Rotations

Why?

Traditionally, the energy generated by a windmill was used to pump water or grind grain into flour. Modern windmill technology may be an important alternative to fossil fuels. Windmills convert the wind’s energy into electricity through the rotation of turbine blades.

Draw Rotations In Lesson 4-7, you learned that a rotation or turn moves every point of a preimage through a specified angle and direction about a fixed point.

Key Concept

A rotation about a fixed point, called the **center of rotation**, through an angle of \( x \)° maps a point to its image such that

- if the point is the center of rotation, then the image and preimage are the same point, or
- if the point is not the center of rotation, then the image and preimage are the same distance from the center of rotation and the measure of the **angle of rotation** formed by the preimage, center of rotation, and image points is \( x \).

The direction of a rotation can be either clockwise or counterclockwise. From this point forward, all rotations will be counterclockwise, unless stated otherwise.

**EXAMPLE 1** Draw a Rotation

Copy \( \triangle ABC \) and point \( K \). Then use a protractor and ruler to draw a 140° rotation of \( \triangle ABC \) about point \( K \).

**Step 1** Draw a segment from vertex \( A \) to point \( K \).

**Step 2** Draw a 140° angle using \( KA \) as one side.

**Step 3** Use a ruler to draw \( A' \) such that \( KA' = KA \).

**Step 4** Repeat Steps 1–3 for vertices \( B \) and \( C \) and draw \( \triangle A'B'C' \).
Draw Rotations in the Coordinate Plane When a point is rotated 90°, 180°, or 270° counterclockwise about the origin, you can use the following rules.

**Key Concept**

<table>
<thead>
<tr>
<th>Rotation</th>
<th>Example</th>
<th>Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>90° Rotation</strong></td>
<td>To rotate a point 90° counterclockwise about the origin, multiply the y-coordinate by -1 and then interchange the x- and y-coordinates.</td>
<td>$(x, y) \rightarrow (-y, x)$</td>
</tr>
<tr>
<td><strong>180° Rotation</strong></td>
<td>To rotate a point 180° counterclockwise about the origin, multiply the x- and y-coordinates by -1.</td>
<td>$(x, y) \rightarrow (-x, -y)$</td>
</tr>
<tr>
<td><strong>270° Rotation</strong></td>
<td>To rotate a point 270° counterclockwise about the origin, multiply the x-coordinate by -1 and then interchange the x- and y-coordinates.</td>
<td>$(x, y) \rightarrow (y, -x)$</td>
</tr>
</tbody>
</table>

### EXAMPLE 1

**Study Tip**

**Clockwise Rotation** Clockwise rotation can be designated by a negative angle measure. For example, a rotation of -90° about the origin is a rotation 90° clockwise about the origin.

**Example**

To rotate a point 90° counterclockwise about the origin, multiply the y-coordinate by -1 and then interchange the x- and y-coordinates.

Symbols $(x, y) \rightarrow (-y, x)$

**Example**

To rotate a point 180° counterclockwise about the origin, multiply the x- and y-coordinates by -1.

Symbols $(x, y) \rightarrow (-x, -y)$

**Example**

To rotate a point 270° counterclockwise about the origin, multiply the x-coordinate by -1 and then interchange the x- and y-coordinates.

Symbols $(x, y) \rightarrow (y, -x)$

### EXAMPLE 2

**Rotations in the Coordinate Plane**

Triangle $PQR$ has vertices $P(1, 1)$, $Q(4, 5)$, and $R(5, 1)$. Graph $\triangle PQR$ and its image after a rotation 90° about the origin.

Multiply the y-coordinate of each vertex by -1 and interchange.

$(x, y) \rightarrow (-y, x)$

$P(1, 1) \rightarrow P'(-1, 1)$

$Q(4, 5) \rightarrow Q'(-5, 4)$

$R(5, 1) \rightarrow R'(-1, 5)$

Graph $\triangle PQR$ and its image $\triangle P'Q'R'$.

### Check Your Progress

2. Parallelogram $FGHJ$ has vertices $F(2, 1)$, $G(7, 1)$, $H(6, -3)$, and $J(1, -3)$. Graph $FGHJ$ and its image after a rotation 180° about the origin.
Triangle $JKL$ is shown at the right. What is the image of point $J$ after a rotation $270^\circ$ counterclockwise about the origin?

A. $(-3, -7)$
B. $(-7, 3)$
C. $(-7, -3)$
D. $(7, -3)$

**Read the Test Item**

You are given that $\triangle JKL$ has coordinates $J(3, -7)$, $K(1, -1)$, and $L(5, -3)$ and are then asked to identify the coordinates of the image of point $J$ after a $270^\circ$ counterclockwise rotation about the origin.

**Solve the Test Item**

To find the coordinates of point $J$ after a $270^\circ$ counterclockwise rotation about the origin, multiply the $x$-coordinate by $-1$ and then interchange the $x$- and $y$-coordinates.

$$(x, y) \rightarrow (y, -x) \quad (3, -7) \rightarrow (-7, -3)$$

The answer is choice C.

**Check Your Progress**

3. Parallelogram $WXYZ$ is rotated $180^\circ$ counterclockwise about the origin. Which of these graphs represents the resulting image?

**Test-Taking Tip**

Solve a Simpler Problem Instead of checking all four vertices of parallelogram $WXYZ$ in each graph, check just one vertex, such as $X$. 

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Check Your Understanding

Example 1  
p. 632
Copy each polygon and point K. Then use a protractor and ruler to draw the specified rotation of each figure about point K.

1. 45°
2. 120°

Example 2  
p. 633
Triangle \( DFG \) has vertices \( D(-2, 6), F(2, 8), \) and \( G(2, 3) \). Graph \( \triangle DFG \) and its image after a rotation 180° about the origin.

Example 3  
p. 634
4. MULTIPLE CHOICE  For the transformation shown, what is the measure of the angle of rotation of \( ABCD \) about the origin?
   A 90°
   B 180°
   C 270°
   D 360°

Practice and Problem Solving

Example 1  
p. 632
Copy each polygon and point K. Then use a protractor and ruler to draw the specified rotation of each figure about point K.

5. 90°
6. 15°
7. 145°
8. 30°
9. 260°
10. 50°

PINWHEELS  Find the angle of rotation to the nearest tenth of a degree that maps \( P \) onto \( P' \). Explain your reasoning.

11.
12.
13.
Graph each figure and its image after the specified rotation about the origin.

14. $\triangle JKL$ has vertices $J(2, 6), K(5, 2),$ and $L(7, 5); 90^\circ$
15. rhombus $WXYZ$ has vertices $W(-3, 4), X(0, 7), Y(3, 4),$ and $Z(0, 1); 90^\circ$
16. $\triangle FGH$ has vertices $F(2, 4), G(5, 6),$ and $H(7, 2); 180^\circ$
17. trapezoid $ABCD$ has vertices $A(-7, -2), B(-6, -6), C(-1, -1),$ and $D(-5, 0); 180^\circ$
18. $\triangle RST$ has vertices $R(-6, -1), S(-4, -5),$ and $T(-2, -1); 270^\circ$
19. parallelogram $MPQV$ has vertices $M(-6, 3), P(-2, 3), Q(-3, -2),$ and $V(-7, -2); 270^\circ$

20. WEATHER A weathervane is used to indicate the direction of the wind. If the vane is pointing northeast and rotates $270^\circ$, what is the new wind direction?

21. PHOTOGRAPHY The photograph of the Grande Roue, or Big Wheel, at the left appears blurred because of the camera’s shutter speed—the length of time the camera’s shutter was open.
   a. Estimate the angle of rotation in the photo. (Hint: Use points $A$ and $A'$)
   b. If the Ferris wheel makes one revolution per minute, use your estimate from part a to estimate the camera’s shutter speed.

Each figure shows a preimage and its image after a rotation about point $P$. Copy each figure, locate point $P$, and find the angle of rotation.

22. 
23. 

**ALGEBRA** Give the equation of the line $y = -x - 2$ after a rotation about the origin through the given angle. Then describe the relationship between the equations of the image and preimage.

24. $90^\circ$
25. $180^\circ$
26. $270^\circ$
27. $360^\circ$

**ALGEBRA** Rotate the line the specified number of degrees about the $x$- and $y$-intercepts and find the equation of the resulting image.

28. $y = x - 5; 90^\circ$
29. $y = 2x + 4; 180^\circ$
30. $y = 3x - 2; 270^\circ$

**RIDES** An amusement park ride consists of four circular cars. The ride rotates at a rate of 0.25 revolution per second. In addition, each car rotates 0.5 revolution per second. If Jane is positioned at point $P$ when the ride begins, what coordinates describe her position after 31 seconds?
32. **BICYCLE RACING** Brandon and Nestor are participating in a bicycle race on a circular track with a radius of 200 feet.

a. If the race starts at (200, 0) and both complete one rotation in 30 seconds, what are their coordinates after 5 seconds?

b. Suppose the length of race is 50 laps and Brandon continues the race at the same rate. If Nestor finishes in 26.2 minutes, who is the winner?

33. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate reflections over a pair of intersecting lines.

a. **GEOMETRIC** On a coordinate plane, draw a triangle and a pair of intersecting lines. Label the triangle $ABC$ and the lines $\ell$ and $m$. Reflect $\triangle ABC$ in the line $\ell$. Then reflect $\triangle A'B'C'$ in the line $m$. Label the final image $A''B''C''$.

b. **GEOMETRIC** Repeat the process in part a two more times in two different quadrants. Label the second triangle $DEF$ and reflect it in intersecting lines $n$ and $p$. Label the third triangle $MNP$ and reflect it in intersecting lines $q$ and $r$.

c. **TABULAR** Measure the angle of rotation of each triangle about the point of intersection of the two lines. Copy and complete the table below.

<table>
<thead>
<tr>
<th>Angle of Rotation Between Figures</th>
<th>Angle Between Intersecting Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\triangle ABC$ and $\triangle A'B'C'$</td>
<td>$\ell$ and $m$</td>
</tr>
<tr>
<td>$\triangle DEF$ and $\triangle D'E'F'$</td>
<td>$n$ and $p$</td>
</tr>
<tr>
<td>$\triangle MNP$ and $\triangle M''N''P$</td>
<td>$q$ and $r$</td>
</tr>
</tbody>
</table>

d. **VERBAL** Make a conjecture about the angle of rotation of a figure about the intersection of two lines after the figure is reflected in both lines.

34. **WRITING IN MATH** Are collinearity and betweenness of points maintained under rotation? Explain.

35. **CHALLENGE** Point $C$ has coordinates $C(5, 5)$. The image of this point after a rotation of 100° about a certain point is $C'(-5, 7.5)$. Find the coordinates of the center of this rotation. Explain.

36. **OPEN ENDED** Draw a figure on the coordinate plane. Describe a nonzero rotation that maps the image onto the preimage with no change in orientation.

37. **REASONING** Is the reflection of a figure in the $x$-axis equivalent to the rotation of that same figure 180° about the origin? Explain.

38. **WRITING IN MATH** Do invariant points sometimes, always, or never occur in a rotation? Explain your reasoning.