In today’s technology-driven society, mathematical skills and understanding are more important than ever. The need to use mathematics with fluency and comfort occurs daily—not just for those in the scientific and technical community, but in the workplace and in everyday situations. Those who understand and can use mathematics will have significantly enhanced opportunities and options for shaping their careers and futures. The past two decades have seen an increased recognition of the importance of mathematics for every student and accompanying need for creating uniform national standards in mathematics education. The National Council of Teachers of Mathematics (NCTM) has led this reform movement from its beginning.

NCTM is the world’s largest mathematics education organization, with more than 100,000 members and 250 affiliates throughout the United States and Canada. Between 1989 and 1995, NCTM released a trio of publications on curriculum and evaluation, assessment, and professional standards to articulate goals for mathematics teachers and policymakers. Since the release of these publications, they have given focus, organization, and fresh ideas to efforts to improve mathematics education.

In 2000, NCTM released its most comprehensive project—the Principles and Standards for School Mathematics. The Principles and Standards for School Mathematics represents the culmination of five years of development by the NCTM’s Commission on the Future of the Standards and the Standards 2000 Writing Group.

**History of the Mathematics Education Reform Movement**

1989 National Council of Teachers of Mathematics (NCTM) publishes *Curriculum and Evaluation Standards for School Mathematics*

1991 NCTM publishes *Professional Standards for Teaching Mathematics*

1995 NCTM publishes *Assessment Standards for School Mathematics*

1995 NCTM appoints the Commission on the Future of Standards to oversee the Standards 2000 project


1997 to 1999 The Standards 2000 Writing Group, with input from Association Review Groups, the NCTM Research Advisory Committee, the National Research Council, and more than 650 individuals and 70 groups, writes the *Principles and Standards for School Mathematics*

2000 NCTM publishes the *Principles and Standards for School Mathematics*

All Standards documents are available at www.nctm.org.
Group. The Standards 2000 Writing Group included teachers, teacher educators, administrators, researchers, and mathematicians with a wide range of expertise. The first draft was released in 1998. Over 650 individuals and more than 70 groups, including a committee of experts from the National Research Council, provided assistance and feedback, and the final version of the Principles and Standards for School Mathematics was released in 2000.

The Principles set forth important overall characteristics of mathematics programs, and the Standards describe the mathematical content that students should learn. Together, the Principles and Standards for School Mathematics constitutes a vision to guide educators as they strive for the continual improvement of mathematics education in classrooms, schools, and educational systems. The Principles and Standards for School Mathematics are consistent with the best and most recent evidence on teaching and learning mathematics; they are chosen through a complex process that involves past practice, research findings, societal expectations, and the vision of the professional field (Heibert, 1999).

The vision for mathematics education described in the Principles and Standards for School Mathematics is highly ambitious. Achieving this vision requires committed, competent, and knowledgeable teachers who can integrate instruction with assessment, administrative policies that support learning and access to technology, and solid mathematics curricula.

ACHIEVING THE NCTM PRINCIPLES AND STANDARDS WITH GLENCOE MATHEMATICS: APPLICATIONS AND CONCEPTS

Realizing the Principles and Standards for School Mathematics requires raising expectations for students’ learning, developing effective methods of supporting the learning of mathematics by all students, and providing students and teachers with the resources and curricula they need. A school’s or district’s choice of mathematics curriculum can be a strong determinant of what students have an opportunity to learn.

Glencoe/McGraw-Hill, one of the nation’s largest textbook developers, has risen to the challenge set by the Principles and Standards for School Mathematics and developed the Mathematics: Applications and Concepts series. This series was specifically designed with several key characteristics recommended by the Principles and Standards for School Mathematics for effective curricula.

- Different topical strands, such as algebra, geometry, and statistics, that are highly interconnected;
- Central mathematical ideas that are organized and integrated, so that students can see how the ideas build on, or connect with, other ideas;

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**NCTM Principles for School Mathematics**

*Equity* Excellence in mathematics education requires equity—high expectations and strong support for all students.

*Curriculum* A curriculum is more than a collection of activities; it must be coherent, focused on important mathematics, and well articulated across the grades.

*Teaching* Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.

*Learning* Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

*Assessment* Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.

*Technology* Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning.

Principles and Standards for School Mathematics is available at www.nctm.org.
Foundational ideas such as equivalence, proportionality, functions, and rate of change;
Activities to facilitate development of mathematical thinking and reasoning skills, including making conjectures and developing sound deductive arguments;
Opportunities for experiences that demonstrate mathematics’ usefulness in modeling and predicting real-world phenomena;
Guidance for teachers on the depth of study warranted at particular times and when closure is expected for particular skills or concepts;
Emphasis on the mathematics processes and skills that support the quantitative literacy of students, such as judging claims, finding fallacies, evaluating risks, and weighing evidence.

PRINCIPLES
The Glencoe Mathematics: Applications and Concepts series was designed to meet all six of the Principles set forth in the Principles and Standards for School Mathematics.

Equity The Glencoe Mathematics: Applications and Concepts encourages high achievement at all academic levels. Prerequisite Skills, at the beginning of each chapter and in each lesson in the Student Edition help teachers assess student readiness for the upcoming concepts. Daily Intervention features in the Teacher Wraparound Edition provide suggestions for addressing various learning styles and helping students who are having difficulty. Numerous teacher support materials provide activities for differentiated instruction, promotion of reading and writing skills, pacing for individual levels of achievement, and other intervention opportunities.

Curriculum The Glencoe Mathematics: Applications and Concepts program was developed with a philosophy and scope and sequence to ensure a continuum of mathematical learning that builds on prior knowledge and extends concepts toward more advanced mathematical thinking. The texts offer a balanced approach of real-world applications, hands-on labs, writing exercises, and practice that enables students to develop both conceptual understanding and procedural knowledge.

Teaching Glencoe recognizes that the teacher is a vital factor in students’ academic achievement. The Glencoe Mathematics: Applications and Concepts series was designed with Teacher Wraparound Editions and multiple supplementary resources to provide teachers with extensive support for excellence in teaching. The comprehensive Teacher Wraparound Editions provide Mathematical Content and Teaching Strategies at the beginning of each chapter to summarize mathematical content and skills in each lesson. The Continuity of Instruction section describes prerequisite student knowledge, the chapter content, and future connections to more advanced concepts. Teacher to Teacher presents suggestions from experienced teachers, and Tips for New Teachers throughout the chapters alert new teachers to potential difficulties and suggests instructional strategies.

Learning The Glencoe Mathematics: Applications and Concepts series offers extensive support to help all students achieve success in mathematics. Key Concept and Concept Summary boxes in the Student Editions help students identify main concepts, and Study Tips in the margins help students understand new material. Foldables™ Study Organizers help students focus on the organization and analysis of main ideas and vocabulary. How to . . . Study Skills and How to Use Your Math Textbook help students organize, understand, and retain course information to get the most from the Student Editions.

Assessment The Glencoe Mathematics: Applications and Concepts series offers multiple strategies for the teacher to assess student learning, as well as student self-assessment. Mid-Chapter Practice Tests and Chapter Practice Tests provide ways for students to check their own progress. Online study tools, such as Self-Check Quizzes, offer a unique way for students to use the Internet to monitor their progress. The assessment tools in the Fast
The Glencoe Mathematics: Applications and Concepts series was also designed to meet all of the Principles and Standards for School Mathematics’ Content Standards. The Content Standards state that instructional programs from pre-kindergarten through grade 12 should enable all students to master specific skills in ten content areas.

RESEARCH-BASED INSTRUCTIONAL STRATEGIES USED IN GLENCOE MATHEMATICS: APPLICATIONS AND CONCEPTS

In addition to responding to the goals set by the Principles and Standards for School Mathematics, extensive efforts were undertaken to ensure that the latest research on best practices in mathematics education was used in the development of the Glencoe Mathematics: Applications and Concepts series. Educational research serves as a basis for many of the assertions made throughout about what is possible for students to learn about certain content areas at certain levels and under certain pedagogical conditions.

Glencoe Mathematics: Applications and Concepts was specifically designed to utilize several important research-based instructional strategies that reinforce the Principles and Standards for School Mathematics.

How Glencoe Meets the Standards

<table>
<thead>
<tr>
<th>Content Area and Specific Standards from NCTM Principles and Standards for School Mathematics</th>
<th>Examples from Glencoe Mathematics: Applications and Concepts (page numbers)</th>
</tr>
</thead>
</table>
| **Numbers and Operations**  
• Understand numbers, ways of representing numbers, relationships among numbers, and number systems  
• Understand the meaning of operations and how they relate to each other  
### Content Area and Specific Standards from NCTM Principles and Standards for School Mathematics

#### Algebra
- Understand patterns, relations, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships
- Analyze change in various contexts

#### Geometry
- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- Apply transformations and use symmetry to analyze mathematical situations
- Use visualization, spatial reasoning, and geometric modeling to solve problems

#### Measurement
- Understand measurable attributes of objects and the units, systems and processes of measurement
- Apply appropriate techniques, tools and formulas to determine measurements

#### Data Analysis and Probability
- Formulate questions that can be addressed with data and collect, organize and display relevant data to answer them
- Select and use appropriate statistical methods to analyze data
- Develop and evaluate inferences and predictions that are based on data
- Understand and apply basic concepts of probability

#### Problem Solving
- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

### Examples from Glencoe Mathematics: Applications and Concepts (page numbers)

#### Course 1:

#### Course 2:

#### Course 3:
<table>
<thead>
<tr>
<th>Content Area and Specific Standards from NCTM Principles and Standards for School Mathematics</th>
<th>Examples from Glencoe Mathematics: Applications and Concepts (page numbers)</th>
</tr>
</thead>
</table>
| **Reasoning and Proof**  
• Recognize reasoning and proof as fundamental aspects of mathematics  
• Make and investigate mathematical conjectures  
• Develop and evaluate mathematical arguments and proofs  
| **Communication**  
• Organize and consolidate their mathematical thinking through communication  
• Communicate their mathematical thinking coherently and clearly to peers, teachers, and others  
• Analyze and evaluate the mathematical thinking and strategies of others  
| **Connections**  
• Recognize and use connections among mathematical ideas  
• Understand how mathematical ideas build on one another to produce a coherent whole  
<table>
<thead>
<tr>
<th>Content Area and Specific Standards from NCTM Principles and Standards for School Mathematics</th>
<th>Examples from Glencoe Mathematics: Applications and Concepts (page numbers)</th>
</tr>
</thead>
</table>
1. **Balancing implicit and explicit learning**

Research shows that teachers cannot simply transfer knowledge to students by lecturing. Students have to take an active role in their own learning, and to accomplish this, mathematics programs must include ample opportunity to explore, question, discuss, and discover. This is not to say that teachers are removed from the educational process. Rather, the learning experience should include a balance of implicit and explicit instruction. Implicit instruction occurs when students figure out for themselves how to grapple with problems and construct conceptual knowledge (Pressley, Harris, & Marks, 1992; Shulman & Keislar, 1996). Explicit instruction occurs when teachers and textbooks clearly explain problem-solving strategies to students in a direct, low-inference fashion (Duffy, 2002).

The *Glencoe Mathematics: Applications and Concepts* series offers a balanced approach of real-world applications, hands-on labs, writing exercises, and practice (see Figure 1). Application problems give students the opportunity to use the skills they have learned in a real-world setting, and Critical Thinking exercises require students to explain, make conjectures, and prove mathematical relationships. Calculator and Spreadsheet Investigations use technology to promote the discovery of patterns and relationships, and Modeling, Speaking, and Writing activities in the Assess sections of the Teacher Wraparound Editions require students to summarize what they have learned by responding to open-ended prompts.

2. **Using prior knowledge to learn new information**

Prior knowledge strategies help students retrieve information stored in their long-term memories to learn new, related information. These strategies include recalling remembered information, asking questions, and elaborating on textbook and teacher information, and referring students to the textbook and other meaningful information (including use of analogies). The *Glencoe Mathematics: Applications and Concepts* series intertwines concepts and continuously refers to material in previous chapters and in students’ personal experiences to make mathematics more relevant.

Asking students to use prior knowledge located in a text may remind them of information already in their long-term memory that, for some reason, is not easily
Research shows that mastering a skill provides an overview of the mathematics in each lesson and links to prior knowledge and future concepts (see Figure 2).

### 3. Practicing important tasks and skills

Providing students with practice on important tasks has long been considered a successful strategy to improve understanding and memory. Practicing helps students acquire additional information as they search and productively struggle, with teacher guidance, for understanding and application of mathematical information. Research shows that mastering a skill requires focused practice. During practice, students adapt and shape what they have learned. In doing so, they increase their conceptual understanding of the skill (Clement, Lockhead, & Mink, 1979; Davis, R.B., 1984; Mathematical Science Education Board, 1990; Romberg & Carpenter, 1986).

In Glencoe Mathematics: Applications and Concepts, Practice and Application Exercises correspond to Guided Practice exercises, which are structured so that students practice the same concepts whether they are assigned odd- or even-numbered problems. Homework Help refers students to examples as they complete these exercises. Each lesson also contains Standardized Test Practice problems correlated to content in the lesson or to content in previous chapters. This enables students to practice skills in different forms and to gain experience in taking standardized tests. (See Figure 3.)

### 4. Note-Taking

In the process of note-taking, students identify the important items from reading and write that information in an organized format. While writing and drawing
notes, students see relationships within the information. Notes need not be verbatim; note-taking is most valuable when students learn to analyze information and select the important points (Bretzing & Kulhary, 1979). When study skills, such as note-taking, are taught within the teaching of content, they promote learner activity and improve metacognition (Hattie et al., 1996; Robinson & Kiewra, 1995).

The Glencoe Mathematics: Applications and Concepts texts include instructions for study organizers, called Foldables™, created by Dinah Zike. These are handmade paper booklets, folded and cut into tabs (see Figure 4). Designed to fit each chapter’s content, the Foldable™ guides students in choosing the important concepts and recording them in an organized format. Since students make their own three-dimensional Foldables™ as well as enter the notes, they feel a sense of ownership. The Study Guide and Review feature at the end of each chapter consists of step-by-step examples and practice exercises. The Lesson-by-Lesson Exercises and Examples present a clear picture of the important concepts in each lesson, reviews vocabulary, and provides students with a model for how they might take notes. The How to . . . Study Skill pages include strategies that students can use to read, study, and comprehend the mathematics they are learning.

5. Feedback

Providing students individual feedback on their practice helps in monitoring and fostering their mathematical learning, and having students continue working on a task until they succeed can increase self-confidence (Bangert-Drowns, Kulik, Kulik & Morgan, 1991; Crooks, 1988). Feedback is most useful when students are told how they are performing relative to a specific learning objective, rather than in relationship to other students (Crooks, 1988), and the greatest improvement occurs when feedback is given immediately after a test or activity (Bangert-Drowns, Kulik, Kulik & Morgan, 1991; Crooks, 1988). Students can also provide some of their own feedback by tracking their performance and comparing answers to answer keys (Wiggins, 1993).
immediate feedback of their understanding of concepts. In addition, students may monitor their own performance using the selected answers in the back of the Student Editions. Self-Check Quizzes for every lesson are available on Glencoe’s Web site, and immediate feedback helps students check their progress and find specific pages and examples in the Student Editions.

6. Questions and Cues

Questions and cues are teaching strategies to help students recall and use what they already know about a topic. Cues are hints about what students will do or learn; for example, a brief description of a hands-on lab activity before students begin. Activation of prior knowledge through cues or questions is critical to learning. Asking students questions before a learning activity helps them process information (Pressley et al., 1992).

Each Unit in the Glencoe Mathematics: Applications and Concepts series begins with a Unit Opener, a brief description of what students will learn in the unit. The opening question and answer in each chapter provide a cue, giving a sample of the applications of math concepts in that chapter. Many lessons also begin with “When am I ever going to use this?” questions, which include application problems that will be solved during that lesson (see Figure 6). Each lesson finishes with “Getting Ready for the Next Lesson” to prompt students to recall prerequisite skills that will apply to the next lesson.

7. Cooperative Learning

Cooperative learning occurs when students work in pairs or groups of three or four to complete tasks. Research shows that cooperative learning provides practice at valuable skills, such as positive interdependence, face-to-face interactions, individual and group accountability, interpersonal skills, and group processing (Johnson & Johnson, 1999). Cooperative learning has a highly positive effect when compared with strategies in which students compete with each other and strategies in which students work on tasks individually (Johnson, Maruyama, Johnson, Nelson & Skon, 1981). There needs to be a balance of cooperative learning and individual learning, however, because students need time to practice skills independently (Anderson, Keder, & Simon, 1997).

Glencoe Mathematics: Applications and Concepts texts were designed to provide a mix of individual and cooperative learning opportunities. Hands-On Labs and Mini-Labs, located throughout the texts, direct students to work with other students in carefully structured activities (see Figure 7 on the next page). The Daily Intervention feature in the Teacher Wraparound Editions includes Flexible Grouping suggestions for specific activities within lessons, such as think-pair-share or teams in a tournament. MindJogger Video-quizzes offer practice in an interactive, game-show format, with students working in teams to earn points. WebQuest Internet Projects also offer the opportunity for team projects, in which students do research on the Internet, gather data, and make presentations.

8. Identifying similarities and differences

Comparison and classification skills are vital in mathematics and science; when identifying similarities and differences, students determine how the current problem and previously solved problems are alike
and different. Research tells us that comparing and classifying are effective ways to identify similarities and differences (Chen, 1996; English, 1997; and Newby et al., 1995). The most effective methods of working with similarities and differences are to have students identify similarities and differences on their own (Chen, 1996; Mason & Sorzio, 1996), use graphic and symbolic representations as well as words (Mason, 1994), and begin with concrete examples and then move toward abstract knowledge (Reeves & Weisberg, 1993).

Glencoe Mathematics: Applications and Concepts utilizes the “Solve a Simpler Problem” feature to identify a simpler problem that shares salient characteristics with the given problem. In activities and assignments, students must often explain the difference between two related concepts, such as a sequence and a series, or comparing and contrasting two equations or two graphs. Students also read and create Venn diagrams and other diagrams showing classification (see Figure 8).

9. Use of high-quality visuals to communicate, organize, and reinforce mathematical learning

Visuals—such as complex diagrams and elaborate drawings—used in conjunction with verbal description increase students’ chances of learning, understanding, and remembering relationships and properties of mathematics concepts. Visuals are often the only way to effectively communicate ideas that explain central concepts needed to understand mathematical strands such as algebra and geometry. Research shows that students are better able to organize and group ideas when visuals illustrate different and common characteristics (Hegarty, Carpenter, & Just, 1991). Also, the mental images that high-quality visuals encourage are an indispensable tool for recalling information, especially compared to information presented with only text or lower-quality visuals (Willows & Houghton, 1987).
**SUMMARY**

Glencoe/McGraw-Hill is committed to the idea that curricula should strive to reach all of the *Principles and Standards for School Mathematics*, thereby providing road maps that help teachers guide students to increasing levels of sophistication and depths of knowledge. The NCTM *Principles and Standards for School Mathematics* were developed to accomplish several goals, including guiding the development of curriculum frameworks, assessments, and other instructional materials. Attaining the vision of the *Principles and Standards for School Mathematics* will require the talents, energy, and attention of many individuals, including students, teachers, school administrators, policy makers, teacher educators, parents, mathematicians, local communities, and curriculum developers. Glencoe is proud to provide the *Mathematics: Applications and Concepts* series as an informed road map to excellence in mathematics education in the 21st century.
REFERENCES


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