11-3

Circles and Circumference

GET READY for the Lesson

CLOCKS Big Ben is a famous clock tower in London, England. The diameter of the clock face is 23 feet.

1. Which point appears at the middle of Big Ben?

2. How does the distance from $A$ to $C$ compare to the distance from $B$ to $D$?

3. Find the distance from $D$ to $C$.

A circle is the set of all points in a plane that are the same distance from a given point, called the center.

The diameter $d$ is the distance across a circle through its center.

The circumference $C$ is the distance around a circle.

The radius $r$ is the distance from the center to any point on a circle.

The diameter of a circle is 2 times the radius, or $d = 2r$. Another relationship that is true of all circles is $\frac{C}{d} = 3.1415926\ldots$. This nonterminating and nonrepeating number is represented by the Greek letter $\pi$ (pi). An approximation often used for $\pi$ is 3.14.

Circumference of a Circle

Words The circumference $C$ of a circle is equal to its diameter $d$ times $\pi$, or 2 times its radius $r$ times $\pi$.

Symbols $C = \pi d$ or $C = 2\pi r$

When finding the circumference of a circle, it is necessary to use an approximation of $\pi$ since its exact value cannot be determined.
Real-World EXAMPLE  

Find Circumference

CLOCKs  Refer to the lesson opener. Find the circumference of Big Ben’s clock face.

**Estimate**  \( C = 3 \cdot 23 \) or 69 ft

\[ C = \pi d \quad \text{Circumference of a circle} \]

\[ C \approx 3.14(23) \] Replace \( \pi \) with 3.14 and \( d \) with 23.

\[ C \approx 72.22 \quad \text{Multiply.} \]

So, the distance around the clock is about 72.22 feet.

**Check for Reasonableness**  \( 72.22 \approx 69 \)

CHECK Your Progress

Find the circumference of each circle. Use 3.14 for \( \pi \). Round to the nearest tenth if necessary.

a. \( \text{4 ft} \)

b. \( \text{63 in.} \)

Another approximation for \( \pi \) is \( \frac{22}{7} \). Use this value when the radius or diameter is a multiple of 7 or has a multiple of 7 in its numerator if the radius is a fraction.

EXAMPLE  

Find Circumference

Find the circumference of a circle with a radius of 21 inches.

Since 21 is a multiple of 7, use \( \frac{22}{7} \) for \( \pi \).

\[ C = 2\pi r \quad \text{Circumference of a circle} \]

\[ C \approx 2 \cdot \frac{22}{7} \cdot 21 \] Replace \( \pi \) with \( \frac{22}{7} \) and \( r \) with 21.

\[ C \approx 2 \cdot \frac{22}{7} \cdot \frac{21}{1} \] Divide by the GCF, 7.

\[ C \approx 132 \quad \text{Simplify.} \]

The circumference of the circle is about 132 inches.

CHECK Your Progress

Find the circumference of each circle. Use \( \frac{22}{7} \) for \( \pi \).

c. \( \text{70 in.} \)

d. \( \text{7 ft} \)
Find the circumference of each circle. Use $3.14$ or $\frac{22}{7}$ for $\pi$. Round to the nearest tenth if necessary.

1. $5$ ft
2. $11.7$ m
3. $14$ m
4. $42$ in.

5. **CLOCKS** To the nearest tenth, how many centimeters does the tip of the minute hand of the clock shown travel each hour?

Find the circumference of each circle. Use $3.14$ or $\frac{22}{7}$ for $\pi$. Round to the nearest tenth if necessary.

6. $16$ m
7. $8$ ft
8. $5.8$ km
9. $7.2$ cm
10. $7$ yd
11. $21$ ft

12. radius $= \frac{3}{4}$ in.
13. diameter $= 10\frac{1}{2}$ in.
14. diameter $= 15.1$ m
15. diameter $= 10.8$ km
16. radius $= \frac{5}{8}$ in.
17. diameter $= 12\frac{1}{4}$ mi

18. **SPORTS** A flying disc has a diameter of $9\frac{5}{8}$ inches. Find its circumference.

19. **WHEELS** A hamster wheel has a radius of $4\frac{1}{2}$ inches. How far will the wheel turn in one revolution?

20. **PATCHES** A National Guard military patch has a diameter of $2.5$ inches. What is its circumference to the nearest tenth?

21. **POOLS** The Cole family owns an above-ground circular swimming pool that has walls made of aluminum. Find the length $\ell$ of aluminum surrounding the pool as shown if the radius is $15$ feet. Round to the nearest tenth.
**LABELS** Determine the length of each can’s label.

22. 10 cm
23. 1.75 in.
24. \( \frac{5}{3} \) in.

**MEASUREMENT** For Exercises 25–28, perform each of the following steps.

a. Use a centimeter ruler to measure the diameter of each circular object listed.

b. Estimate to find the approximate circumference of each circle.

c. Calculate the circumference of each circle. Use 3.14 for \( \pi \).

d. Cut a piece of string the length of the circumference of each circle. Use a centimeter ruler to measure the length of the string to the nearest tenth of a centimeter. Compare this actual length to the calculated length.

25. soup can lid
26. quarter
27. CD or DVD
28. button

**ALGEBRA** Find the diameter or radius of each circle. Use 3.14 for \( \pi \). Round to the nearest tenth if necessary.

29. \( C = 25 \text{ ft} \), diameter = \( \Box \text{ ft} \)
30. \( C = 54 \text{ cm} \), diameter = \( \Box \text{ cm} \)
31. \( C = 30 \text{ yd} \), radius = \( \Box \text{ yd} \)
32. \( C = 48 \text{ km} \), radius = \( \Box \text{ km} \)

**UNICYCLES** A unicycle wheel has a radius of 10 inches. How many feet will the unicycle travel in 5 revolutions? Explain how you solved this problem.

**ROUNDABOUTS** A roundabout is a one-way circular intersection. About how many feet would a car travel if it drove once around the roundabout?

**FIND THE DATA** Refer to the Data File on pages 16–19. Choose some data and write a real-world problem in which you would find the circumference of a circle.

**H.O.T. Problems**

36. **OPEN ENDED** Select a real-world situation in which finding the circumference of a circle would be useful.

37. **FIND THE ERROR** Logan and Elsa are finding the circumference of a circle with a radius of 7 inches. Who is correct? Explain.

Logan: \( C = \pi \cdot 7 \) or \( 7\pi \)

Elsa: \( C = 2 \cdot \pi \cdot 7 \) or \( 14\pi \)
CHALLENGE For Exercises 38 and 39, use the circle at the right.
38. How many lengths $x$ will fit on the circle’s circumference?
39. If the value of $x$ is doubled, what effect will this have on the diameter? on the circumference? Explain your reasoning.

40. WRITING IN MATH A constant is a quantity whose value never changes. In the formula for the circumference of a circle, identify any constants. Justify your response.

TEST PRACTICE

41. Malik’s bike tire has a radius of 8 inches. Which equation could be used to find the circumference of the tire in inches?
   - A $C = \pi \cdot 4$
   - B $C = \pi \cdot 16 \times 2$
   - C $C = \pi \cdot 16$
   - D $C = \pi \cdot 8$

42. Each wheel on Nina’s car has a diameter of 18 inches. Which expression could be used to find the circumference of the wheel?
   - F $2 \times 9 \times \pi$
   - G $2 \times 18 \times \pi$
   - H $9 \times 9 \times \pi$
   - J $18 \times 18 \times \pi$

43. Which measure is closest to the circumference of the dreamcatcher shown below?

Spiral Review

MEASUREMENT Find the area of each figure. Round to the nearest tenth if necessary. (Lesson 11-2)

44. 9.8 km
45. 2.4 ft

46. MEASUREMENT Find the area of a parallelogram with base 6.5 meters and height 7.0 meters. (Lesson 11-1)

47. PROBABILITY Jorge rolled a number cube several times and recorded the results in the table shown. Find the experimental probability that an odd number turned up. (Lesson 9-7)

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>III</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
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<tr>
<td>5</td>
<td>II</td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

GET READY for the Next Lesson

PREREQUISITE SKILL Use a calculator to find each product to the nearest tenth. (Lesson 1-4)

48. $\pi \cdot 5^2$ 51. $\pi \cdot (4.5)^2$
49. $\pi \cdot 7^2$ 50. $\pi \cdot (2.4)^2$