standardized Test Practice

58. Jerry's electric bill is \$23 less than his natural gas bill. The two bills are a total of \$109. Which of the following equations can be used to find the amount of his natural gas bill?

A $g + g = 109$ **C** $g - 23 = 109$

B $23 + 2g = 109$ **D** $2g - 23 = 109$

59. Solve $a^2 - 2a + 1 = 25$.

F $-4, -6$ **H** $-4, 6$

G $4, -6$ **J** $4, 6$

60. The expression $\sqrt{160x^2y^5}$ is equivalent to which of the following?

A $16|x|y^2\sqrt{10y}$ **C** $4|x|y^2\sqrt{10y}$

B $|x|y^2\sqrt{160y}$ **D** $10|x|y^2\sqrt{4y}$

61. GRIDDED RESPONSE Miki earns \$10 an hour and 10% commission on sales. If Miki worked 38 hours and had a total sales of \$1275 last week, how much did she make?

Spiral Review

Graph each function. Compare to the parent graph. State the domain and range. (Lesson 10-1)

62. $y = 2\sqrt{x} - 1$ **63.** $y = \frac{1}{2}\sqrt{x}$ **64.** $y = 2\sqrt{x + 2}$

65. $y = -\sqrt{x + 1}$ **66.** $y = -3\sqrt{x - 3}$ **67.** $y = -2\sqrt{x} + 1$

Look for a pattern in each table of values to determine which kind of model best describes the data. (Lesson 9-9)

68.

x	0	1	2	3	4
y	1	3	9	27	81

69.

x	−3	−2	−1	0	1
y	18	8	2	0	2

70.

x	1	2	3	4	5
y	1	3	5	7	9

71. POPULATION The country of Latvia has been experiencing a 1.1% annual decrease in population. In 2005, its population was 2,290,237. If the trend continues, predict Latvia's population in 2015. (Lesson 9-7)

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. (Lesson 9-5)

72. $x^2 - 25 = 0$ **73.** $r^2 + 25 = 0$ **74.** $4w^2 + 100 = 40w$

75. $2r^2 + r - 14 = 0$ **76.** $5v^2 - 7v = 1$ **77.** $11z^2 - z = 3$

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*. (Lesson 8-5)

78. $n^2 - 81$ **79.** $4 - 9a^2$ **80.** $2x^5 - 98x^3$

81. $32x^4 - 2y^4$ **82.** $4t^2 - 27$ **83.** $x^3 - 3x^2 - 9x + 27$

84. GARDENING Cleveland is planting 120 jalapeno pepper plants in a rectangular arrangement in his garden. In what ways can he arrange them so that he has at least 4 rows of plants, the same number of plants in each row, and at least 6 plants in each row? (Lesson 8-1)

Skills Review

Write the prime factorization of each number. (Concepts and Skills Bank Lesson 6)

85. 24 **86.** 88 **87.** 180

88. 31 **89.** 60 **90.** 90