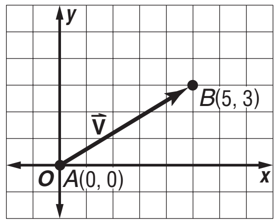
NAME ______________________________ DATE ____________ PERIOD _____

Study Guide and Intervention

Vectors

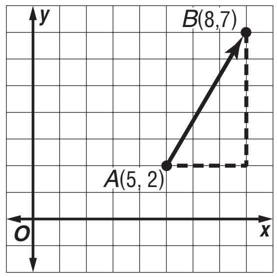
Magnitude and Direction A vector is a directed segment representing a quantity that has both **magnitude**, or length, and **direction**. For example, the speed and direction of an airplane can be represented by a vector. In symbols, a vector is written as $\overrightarrow{AB}$, where A is the initial point and B is the endpoint, or as $\vec{\mathbf{v}}$.

A vector in **standard position** has its initial point at (0, 0) and can be represented by the ordered pair for point B. The vector at the right can be expressed as $\vec{\mathbf{v}} = \langle 5, 3 \rangle$.



You can use the Distance Formula to find the magnitude $|\overrightarrow{AB}|$ of a vector. You can describe the direction of a vector by measuring the angle that the vector forms with the positive x-axis or with any other horizontal line.

Example **Find the magnitude and direction of $\overrightarrow{AB}$ for $A(5, 2)$ and $B(8, 7)$.**



Find the magnitude.

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(8 - 5)^2 + (7 - 2)^2}$$
$$= \sqrt{34} \text{ or about 5.8 units}$$

To find the direction, use the tangent ratio.

$\tan A = \frac{5}{3}$ The tangent ratio is opposite over adjacent.

$m\angle A \approx 59.0$ Use a calculator.

The magnitude of the vector is about 5.8 units and its direction is 59°.

Exercises

Find the magnitude and direction of $\overrightarrow{AB}$ for the given coordinates. Round to the nearest tenth.

1. $A(3, 1)$, $B(-2, 3)$

2. $A(0, 0)$, $B(-2, 1)$

3. $A(0, 1)$, $B(3, 5)$

4. $A(-2, 2)$, $B(3, 1)$

5. $A(3, 4)$, $B(0, 0)$

6. $A(4, 2)$, $B(0, 3)$

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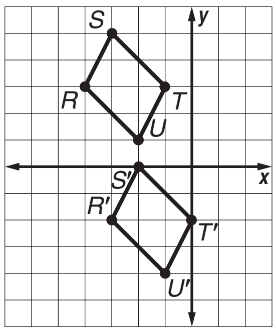
Study Guide and Intervention *(continued)*

Vectors

Translations with Vectors Recall that the transformation $(a, b) \rightarrow (a + 2, b - 3)$ represents a translation right 2 units and down 3 units. The vector $\langle 2, -3 \rangle$ is another way to describe that translation. Also, two vectors can be added: $\langle a, b \rangle + \langle c, d \rangle = \langle a + c, b + d \rangle$. The sum of two vectors is called the **resultant**.

Example **Graph the image of parallelogram *RSTU* under the translation by the vectors $\overrightarrow{\mathbf{m}} = \langle 3, -1 \rangle$ and $\overrightarrow{\mathbf{n}} = \langle -2, -4 \rangle$.**



Find the sum of the vectors.

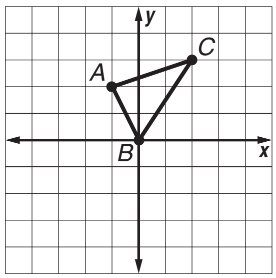
$$\begin{aligned} \overrightarrow{\mathbf{m}} + \overrightarrow{\mathbf{n}} &= \langle 3, -1 \rangle + \langle -2, -4 \rangle \\ &= \langle 3 - 2, -1 - 4 \rangle \\ &= \langle 1, -5 \rangle \end{aligned}$$

Translate each vertex of parallelogram *RSTU* right 1 unit and down 5 units.

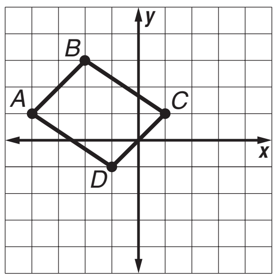
Exercises

Graph the image of each figure under a translation by the given vector(s).

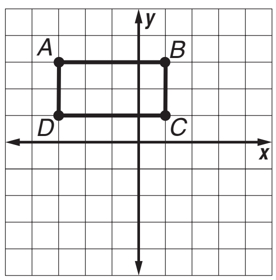
1. $\triangle ABC$ with vertices $A(-1, 2)$, $B(0, 0)$, and $C(2, 3)$; $\overrightarrow{\mathbf{m}} = \langle 2, -3 \rangle$



2. *ABCD* with vertices $A(-4, 1)$, $B(-2, 3)$, $C(1, 1)$, and $D(-1, -1)$; $\overrightarrow{\mathbf{n}} = \langle 3 \ -3 \rangle$



3. *ABCD* with vertices $A(-3, 3)$, $B(1, 3)$, $C(1, 1)$, and $D(-3, 1)$; the sum of $\overrightarrow{\mathbf{p}} = \langle -2, 1 \rangle$ and $\overrightarrow{\mathbf{q}} = \langle 5, -4 \rangle$



Given $\overrightarrow{\mathbf{m}} = \langle 1, -2 \rangle$ and $\overrightarrow{\mathbf{n}} = \langle -3, -4 \rangle$, represent each of the following as a single vector.

4. $\overrightarrow{\mathbf{m}} + \overrightarrow{\mathbf{n}}$

5. $\overrightarrow{\mathbf{n}} - \overrightarrow{\mathbf{m}}$

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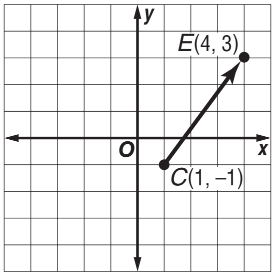
Skills Practice

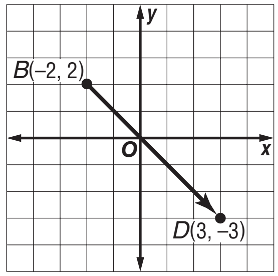
Vectors

Write the component form of each vector.

1.



2.



Find the magnitude and direction of $\overrightarrow{RS}$ for the given coordinates. Round to the nearest tenth.

3. $R(2, -3)$, $S(4, 9)$

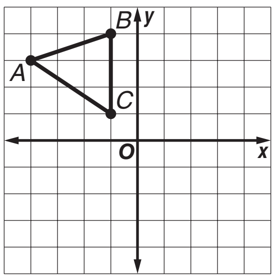
4. $R(0, 2)$, $S(3, 12)$

5. $R(5, 4)$, $S(-3, 1)$

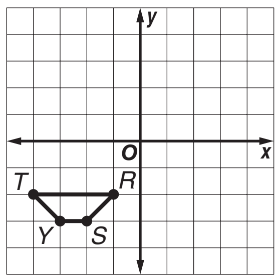
6. $R(1, 5)$, $S(-4, -6)$

Graph the image of each figure under a translation by the given vector(s).

7. $\triangle ABC$ with vertices $A(-4, 3)$, $B(-1, 4)$, $C(-1, 1)$; $\overrightarrow{\mathbf{t}} = \langle 4, -3 \rangle$



8. trapezoid with vertices $T(-4, -2)$, $R(-1, -2)$, $S(-2, -3)$, $Y(-3, -3)$; $\overrightarrow{\mathbf{a}} = \langle 3, 1 \rangle$ and $\overrightarrow{\mathbf{b}} = \langle 2, 4 \rangle$



Find the magnitude and direction of each resultant for the given vectors.

9. $\overrightarrow{\mathbf{y}} = \langle 7, 0 \rangle$, $\overrightarrow{\mathbf{z}} = \langle 0, 6 \rangle$

10. $\overrightarrow{\mathbf{b}} = \langle 3, 2 \rangle$, $\overrightarrow{\mathbf{c}} = \langle -2, 3 \rangle$

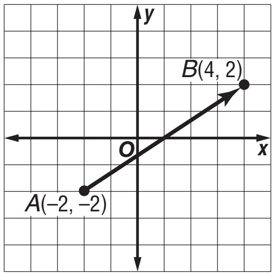
NAME ____ DATE ____________ PERIOD _____

Practice

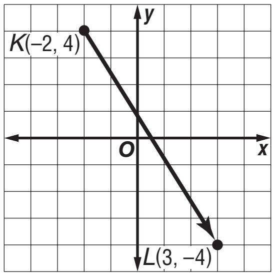
Vectors

Write the component form of each vector.

1.



2.



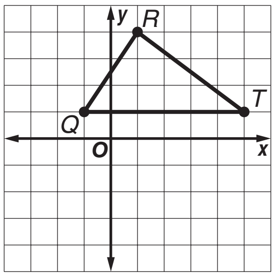
Find the magnitude and direction of $\overrightarrow{FG}$ for the given coordinates. Round to the nearest tenth.

3. $F(-8, -5)$, $G(-2, 7)$

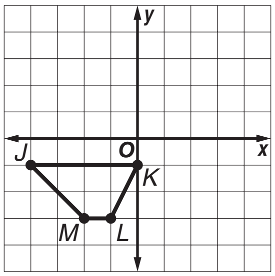
4. $F(-4, 1)$, $G(5, -6)$

Graph the image of each figure under a translation by the given vector(s).

5. $\triangle QRT$ with vertices $Q(-1, 1)$, $R(1, 4)$, $T(5, 1)$; $\vec{\mathbf{s}} = \langle -2, -5 \rangle$



6. trapezoid with vertices $J(-4, -1)$, $K(0, -1)$, $L(-1, -3)$, $M(-2, -3)$; $\vec{\mathbf{c}} = \langle 5, 4 \rangle$ and $\vec{\mathbf{d}} = \langle -2, 1 \rangle$



Find the magnitude and direction of each resultant for the given vectors.

7. $\vec{\mathbf{a}} = \langle -6, 4 \rangle$, $\vec{\mathbf{b}} = \langle 4, 6 \rangle$

8. $\vec{\mathbf{e}} = \langle -4, -5 \rangle$, $\vec{\mathbf{f}} = \langle -1, 3 \rangle$

AVIATION For Exercises 9 and 10, use the following information.

A jet begins a flight along a path due north at 300 miles per hour. A wind is blowing due west at 30 miles per hour.

9. Find the resultant velocity of the plane.

10. Find the resultant direction of the plane.

NAME ____ DATE ____________ PERIOD _____

Reading to Learn Mathematics

Vectors

Reading the Lesson

1. Supply the missing words or phrases to complete the following sentences.

a. A _________________ is a directed segment representing a quantity that has both magnitude and direction.

b. The length of a vector is called its _________________.

c. Two vectors are parallel if and only if they have the same or _________________ direction.

d. A vector is in _____________________ if it is drawn with initial point at the origin.

e. Two vectors are equal if and only if they have the same _________________ and the same_________________.

f. The sum of two vectors is called the _________________.

g. A vector is written in _____________________ if it is expressed as an ordered pair.

h. The process of multiplying a vector by a constant is called ________________________.

2. Write each vector described below in component form.

a. a vector in standard position with endpoint (a, b)

b. a vector with initial point (a, b) and endpoint (c, d)

c. a vector in standard position with endpoint $(-3, 5)$

d. a vector with initial point $(2, -3)$ and endpoint $(6, -8)$

e. $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ if $\vec{\mathbf{a}} = \langle -3, 5\rangle$ and $\vec{\mathbf{b}} = \langle 6, -4\rangle$

f. $5\vec{\mathbf{u}}$ if $\vec{\mathbf{u}} = \langle 8, -6\rangle$

g. $-\frac{1}{3}\vec{\mathbf{v}}$ if $\vec{\mathbf{v}} = \langle -15, 24\rangle$

h. $0.5\vec{\mathbf{u}} + 1.5\vec{\mathbf{v}}$ if $\vec{\mathbf{u}} = \langle 10, -10\rangle$ and $\vec{\mathbf{v}} = \langle -8, 6\rangle$

Helping You Remember

3. A good way to remember a new mathematical term is to relate it to a term you already know. You learned about *scale factors* when you studied similarity and dilations. How is the idea of a *scalar* related to *scale factors*?

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Enrichment

Reading Mathematics

Many quantities in nature can be thought of as vectors. The science of physics involves many vector quantities. In reading about applications of mathematics, ask yourself whether the quantities involve only magnitude or both magnitude and direction. The first kind of quantity is called **scalar**. The second kind is a **vector**.

Classify each of the following. Write *scalar* or *vector*.

1. the mass of a book

2. a car traveling north at 55 mph

3. a balloon rising 24 feet per minute

4. the size of a shoe

5. a room temperature of 22 degrees Celsius

6. a west wind of 15 mph

7. the batting average of a baseball player

8. a car traveling 60 mph

9. a rock falling at 10 mph

10. your age

11. the force of Earth's gravity acting on a moving satellite

12. the area of a record rotating on a turntable

13. the length of a vector in the coordinate plane

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Study Guide and Intervention

Vectors

Magnitude and Direction A vector is a directed segment representing a quantity that has both **magnitude**, or length, and **direction**. For example, the speed and direction of an airplane can be represented by a vector. In symbols, a vector is written as $\overrightarrow{AB}$, where A is the initial point and B is the endpoint, or as $\vec{v}$.

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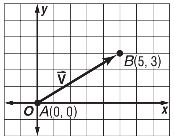

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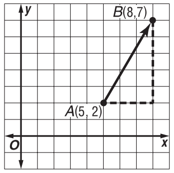
Example **Find the magnitude and direction of $\overrightarrow{AB}$ for $A(5, 2)$ and $B(8, 7)$.**



Find the magnitude.

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8 - 5)^2 + (7 - 2)^2}$

$= \sqrt{34}$ or about 5.8 units

To find the direction, use the tangent ratio.

$\tan A = \frac{5}{3}$ The tangent ratio is opposite over adjacent.

$m\angle A \approx 59.0$ Use a calculator.

The magnitude of the vector is about 5.8 units and its direction is 59°.

Exercises

Find the magnitude and direction of $\overrightarrow{AB}$ for the given coordinates. Round to the nearest tenth.

1. $A(3, 1), B(-2, 3)$ **5.4; 158.2°**
2. $A(0, 0), B(-2, 1)$ **2.2; 153.4°**
3. $A(0, 1), B(3, 5)$ **5; 53.1°**
4. $A(-2, 2), B(3, 1)$ **5.1; 348.7°**
5. $A(3, 4), B(0, 0)$ **5; 233.1°**
6. $A(4, 2), B(0, 3)$ **4.1; 166.0°**

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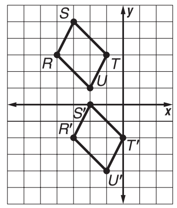
Study Guide and Intervention *(continued)*

Vectors

Translations with Vectors Recall that the transformation $(a, b) \rightarrow (a + 2, b - 3)$ represents a translation right 2 units and down 3 units. The vector $\langle 2, -3\rangle$ is another way to describe that translation. Also, two vectors can be added: $\langle a, b\rangle + \langle c, d\rangle = \langle a + c, b + d\rangle$. The sum of two vectors is called the **resultant**.

Example **Graph the image of parallelogram $RSTU$ under the translation by the vectors $\vec{m} = \langle 3, -1\rangle$ and $\vec{n} = \langle -2, -4\rangle$.**



Find the sum of the vectors.

$\vec{m} + \vec{n} = \langle 3, -1\rangle + \langle -2, -4\rangle$

$= \langle 3 - 2, -1 - 4\rangle$

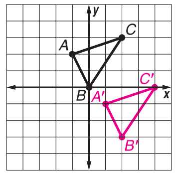
$= \langle 1, -5\rangle$

Translate each vertex of parallelogram $RSTU$ right 1 unit and down 5 units.

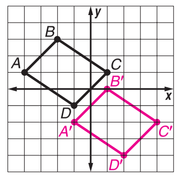
Exercises

Graph the image of each figure under a translation by the given vector(s).

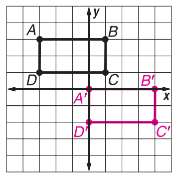
1. $\triangle ABC$ with vertices $A(-1, 2), B(0, 0)$, and $C(2, 3)$; $\vec{m} = \langle 2, -3\rangle$



2. $ABCD$ with vertices $A(-4, 1), B(-2, 3)$, $C(1, 1)$, and $D(-1, -1)$; $\vec{n} = \langle 3\ -3\rangle$



3. $ABCD$ with vertices $A(-3, 3), B(1, 3), C(1, 1)$, and $D(-3, 1)$; the sum of $\vec{p} = \langle -2, 1\rangle$ and $\vec{q} = \langle 5, -4\rangle$



Given $\vec{m} = \langle 1, -2\rangle$ and $\vec{n} = \langle -3, -4\rangle$, represent each of the following as a single vector.

4. $\vec{m} + \vec{n}$ **⟨−2, −6⟩**
5. $\vec{n} - \vec{m}$ **⟨−4, −2⟩**

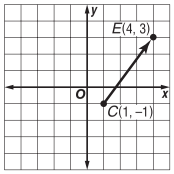
NAME ____________________ DATE __________ PERIOD _____

Skills Practice

Vectors

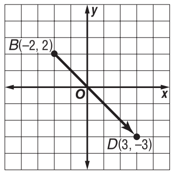
Write the component form of each vector.

1.



⟨3, 4⟩

2.



⟨5, –5⟩

Find the magnitude and direction of $\overrightarrow{RS}$ for the given coordinates. Round to the nearest tenth.

3. $R(2, -3)$, $S(4, 9)$
$2\sqrt{37} \approx 12.2, 80.5°$

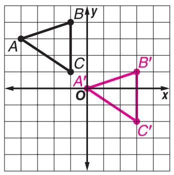
4. $R(0, 2)$, $S(3, 12)$
$\sqrt{109} \approx 10.4, 73.3°$

5. $R(5, 4)$, $S(-3, 1)$
$\sqrt{73} \approx 8.5, 200.6°$

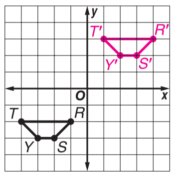
6. $R(1, 5)$, $S(-4, -6)$
$\sqrt{146} \approx 12.1, 245.6°$

Graph the image of each figure under a translation by the given vector(s).

7. $\triangle ABC$ with vertices $A(-4, 3)$, $B(-1, 4)$, $C(-1, 1)$; $\vec{t} = \langle 4, -3 \rangle$



8. trapezoid with vertices $T(-4, -2)$, $R(-1, -2)$, $S(-2, -3)$, $Y(-3, -3)$; $\vec{a} = \langle 3, 1 \rangle$ and $\vec{b} = \langle 2, 4 \rangle$



Find the magnitude and direction of each resultant for the given vectors.

9. $\vec{y} = \langle 7, 0 \rangle$, $\vec{z} = \langle 0, 6 \rangle$
$\sqrt{85} \approx 9.2, 40.6°$

10. $\vec{b} = \langle 3, 2 \rangle$, $\vec{c} = \langle -2, 3 \rangle$
$\sqrt{26} \approx 5.1, 78.7°$

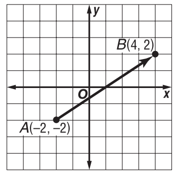
NAME ____________________ DATE __________ PERIOD _____

Practice (Average)

Vectors

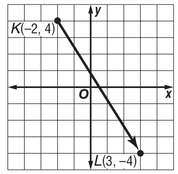
Write the component form of each vector.

1.



⟨6, 4⟩

2.



⟨5, –8⟩

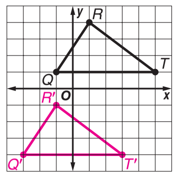
Find the magnitude and direction of $\overrightarrow{FG}$ for the given coordinates. Round to the nearest tenth.

3. $F(-8, -5)$, $G(-2, 7)$
$6\sqrt{5} \approx 13.4, 63.4°$

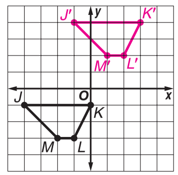
4. $F(-4, 1)$, $G(5, -6)$
$\sqrt{130} \approx 11.4, 322.1°$

Graph the image of each figure under a translation by the given vector(s).

5. $\triangle QRT$ with vertices $Q(-1, 1)$, $R(1, 4)$, $T(5, 1)$; $\vec{s} = \langle -2, -5 \rangle$



6. trapezoid with vertices $J(-4, -1)$, $K(0, -1)$, $L(-1, -3)$, $M(-2, -3)$; $\vec{c} = \langle 5, 4 \rangle$ and $\vec{d} = \langle -2, 1 \rangle$



Find the magnitude and direction of each resultant for the given vectors.

7. $\vec{a} = \langle -6, 4 \rangle$, $\vec{b} = \langle 4, 6 \rangle$
$2\sqrt{26} \approx 10.2, 101.3°$

8. $\vec{e} = \langle -4, -5 \rangle$, $\vec{f} = \langle -1, 3 \rangle$
$\sqrt{29} \approx 5.4, 201.8°$

AVIATION For Exercises 9 and 10, use the following information.
A jet begins a flight along a path due north at 300 miles per hour. A wind is blowing due west at 30 miles per hour.

9. Find the resultant velocity of the plane. about 301.5 mph

10. Find the resultant direction of the plane. about 5.7° west of due north

NAME ______________________ DATE ____________ PERIOD _____

Reading to Learn Mathematics

Vectors

Reading the Lesson

1. Supply the missing words or phrases to complete the following sentences.

a. A ___**vector**___ is a directed segment representing a quantity that has both magnitude and direction.

b. The length of a vector is called its ___**magnitude**___.

c. Two vectors are parallel if and only if they have the same or ___**opposite**___ direction.

d. A vector is in ___**standard position**___ if it is drawn with initial point at the origin.

e. Two vectors are equal if and only if they have the same ___**magnitude**___ and the same ___**direction**___.

f. The sum of two vectors is called the ___**resultant**___.

g. A vector is written in ___**component form**___ if it is expressed as an ordered pair.

h. The process of multiplying a vector by a constant is called ___**scalar multiplication**___.

2. Write each vector described below in component form.

a. a vector in standard position with endpoint (a, b) $\langle a, b\rangle$

b. a vector with initial point (a, b) and endpoint (c, d) $\langle c - a, d - b\rangle$

c. a vector in standard position with endpoint $(-3, 5)$ $\langle -3, 5\rangle$

d. a vector with initial point $(2, -3)$ and endpoint $(6, -8)$ $\langle 4, -5\rangle$

e. $\overline{\mathbf{a}} + \overline{\mathbf{b}}$ if $\overline{\mathbf{a}} = \langle -3, 5\rangle$ and $\overline{\mathbf{b}} = \langle 6, -4\rangle$ $\langle 3, 1\rangle$

f. $5\overline{\mathbf{u}}$ if $\overline{\mathbf{u}} = \langle 8, -6\rangle$ $\langle 40, -30\rangle$

g. $-\frac{1}{3}\overline{\mathbf{v}}$ if $\overline{\mathbf{v}} = \langle -15, 24\rangle$ $\langle 5, -8\rangle$

h. $0.5\overline{\mathbf{u}} + 1.5\overline{\mathbf{v}}$ if $\overline{\mathbf{u}} = \langle 10, -10\rangle$ and $\overline{\mathbf{v}} = \langle -8, 6\rangle$ $\langle -7, 4\rangle$

Helping You Remember

3. A good way to remember a new mathematical term is to relate it to a term you already know. You learned about *scale factors* when you studied similarity and dilations. How is the idea of a *scalar* related to *scale factors*? **Sample answer: A *scalar* is the term used for a constant (a specific real number) when working with vectors. A vector has both magnitude and direction, while a scalar is just a magnitude. Multiplying a vector by a positive scalar changes the magnitude of the vector, but not the direction, so it represents a change in scale.**

NAME ______________________ DATE ____________ PERIOD _____

Enrichment

Reading Mathematics

Many quantities in nature can be thought of as vectors. The science of physics involves many vector quantities. In reading about applications of mathematics, ask yourself whether the quantities involve only magnitude or both magnitude and direction. The first kind of quantity is called **scalar**. The second kind is a **vector**.

Classify each of the following. Write *scalar* or *vector*.

1. the mass of a book **scalar**

2. a car traveling north at 55 mph **vector**

3. a balloon rising 24 feet per minute **vector**

4. the size of a shoe **scalar**

5. a room temperature of 22 degrees Celsius **scalar**

6. a west wind of 15 mph **vector**

7. the batting average of a baseball player **scalar**

8. a car traveling 60 mph **vector**

9. a rock falling at 10 mph **vector**

10. your age **scalar**

11. the force of Earth's gravity acting on a moving satellite **vector**

12. the area of a record rotating on a turntable **scalar**

13. the length of a vector in the coordinate plane **scalar**