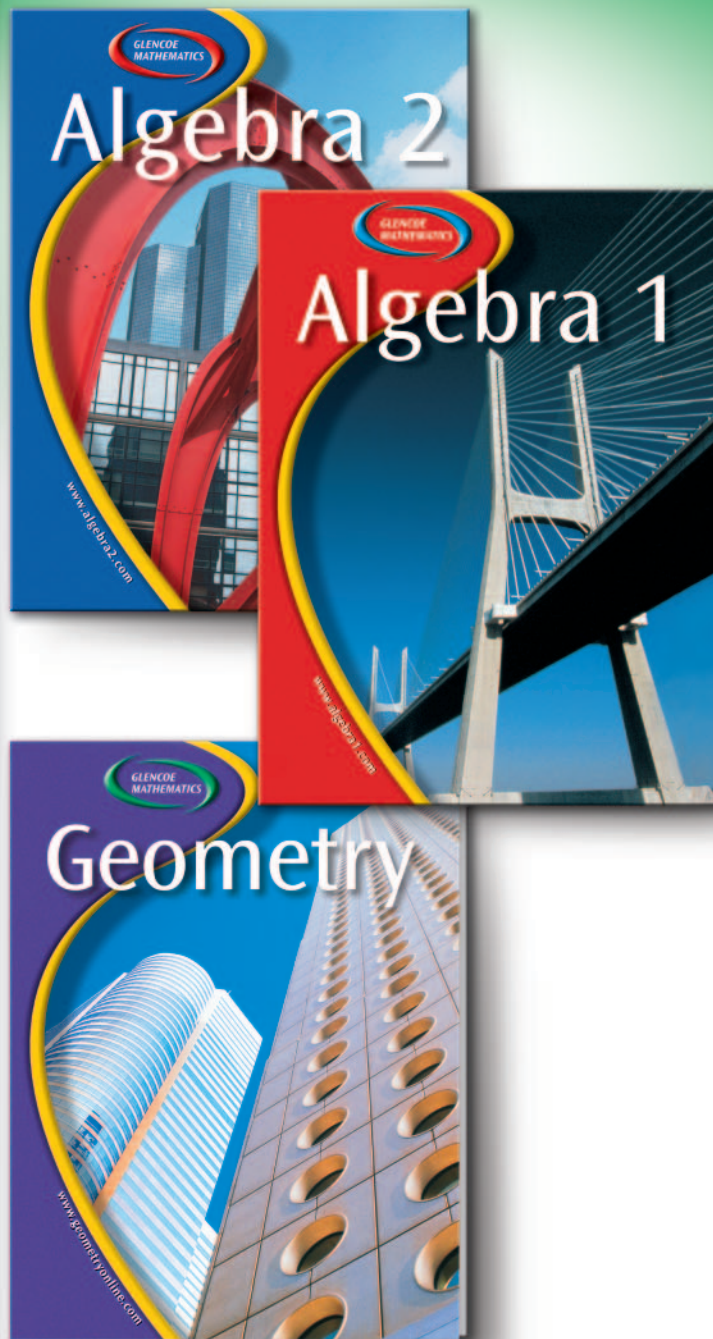


Research- Based Strategies Used to Develop



Glencoe Algebra 1
Glencoe Algebra 2
Glencoe Geometry

MATHEMATICS WITH UNDERSTANDING

The NCTM *Principles and Standards for School Mathematics*

In today's technology-driven society, mathematical skills and understanding are more important than ever. The need to use mathematics with fluency and comfort occurs daily—not just for those in the scientific and technical community, but in the workplace and in everyday situations. Those who understand and can use mathematics will have significantly enhanced opportunities and options for shaping their careers and futures. The past two decades have seen an increased recognition of the importance of mathematics for every student and accompanying need for creating uniform national standards in mathematics education. The National Council of Teachers of Mathematics

(NCTM) has led this reform movement from its beginning.

NCTM is the world's largest mathematics education organization, with more than 100,000 members and 250 affiliates throughout the United States and Canada. Between 1989 and 1995, NCTM released a trio of publications on curriculum and evaluation, assessment, and professional standards to articulate goals for mathematics teachers and policymakers. Since the release of these publications, they have given focus, organization, and fresh ideas to efforts to improve mathematics education.

In 2000, NCTM released its most comprehensive project—the *Principles and Standards for School Mathematics*. The *Principles and*

History of the Mathematics Education Reform Movement

- 1989 National Council of Teachers of Mathematics (NCTM) publishes *Curriculum and Evaluation Standards for School Mathematics*
- 1991 NCTM publishes *Professional Standards for Teaching Mathematics*
- 1995 NCTM publishes *Assessment Standards for School Mathematics*
- 1995 NCTM appoints the Commission on the Future of Standards to oversee the *Standards 2000* project
- 1997 The Commission on the Future of Standards appoints the Standards 2000 Writing Group and the Standards 2000 Electronic Format Group
- 1997 to 1999 The Standards 2000 Writing Group, with input from Association Review Groups, the NCTM Research Advisory Committee, the National Research Council, and more than 650 individuals and 70 groups, writes the *Principles and Standards for School Mathematics*
- 2000 NCTM publishes the *Principles and Standards for School Mathematics*

All *Standards* documents are available at www.nctm.org.

Standards for School Mathematics represent the culmination of five years of development by the NCTM's Commission on the Future of the Standards and their Standards 2000 Writing Group. The Standards 2000 Writing Group included teachers, teacher educators, administrators, researchers, and mathematicians with a wide range of expertise. The first draft was released in 1998. Over 650 individuals and more than 70 groups, including a committee of experts from the National Research Council, provided assistance and feedback, and the final version of the *Principles and Standards for School Mathematics* was released in 2000.

The *Principles* set forth important overall characteristics of mathematics programs, and the *Standards* describe the mathematical content that students should learn. Together, the *Principles and Standards for School Mathematics* constitute a vision to guide educators as they strive for the continual improvement of mathematics education in classrooms, schools, and educational systems. The *Principles and Standards for School Mathematics* are consistent with the best and most recent evidence on teaching and learning mathematics; they are chosen through a complex process that involves past practice, research findings, societal expectations, and the vision of the professional field (Heibert, 1999).

The vision for mathematics education described in *Principles and Standards for School Mathematics* is highly ambitious.

Achieving it requires committed, competent and knowledgeable teachers who can integrate instruction with assessment, administrative policies that support learning and access to technology, and solid mathematics curricula.

ACHIEVING THE NCTM PRINCIPLES AND STANDARDS WITH GLENCOE'S ALGEBRA 1, ALGEBRA 2, AND GEOMETRY SERIES

Realizing the *Principles and Standards for School Mathematics* requires raising expectations for students' learning, developing effective methods of supporting the learning of mathematics by all students, and providing students and teachers with the resources and curricula they need. A school or district's choice of math curriculum can be a strong determinant of what students have an opportunity to learn.

Glencoe/McGraw-Hill, one of the nation's largest textbook developers, has risen to the challenge set by the *Principles and Standards for School Mathematics* and developed Glencoe's *Algebra 1*, *Algebra 2*, and *Geometry* series. This series was specifically designed with several key characteristics recommended by *Principles and Standards for School Mathematics* for effective curricula:

- Different topical strands, such as algebra and geometry, that are highly interconnected;

NCTM Principles for School Mathematics

Equity Excellence in mathematics education requires equity—high expectations and strong support for all students.

Curriculum A curriculum is more than a collection of activities; it must be coherent, focused on important mathematics, and well articulated across the grades.

Teaching Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.

Learning Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

Assessment Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.

Technology Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

Principles and Standards for School Mathematics is available at www.nctm.org.

- Central mathematical ideas that are organized and integrated, so that students can see how the ideas build on, or connect with, other ideas;
- Foundational ideas such as equivalence, proportionality, function, and rate of change;
- Activities to facilitate development of mathematical thinking and reasoning skills, including making conjectures and developing sound deductive arguments;
- Opportunities for experiences that demonstrate mathematics' usefulness in modeling and predicting real-world phenomena;
- Guidance for teachers on the depth of study warranted at particular times and when closure is expected for particular skills or concepts;
- Emphasis on the mathematics processes and skills that support the quantitative literacy of students, such as judging claims, finding fallacies, evaluating risks, and weighing evidence.

PRINCIPLES

Glencoe's *Algebra 1*, *Algebra 2*, and *Geometry* series was designed to meet all six of the Principles set forth in the *Principles and Standards for School Mathematics*.

- **Equity** Glencoe's texts encourage high achievement at all levels. Numerous teacher support materials provide activities for differentiated instruction, promotion of reading and writing skills, pacing for individual levels of achievement, and daily intervention opportunities.
- **Curriculum** Glencoe's *Algebra 1*, *Algebra 2*, and *Geometry* were developed with a philosophy, scope and sequence to ensure a continuum of mathematical learning that builds on prior knowledge and extends concepts toward more advanced mathematical thinking.

- **Teaching** The comprehensive Teacher Wraparound Editions provide mathematical background, teaching tips, resource management guidelines, and tips for new teachers.

- **Learning** The Teacher Wraparound Editions include instruction on building from prior knowledge with materials in each interleaf and in Building On Prior Knowledge features. Find the Error and Unlocking Misconception teaching tips help to evaluate how students are thinking and learning.

- **Assessment** The Practice Quizzes and Chapter Practice Tests provide ways for students to check their own progress. Online Study Tools, such as Self-Check Quizzes, offer a unique way for students to use Internet access to monitor their progress. The assessment tools in the Chapter Resource Masters contain different levels and formats for all tests, as well as intermediate opportunities for assessment.

- **Technology** The Student Editions offer opportunities to utilize graphing calculators, spreadsheets, and geometry software in the exploration of algebra and geometry concepts. The Teacher Wraparound Editions offer teaching tips on using technology, and the Graphing Calculator and Computer Masters offer additional activities. Also, Glencoe's Web site is constantly updated to meet the needs of students and teachers in excelling in mathematics education.

STANDARDS

Glencoe's *Algebra 1*, *Algebra 2*, and *Geometry* series was also designed to meet all of the *Principles and Standards for School Mathematics'* Content Standards. The Content Standards state that instructional programs from pre-kindergarten through grade 12 should enable all students to master specific skills in ten content areas.

How Glencoe Meets the Standards

Standards from NCTM Principles and Standards for School Mathematics

Examples from *Glencoe Algebra 1*, *Glencoe Algebra 2*, and *Glencoe Geometry* (page numbers)

Numbers and Operations

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems
- Understand the meaning of operations and how they relate to each other
- Compute fluently and make reasonable estimates

Glencoe Algebra 1: 6–36, 43–56, 68–109, 120–178, 232–245, 368, 425–430, 474–479, 567–573, 586–597, 605–622, 642–653, 655–695, 708–728, 731–744, 754–758, 760–788

Glencoe Algebra 2: 6–46, 56–99, 110–144, 154–208, 222–275, 294–299, 301–319, 346–358, 360–399, 492–498, 522–565, 578–617, 632–685, 709–751, 762–781, 786–797, 799–804

Glencoe Geometry: 20–36, 62–66, 282–323, 325–331, 342–348

Algebra

- Understand patterns, relations, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships
- Analyze change in various contexts

Glencoe Algebra 1: 6–36, 43–48, 120–178, 192–203, 205–245, 256–307, 318–358, 369–398, 410–423, 431–463, 474–514, 524–573, 586–592, 598–621, 642–695

Glencoe Algebra 2: 6–10, 20–46, 56–99, 110–144, 154–158, 160–208, 222–275, 286–335, 346–399, 412–416, 419–431, 433–452, 455–460, 472–512, 522–565, 588–592, 594–604, 606–610, 612–621, 700–708, 762–804

Glencoe Geometry: 13–19, 21–43, 45–50, 94–100, 133–157, 159–164, 238–273, 282–287, 289–331, 404–437, 439–451, 463–488, 490–511, 688–706

Geometry

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- Apply transformations and use symmetry to analyze mathematical situations
- Use visualization, spatial reasoning, and geometric modeling to solve problems

Glencoe Algebra 1: 6–9, 155–159, 192–231, 240–245, 256–262, 271–277, 292–297, 416, 501–506, 567–572, 605–630, 759

Glencoe Algebra 2: 19, 167–174, 239–244, 270–275, 286–293, 301–312, 329–335, 360–382, 390–394, 412–460, 522–540, 552–559, 611, 700–751, 769–776

Glencoe Geometry: 6–52, 75–87, 89–114, 126–166, 178–226, 236–273, 289–331, 342–391, 404–451, 462–511, 522–580, 595–627, 636–677, 688–719

Measurement

- Understand measurable attributes of objects and the units, systems and processes of measurement
- Apply appropriate techniques, tools and formulas to determine measurements

Glencoe Algebra 1: 256–277, 339–351

Glencoe Algebra 2: 6–10, 20–27, 63–74, 87–88, 182–188, 360–364, 371–382, 390–394, 432, 523–538, 541–551, 554–565, 611, 709–751, 769–776

Glencoe Geometry: 13–28, 51–52, 522–528, 536–543, 569–580

Data Analysis and Probability

- Formulate questions that can be addressed with data and collect, organize and display relevant data to answer them
- Select and use appropriate statistical methods to analyze data
- Develop and evaluate inferences and predictions that are based on data
- Understand and apply basic concepts of probability

Glencoe Algebra 1: 6–55, 68–109, 120–126, 128–140, 142–178, 192–203, 205–223, 226–245, 256–262, 272, 305, 318, 358, 368–398, 410–430, 432–436, 439–449, 452–463, 474–514, 524–573, 586–621, 623–630, 642–653, 655–695, 708–744, 754–788

Glencoe Algebra 2: 81–95, 154–158, 300, 359, 539–540, 632–686

Glencoe Geometry: 20, 622–627

Problem Solving

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

Glencoe Algebra 1: 37–42, 96–102, 256–262, 278–279, 292–307, 416, 531–532, 545, 622, 759

Glencoe Algebra 2: 20–27, 33–46, 63–99, 110–127, 129–135, 138–144, 160–207, 222–267, 270–275, 286–319, 322–335, 346–399, 412–431, 433–452, 455–460, 472–512, 522–551, 554–565, 578–621, 632–685, 700–751, 762–781, 786–790, 799–804

Glencoe Geometry: 6–43, 45–52, 62–114, 126–131, 133–164, 178–213, 216–226, 238–273, 282–331, 342–391, 404–451, 463–488, 490–511, 522–580, 595–627, 636–676, 688–719

Standards from NCTM Principles and Standards for School Mathematics

**Examples from *Glencoe Algebra 1*,
Glencoe Algebra 2, and *Glencoe Geometry*
(page numbers)**

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs
- Select and use various types of reasoning and methods of proof

Glencoe Algebra 1: 6–56, 68–109, 120–126, 128–140, 142–177, 192–223, 226–231, 233–245, 256–307, 318–358, 369–374, 376–398, 410–415, 417–430, 432–436, 439–449, 452–463, 474–514, 524–552, 554–573, 586–621, 623–630, 642–695, 708–742, 754–788

Glencoe Algebra 2: 56–62, 68–74, 116–122, 138–144, 182–188, 202–207, 222–228, 233–238, 245–256, 270–275, 301–312, 322–328, 346–352, 371–399, 432, 453–454, 522, 531–538, 541–546, 554–559, 578–592, 594–604, 606–611, 618–621, 686, 762–768, 786–797, 799–804

Glencoe Geometry: 51–52, 62–114, 132–138, 151–157, 165–166, 200–226, 255–273, 417–423, 431–437, 439–451

Communication

- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely

Glencoe Algebra 1: 6–56, 68–109, 120–126, 128–140, 142–178, 192–203, 205–223, 226–231, 233–245, 256–277, 280–307, 318–358, 368–374, 376–398, 410–415, 417–430, 432–436, 439–449, 452–463, 474–479, 481–486, 489–514, 524–530, 533–544, 546–552, 554–573, 586–603, 605–621, 623–630, 642–653, 655–695, 708–744, 754–788

Glencoe Algebra 2: 63–86, 89–99, 110–127, 129–135, 138–144, 154–158, 160–207, 222–267, 270–275, 286–335, 346–358, 360–399, 412–452, 455–460, 472–511, 522–551, 554–565, 578–592, 594–621, 632–680, 682–686, 701–751, 762–797, 799–804

Glencoe Geometry: 6–43, 45–52, 62–87, 89–114, 126–157, 159–166, 178–183, 185–213, 216–226, 238–273, 282–323, 325–331, 342–348, 350–383, 385–390, 404–409, 411–437, 439–451, 463–488, 490–511, 522–558, 561–580, 595–627, 636–677, 688–694, 696–719

Connections

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

Glencoe Algebra 1: 6–56, 68–109, 120–126, 128–140, 142–178, 192–203, 205–223, 226–231, 233–245, 256–277, 280–307, 318–358, 368–374, 376–398, 410–415, 417–430, 432–436, 439–449, 452–463, 474–479, 481–486, 489–514, 524–530, 533–544, 546–552, 554–573, 586–603, 605–621, 623–630, 642–653, 655–695, 708–744, 754–788

Glencoe Algebra 2: 6–46, 56–99, 110–127, 129–135, 138–144, 154–158, 160–207, 222–267, 270–275, 286–299, 301–319, 322–335, 346–399, 412–416, 419–431, 433–452, 455–460, 472–490, 492–511, 523–538, 541–551, 554–565, 578–592, 594–610, 612–617, 632–686, 701–715, 717–751, 762–781, 786–797, 799, 804

Glencoe Geometry: 6–19, 21–27, 29–43, 45–50, 62–87, 89–114, 126–131, 133–157, 159–164, 178–183, 185–213, 216–226, 238–254, 261–273, 282–287, 289–323, 325–331, 342–348, 350–383, 385–390, 404–409, 411–437, 439–451, 463–488, 490–511, 522–558, 561–580, 595–627, 636–677, 688–694, 696–719

Representation

- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena

Glencoe Algebra 1: 6–56, 68–109, 120–177, 192–203, 205–231, 233–245, 256–307, 318–358, 368–398, 410–415, 417–463, 474–514, 524–530, 533–544, 546–552, 554–573, 586–603, 605–621, 623–630, 642–653, 655–695, 715–728, 731–744, 754–788

Glencoe Algebra 2: 19, 28–32, 40–46, 56–62, 68–99, 110–127, 129–144, 154–181, 189–208, 222–232, 239–244, 250–256, 263–275, 286–300, 306–312, 320–335, 346–377, 383–399, 412–431, 433–460, 472–490, 492–511, 522–530, 539–540, 583–592, 594–610, 612–617, 632–686, 701–715, 717–751, 762–781, 786, 804

Glencoe Geometry: 6–19, 21–43, 45–50, 62–87, 89–114, 126–131, 133–157, 159–164, 178–226, 236–273, 282–287, 289–323, 325–331, 342–348, 350–383, 385–390, 404–409, 411–437, 439–451, 463–488, 490–511, 522–558, 561–580, 595–627, 636–677, 688–694, 696–719

RESEARCH-BASED INSTRUCTIONAL STRATEGIES USED IN GLENCOE'S ALGEBRA 1, ALGEBRA 2, AND GEOMETRY SERIES

In addition to responding to the goals set by the *Principles and Standards in School Mathematics*, extensive efforts were undertaken to ensure that the latest research on best practices in mathematics education was used in the development of Glencoe's *Algebra 1*, *Algebra 2*, and *Geometry*. Educational research serves as a basis for many of the assertions made throughout about what is possible for students to learn about certain content areas at certain levels and under certain pedagogical conditions. Research demonstrates that a curriculum that enables students to reach high mathematics standards has the following characteristics:

- **Balance**—focuses on conceptual understanding and procedural fluency;
- **Comprehensiveness**—includes all the important content strands of mathematics as well as computation and other procedural skills;
- **Alignment with standards**—includes content and strategies which align with state and national standards, external assessments and instruction;

- **Coordination and coherence within and across grades**—well-developed ideas that build on and connect with other ideas, both within and across grades (Apthorp, Bodrova, Dean & Florian, 2001).

Glencoe's *Algebra 1*, *Algebra 2*, and *Geometry* were specifically designed with all of these characteristics in mind. They were also designed to utilize several important instructional strategies that reinforce the *Principles and Standards for School Mathematics*.

1. Balancing implicit and explicit learning

Research shows us that teachers cannot simply transfer knowledge to students by lecturing. Students have to take an active role in their own learning, and to accomplish this, mathematics programs must include ample opportunity to explore, question, discuss and discover. This is not to say that teachers are removed from the educational process. Rather, the learning experience should include a balance of implicit and explicit instruction. Implicit instruction occurs when students figure out for themselves how to grapple with problems and construct conceptual knowledge (Pressley, Harris, & Marks, 1992; Shulman & Keislar, 1996). Explicit instruction occurs when teachers and textbooks clearly explain problem-solving strategies to students in a direct, low-inference fashion (Duffy, 2002). In Geometry instruction, a combination of expository and discovery methods has been found to be effective, especially when manipulatives are used to help students represent and comprehend geometric concepts (Klausmeier, 1992; Clements, 1999).

Glencoe's *Algebra 1*, *Algebra 2*, and *Geometry* offer a balanced approach of real-world applications, hands-on labs, direct instruction, writing exercises, and practice that enables students to develop both conceptual understanding and procedural knowledge. Lessons begin with real-world problems for students to solve, and then students use multiple representations to explore new concepts. Hands-On Labs, Algebra Activities, and Geometry Activities (see Figure 1) provide carefully designed opportunities for discovery. Calculator and Spreadsheet Investigations use technology to promote discovery of patterns and relationships.

Figure 1

Example 1 Use ASA in Proofs

Write a paragraph proof.

Given: \overline{CP} bisects $\angle BCR$ and $\angle BPR$.

Prove: $\triangle BCP \cong \triangle RCP$

Proof:

Since \overline{CP} bisects $\angle BCR$ and $\angle BPR$, $\angle BCP \cong \angle RCP$ and $\angle BPC \cong \angle RPC$. $\overline{CP} \cong \overline{CP}$ by the Reflexive Property. By ASA, $\triangle BCP \cong \triangle RCP$.

AAS THEOREM Suppose you are given the measures of two angles and a nonincluded side. Is this information sufficient to prove two triangles congruent?

Geometry Activity

Angle-Angle-Side Congruence

Model

1. Draw a triangle on a piece of patty paper. Label the vertices A, B, and C.
2. Copy \overline{AB} , $\angle B$, and $\angle C$ on another piece of patty paper and cut them out.
3. Assemble them to form a triangle in which the side is not the included side of the angles.

Analyze

1. Place the original $\triangle ABC$ over the assembled figure. How do the two triangles compare?
2. **Make a conjecture** about two triangles with two angles and the nonincluded side of one triangle congruent to two angles and the nonincluded side of the other triangle.

This activity leads to the Angle-Angle-Side Theorem, written as AAS.

Theorem 4.5

Angle-Angle-Side Congruence If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.

Abbreviation: AAS

Example: $\triangle KLE \cong \triangle CAB$

Proof Theorem 4.5

Given: $\angle M \cong \angle S$, $\angle J \cong \angle R$, $\overline{MP} \cong \overline{ST}$

Prove: $\triangle JMP \cong \triangle RST$

Proof:

Statements	Reasons
1. $\angle M \cong \angle S$, $\angle J \cong \angle R$, $\overline{MP} \cong \overline{ST}$	1. Given
2. $\angle P \cong \angle T$	2. Third Angle Theorem
3. $\triangle JMP \cong \triangle RST$	3. ASA

208 Chapter 4 Congruent Triangles

2. Using prior knowledge to learn new information

Prior knowledge strategies help students retrieve information stored in their long-term memories to learn new, related information. These strategies include recalling remembered information, asking questions, elaborating on textbook and teacher information, and referring students to the textbook (including use of analogies) and other meaningful information. Glencoe's *Algebra 1*, *Algebra 2*, and *Geometry* intertwine concepts and continuously refer to material in previous chapters (see Figure 2) and in students' personal experiences to make mathematics more relevant. Asking students to use prior knowledge located in a text may remind them of information already in their long-term memory that, for some reason, is not easily remembered (Bransford, 1979; Pressley & McCormick, 1995).

3. Practicing important tasks and skills

Providing students with practice on important tasks has long been considered a successful strategy to improve understanding and memory. Giving students individual feedback on their practice helps in monitoring and fostering their mathematical learning. Practicing helps students acquire additional information as they search and productively struggle, with teacher guidance, for understanding and application of mathematical information. Research shows that mastering a skill requires focused practice. During practice, students adapt and shape what they have learned. In doing so, they increase their conceptual understanding of the skill (Clement, Lockhead, & Mink, 1979; Davis, R.B., 1984; Mathematical Science Education Board, 1990; Romberg & Carpenter, 1986).

The Concept Check in each lesson ensures that students understand the concepts and skills of the lesson. After students correctly complete the Guided Practice, they are ready to work through practice exercises on their own, either in class or as homework. Practice and Applications exercises can be assigned and higher-difficulty exercises are marked in the Teacher Wraparound Editions to provide

4-3

Relations

What You'll Learn

- Represent relations as sets of ordered pairs, tables, mappings, and graphs.
- Find the inverse of a relation.

Vocabulary

- mapping
- inverse

How can relations be used to represent baseball statistics?

Ken Griffey, Jr.'s batting statistics for home runs and strikeouts can be represented as a set of ordered pairs. The number of home runs are the first coordinates, and the number of strikeouts are the second coordinates. You can plot the ordered pairs on a graph to look for patterns.

Year	Home Runs	Strikeouts
1994	49	73
1995	17	53
1996	49	104
1997	56	121
1998	56	121
1999	48	108
2000	48	117
2001	22	72

REPRESENT RELATIONS Recall that a *relation* is a set of ordered pairs. A relation can be represented by a set of ordered pairs, a table, a graph, or a **mapping**. A mapping illustrates how each element of the domain is paired with an element in the range. Study the different representations of the same relation below.

Ordered Pairs

(1, 2)
(-2, 4)
(0, -3)

Table

x	y
1	2
-2	4
0	-3

Graph

Mapping

Example 1 Represent a Relation

a. Express the relation (3, 2), (-1, 4), (0, -3), (-3, 4), (-2, -2) as a table, a graph, and a mapping.

Table

List the set of x-coordinates in the first column and the corresponding y-coordinates in the second column.

x	y
3	2
-1	4
0	-3
-3	4
-2	-2

Graph

Graph each ordered pair on a coordinate plane.

(continued on the next page)
Lesson 4-3 Relations 205

Figure 2

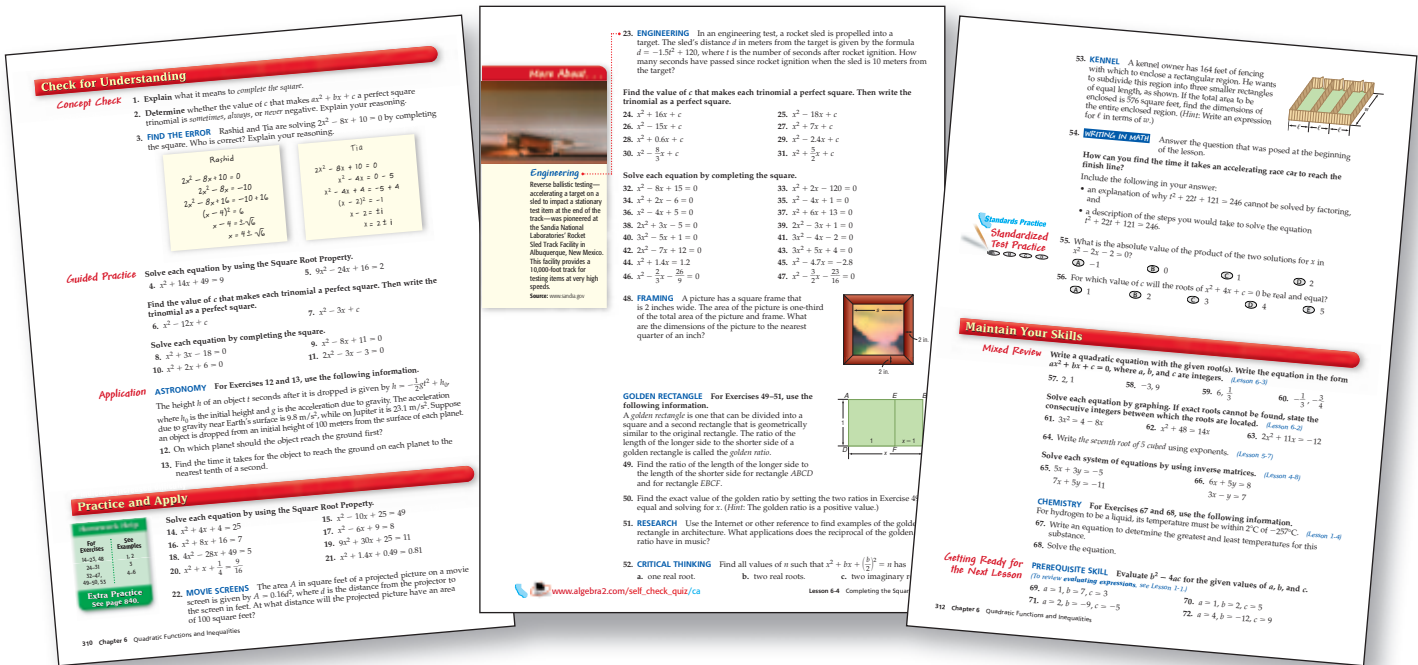
Glencoe Algebra 1, page 205

more challenges and practice for students. The Mixed Review section in every lesson includes spiraled, cumulative exercises from the two previous lessons as well as earlier lessons. Here students practice the important skills learned earlier, building toward mastery. Each lesson also contains at least one Standardized Test problem. (See Figure 3 on the next page.)

Each chapter provides two pages of Standardized Test Practice problems correlated to prerequisite skills and to the content in previous chapters. This enables students to practice skills in a different form, to help in adapting and shaping process, and at the same time to gain experience in taking standardized tests.

4. Note-Taking

In the process of note-taking, students identify the important items from reading and write that information in an organized format. While writing and drawing notes, students see relationships within the information. Notes need not be verbatim; note-taking is most valuable when students learn to analyze information and select the important

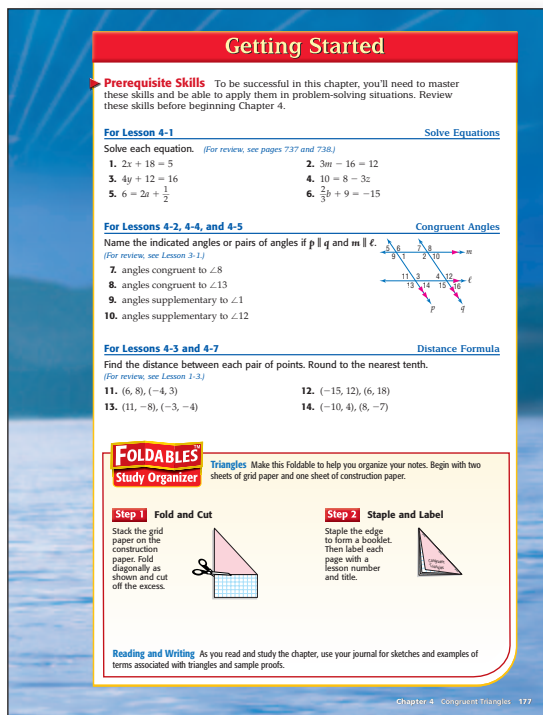


Glencoe Algebra 2, pages 310–312

Figure 3

points (Bretzing & Kulhary, 1979). When study skills, such as note-taking are taught within the teaching of content, they promote learner activity and improve metacognition (Hattie et al., 1996; Robinson & Kiewra, 1995).

Figure 4



Glencoe Geometry, page 177

Glencoe's *Algebra 1*, *Algebra 2*, and *Geometry* include instructions for study organizers, called Foldables™, created by Dinah Zike. (See Figure 4.) Study organizers are handmade paper booklets, folded and cut into tabs. The Foldable, designed to fit that chapter's content, guides students in choosing the important concepts and recording them in an organized format. Since students make their own three-dimensional Foldables as well as enter the notes, they feel a sense of ownership. The Study Guide and Review feature at the end of each chapter consists of notes plus worked examples and practice exercises. The Lesson-by-Lesson Review presents a clear picture of the important concepts in each lesson and provides students with a model for how they might take notes.

5. Cooperative learning

Cooperative learning occurs when students work in pairs or groups of three or four to complete tasks. Research shows that cooperative learning provides practice at valuable skills, such as positive interdependence, face-to-face interactions, individual and group accountability, interpersonal skills, and group processing (Johnson & Johnson, 1999). Cooperative learning has a highly

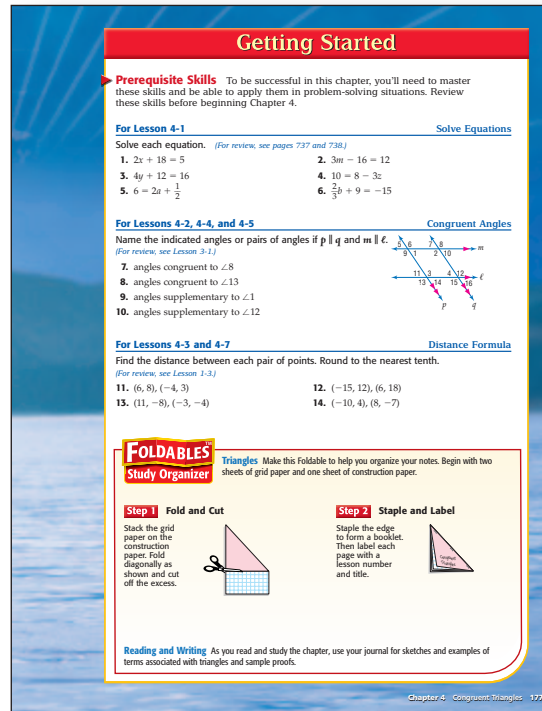
positive effect when compared with strategies in which students compete with each other and strategies in which students work on tasks individually (Johnson, Maruyama, Johnson, Nelson & Skon, 1981). There needs to be a balance of cooperative learning and individual learning, however, because students need time to practice skills independently (Anderson, Keder, & Simon, 1997).

Glencoe's *Algebra 1*, *Algebra 2*, and *Geometry* were designed to provide a mix of individual and cooperative learning opportunities. Hands-On Labs and Mini-Labs, located throughout the texts, direct students to work with other students in carefully structured activities. In Algebra Activity sections, students work together in small groups to collect data, analyze data, and make conjectures. (See Figure 5.) The Daily Intervention section in the Teacher Wrap-around Editions includes Flexible Grouping suggestions for specific activities within lessons, such as think-pair-share or teams in a tournament.

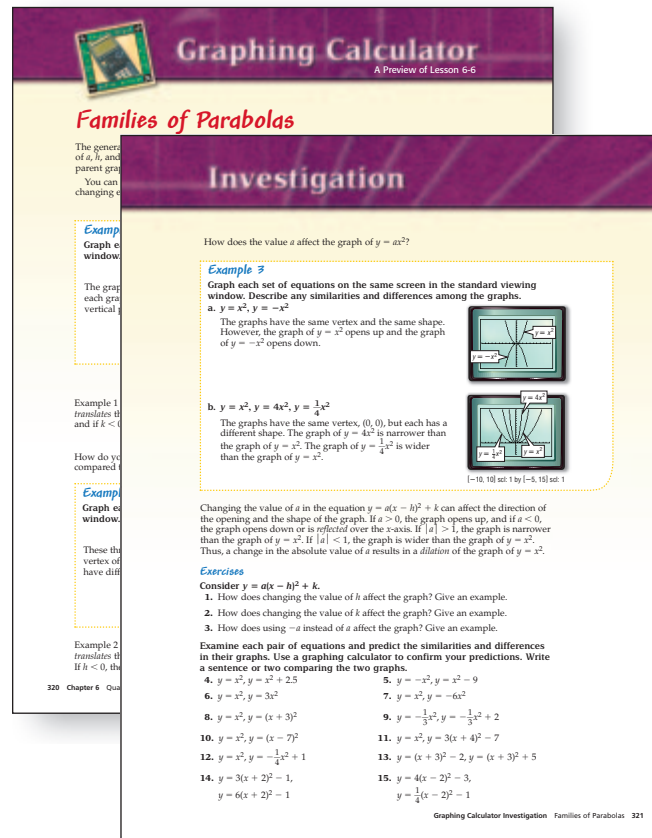
6. Identifying similarities and differences

Comparison and classification skills are vital in mathematics and science; when identifying similarities and differences, students determine how the current problem and previously-solved problems are alike and different. Research tells us that comparing and classifying are effective ways to identify similarities and differences (Chen, 1996; English, 1997; and Newby et al., 1995). The most effective methods of working with similarities and differences are to have students identify similarities and differences on their own (Chen, 1996; Mason & Sorzio, 1996), use graphic and symbolic representations as well as words (Mason, 1994), and begin with concrete examples and then move towards abstract knowledge (Reeves & Weisberg, 1993).

In Glencoe's *Algebra 1*, *Algebra 2*, and *Geometry*, students must often explain the difference between two related concepts, such as a sequence and a series, or compare and contrast graphs or equations that belong to a



Glencoe Algebra 1, page 501



Glencoe Algebra 2, pages 320-321

Figure 5

Figure 6

family of graphs or equations, respectively. (See Figure 6 on the previous page.) Students also interpret and create Venn diagrams and other diagrams showing classification, such as with various types of polygons.

7. Use of visuals to communicate, organize, and reinforce mathematical learning

Visuals—such as complex diagrams and elaborate drawing—used in conjunction with verbal description increase students' chances of learning, understanding and remembering relationships and properties of mathematics concepts. Visuals are often the only way to effectively communicate ideas that explain central concepts needed to understand algebra and geometry. (See Figure 7.) Research shows that students are

better able to organize and group ideas when visuals illustrate different and common characteristics (Hegarty, Carpenter, & Just, 1991). Also, the mental images that high-quality visuals encourage are an indispensable tool for recalling information, especially compared to information presented with only text or lower-quality visuals (Willows & Houghton, 1987). Glencoe's *Algebra 1*, *Algebra 2*, and *Geometry* include high-quality charts, tables, graphs, art and photographs throughout the text. Visuals are often accompanied by caption questions and ideas for effective use of models.

Figure 7

5-1

Slope

What You'll Learn

- Find the slope of a line.
- Use rate of change to solve problems.


Vocabulary

- slope
- rate of change

Why Is slope important in architecture?

The slope of a roof describes how steep it is. It is the number of units the roof rises for each unit of run. In the photo, the roof rises 8 feet for each 12 feet of run.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{8 \text{ ft}}{12 \text{ ft}} \text{ or } \frac{2}{3}$$


FIND SLOPE The **slope** of a line is a number determined by any two points on the line. This number describes how steep the line is. The greater the absolute value of the slope, the steeper the line. Slope is the ratio of the change in the y-coordinates (rise) to the change in the x-coordinates (run) as you move from one point to the other.


The graph shows a line that passes through (1, 3) and (4, 5).

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{\text{change in y-coordinates}}{\text{change in x-coordinates}}$$

$$= \frac{5 - 3}{4 - 1} \text{ or } \frac{2}{3}$$

So, the slope of the line is $\frac{2}{3}$.



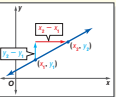
Key Concept

- Words** The slope of a line is the ratio of the rise to the run.
- Symbols** The slope m of a nonvertical line through any two points, (x_1, y_1) and (x_2, y_2) , can be found as follows.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

← change in y
← change in x

Slope of a Line



Study Tip

Reading Math In $m = \frac{y_2 - y_1}{x_2 - x_1}$, the 1 is called a subscript. It is read x sub 1.

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SUMMARY

Glencoe/McGraw-Hill is committed to the idea that curricula should strive to reach all of the *Principles and Standards for School Mathematics*, thereby providing road maps that help teachers guide students to increasing levels of sophistication and depths of knowledge. The NCTM *Principles and Standards for School Mathematics* were developed to accomplish several goals, including guiding the development of curriculum frameworks, assessments and other instructional materials. Attaining the vision of the *Principles and Standards for School Mathematics* will require the talents, energy, and attention of many individuals, including students, teachers, school administrators, policymakers, teacher educators, parents, mathematicians, local communities and curriculum developers. Glencoe is proud to provide their *Algebra 1*, *Algebra 2*, and *Geometry* series as an informed road map to excellence in mathematics education in the 21st century.

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