Intersections of Plane Figures

Recall that there are three different possibilities for the intersection of two lines. The lines can intersect in zero points, one point, or infinitely many points. If the two lines intersect in infinitely many points, then they are the same line.

<table>
<thead>
<tr>
<th>Zero points</th>
<th>One point</th>
<th>Infinitely many points</th>
</tr>
</thead>
</table>

You can also explore the ways that other geometric figures intersect in a plane.

**EXAMPLE**

Intersection of a Line and a Circle

**GEOMETRY** Describe the different possible points of intersection between a line and a circle.

It may be helpful to draw the different ways a line and a circle can intersect.

<table>
<thead>
<tr>
<th>Zero points</th>
<th>One point</th>
<th>Two points</th>
</tr>
</thead>
</table>

A line and a circle can intersect in zero points, one point, or have two points of intersection. It is impossible for a line and a circle to intersect in more than two points.

As with other plane figures, two circles can intersect in several different ways.
You can also explore the ways that other two-dimensional figures intersect in a plane.

### Intersection of a Line and a Triangle

**Geometry** A line and a triangle lie in the same plane. How many points of intersection are possible between the line and the triangle?

Once again, it may be helpful to draw all of the possible ways that a line and a triangle can intersect.

<table>
<thead>
<tr>
<th>Zero points</th>
<th>One point</th>
<th>Two points</th>
<th>Infinitely many points</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
<td><img src="image3" alt="Diagram" /></td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
</tbody>
</table>

A line and a triangle can intersect in zero points, one point, two points, or infinitely many points. A line and a triangle will intersect in infinitely many points if one side of the triangle is part of the line.

You can use what you know about intersecting figures to make conjectures about the ways two or more geometric figures can intersect.

### Conjecture about Intersecting Triangles

**Geometry** Make a conjecture about the number of ways that two distinct triangles can intersect. Justify your conjecture by drawing the possibilities.

Two triangles have many possible points of intersection. To be sure that you draw all of the possibilities, begin by drawing an example of zero points of intersection and increase the intersection points until no additional examples can exist or until the figures intersect in infinitely many points.

- 0 points
- 1 point
- 2 points
- 3 points
- 4 points
- 5 points
- 6 points
- Infinitely many points

All of the possible ways that two distinct triangles can intersect are shown. So, two distinct triangles can intersect in eight different ways.

You can also explore examples of intersection of more than two geometric figures in a plane.
**EXAMPLE** Intersection of More Than Two Geometric Figures

**ART** Juanita has an assignment for art class in which she must arrange a rectangle, a triangle, and a circle into a frame so that the rectangle and the triangle intersect in two points, the rectangle and the circle intersect in two points, and the circle and triangle intersect in two points. Draw a sketch of what her art piece might look like.

Begin by drawing one of the three figures required. Draw the next two figures so that they intersect each of the existing figures as described above. Two possible sketches are shown below.

![Sketches of intersecting figures](image)

**Exercises**

1. **GEOMETRY** A circle and a square lie in the same plane. Describe the possible points of intersection between the circle and the square. Justify your answer by drawing the possibilities.

2. **GEOMETRY** Draw three triangles that intersect at only one point.

3. **GEOMETRY** Draw two hexagons that intersect at two points.

4. **GEOMETRY** Describe the possible points of intersection between two distinct quadrilaterals in a plane.

5. **GEOMETRY** A circle and a triangle intersect in a plane. What is the greatest number of points of intersection possible? Make a drawing to justify your answer.

6. **GEOMETRY** A line, a circle, and a rhombus intersect in a plane. Draw a sketch of what this might look like.

**ART** For Exercises 7–11, refer to the art design at the right.

7. In how many points does the circle intersect the triangle?

8. In how many points do the two rectangles intersect each other?

9. In how many points does rectangle $A$ intersect the triangle?

10. Name two figures that do not intersect one another.

11. Name three figures that all intersect in one single point.