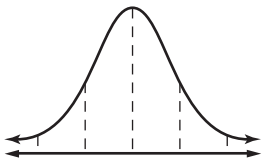
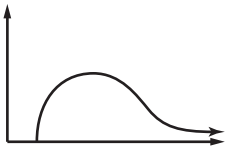
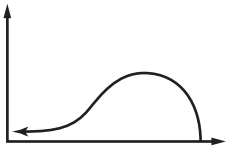
NAME _____ DATE _____ PERIOD _____

Study Guide and Intervention

The Normal Distribution

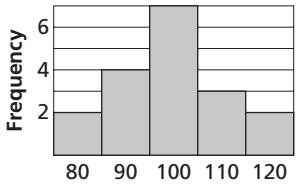
Normal and Skewed Distributions A **continuous probability** distribution is represented by a curve.

Types of Continuous Distributions	Normal	Positively Skewed	Negatively Skewed
			

Example **Determine whether the data below appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.**
{100, 120, 110, 100, 110, 80, 100, 90, 100, 120, 100, 90, 110, 100, 90, 80, 100, 90}

Make a frequency table for the data.

Value	80	90	100	110	120
Frequency	2	4	7	3	2



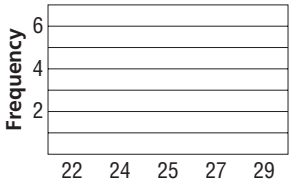
Then use the data to make a histogram.

Since the histogram is roughly symmetric, the data appear to be normally distributed.

Exercises

Determine whether the data in each table appear to be *positively skewed*, *negatively skewed*, or *normally distributed*. Make a histogram of the data.

1. {27, 24, 29, 25, 27, 22, 24, 25, 29, 24, 25, 22, 27, 24, 22, 25, 24, 22}

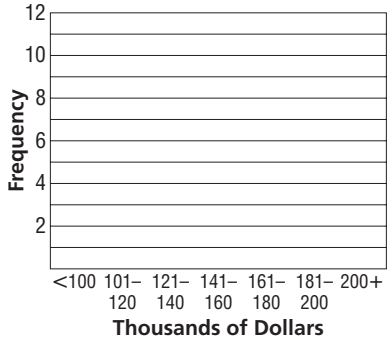


2.

Shoe Size	4	5	6	7	8	9	10
No. of Students	1	2	4	8	5	1	2

3.

Housing Price	No. of Houses Sold
less than $100,000	0
$100,00–$120,000	1
$121,00–$140,000	3
$141,00–$160,000	7
$161,00–$180,000	8
$181,00–$200,000	6
over $200,000	12



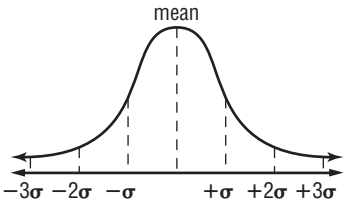
NAME _____ DATE _____ PERIOD _____

Study Guide and Intervention *(continued)*

The Normal Distribution

Use Normal Distributions

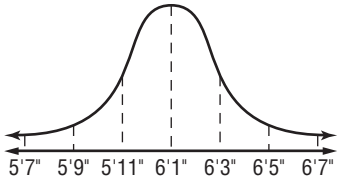
Normal Distribution



Normal distributions have these properties.
The graph is maximized at the mean.
The mean, median, and mode are about equal.
About 68% of the values are within one standard deviation of the mean.
About 95% of the values are within two standard deviations of the mean.
About 99% of the values are within three standard deviations of the mean.

Example **The heights of players in a basketball league are normally distributed with a mean of 6 feet 1 inch and a standard deviation of 2 inches.**

a. What is the probability that a player selected at random will be shorter than 5 feet 9 inches?



Draw a normal curve. Label the mean and the mean plus or minus multiples of the standard deviation.

The value of 5 feet 9 inches is 2 standard deviations below the mean, so approximately 2.5% of the players will be shorter than 5 feet 9 inches.

b. If there are 240 players in the league, about how many players are taller than 6 feet 3 inches?

The value of 6 feet 3 inches is one standard deviation above the mean. Approximately 16% of the players will be taller than this height.

$240 \times 0.16 \approx 38$

About 38 of the players are taller than 6 feet 3 inches.

Exercises

EGG PRODUCTION The number of eggs laid per year by a particular breed of chicken is normally distributed with a mean of 225 and a standard deviation of 10 eggs.

1. About what percent of the chickens will lay between 215 and 235 eggs per year?

2. In a flock of 400 chickens, about how many would you expect to lay more than 245 eggs per year?

MANUFACTURING The diameter of bolts produced by a manufacturing plant is normally distributed with a mean of 18 mm and a standard deviation of 0.2 mm.

3. What percent of bolts coming off of the assembly line have a diameter greater than 18.4 mm?

4. What percent have a diameter between 17.8 and 18.2 mm?

NAME _____ DATE _____ PERIOD _____

Skills Practice

The Normal Distribution

Determine whether the data in each table appear to be *positively skewed*, *negatively skewed*, *or normally distributed*.

1.

Miles Run	Track Team Members
0–4	3
5–9	4
10–14	7
15–19	5
20–23	2

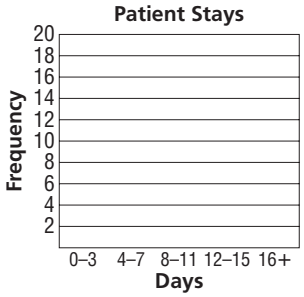
2.

Speeches Given	Political Candidates
0–5	1
6–11	2
12–17	3
18–23	8
24–29	8

For Exercises 3 and 4, use the frequency table that shows the average number of days patients spent on the surgical ward of a hospital last year.

Days	Number of Patients
0–3	5
4–7	18
8–11	11
12–15	9
16+	6

3. Make a histogram of the data.



4. Do the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*? Explain.

DELIVERY For Exercises 5–7, use the following information.

The time it takes a bicycle courier to deliver a parcel to his farthest customer is normally distributed with a mean of 40 minutes and a standard deviation of 4 minutes.

5. About what percent of the courier's trips to this customer take between 36 and 44 minutes?

6. About what percent of the courier's trips to this customer take between 40 and 48 minutes?

7. About what percent of the courier's trips to this customer take less than 32 minutes?

TESTING For Exercises 8–10, use the following information.

The average time it takes sophomores to complete a math test is normally distributed with a mean of 63.3 minutes and a standard deviation of 12.3 minutes.

8. About what percent of the sophomores take more than 75.6 minutes to complete the test?

9. About what percent of the sophomores take between 51 and 63.3 minutes?

10. About what percent of the sophomores take less than 63.3 minutes to complete the test?

NAME _____ DATE _____ PERIOD _____

Practice

The Normal Distribution

Determine whether the data in each table appear to be *positively skewed*, *negatively skewed*, *or normally distributed*.

1.

Time Spent at a Museum Exhibit	
Minutes	**Frequency**
0–25	27
26–50	46
51–75	89
75–100	57
100+	24

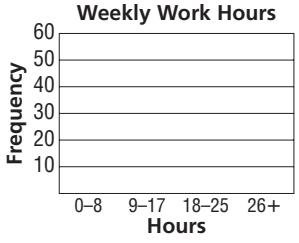
2.

Average Age of High School Principals	
Age in Years	**Number**
31–35	3
36–40	8
41–45	15
46–50	32
51–55	40
56–60	38
60+	4

For Exercises 3 and 4, use the frequency table that shows the number of hours worked per week by 100 high school seniors.

Hours	Number of Students
0–8	30
9–17	45
18–25	20
26+	5

3. Make a histogram of the data.



4. Do the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*? Explain.

TESTING For Exercises 5–10, use the following information.

The scores on a test administered to prospective employees are normally distributed with a mean of 100 and a standard deviation of 15.

5. About what percent of the scores are between 70 and 130?

6. About what percent of the scores are between 85 and 130?

7. About what percent of the scores are over 115?

8. About what percent of the scores are lower than 85 or higher than 115?

9. If 80 people take the test, how many would you expect to score higher than 130?

10. If 75 people take the test, how many would you expect to score lower than 85?

11. TEMPERATURE The daily July surface temperature of a lake at a resort has a mean of 82° and a standard deviation of 4.2°. If you prefer to swim when the temperature is at least 77.8°, about what percent of the days does the temperature meet your preference?

NAME _____ DATE _____ PERIOD _____

Reading to Learn Mathematics

The Normal Distribution

Reading the Lesson

1. Indicate whether each of the following statements is *true* or *false*.

a. In a continuous probability distribution, there is a finite number of possible outcomes.

b. Every normal distribution can be represented by a bell curve.

c. A distribution that is represented by a curve that is high at the left and has a tail to the right is negatively skewed.

d. A normal distribution is an example of a skewed distribution.

2. Ms. Rose gave the same quiz to her two geometry classes. She recorded the following scores.

First-period class:

Score	0	1	2	3	4	5	6	7	8	9	10
Frequency	1	0	1	0	3	4	5	7	4	3	2

Fifth-period class:

Score	0	1	2	3	4	5	6	7	8	9	10
Frequency	0	0	0	0	3	4	9	7	6	1	0

In each class, 30 students took the quiz. The mean score for each class was 6.4. Which set of scores has the greater standard deviation? (Answer this question without doing any calculations.) Explain your answer.

Helping You Remember

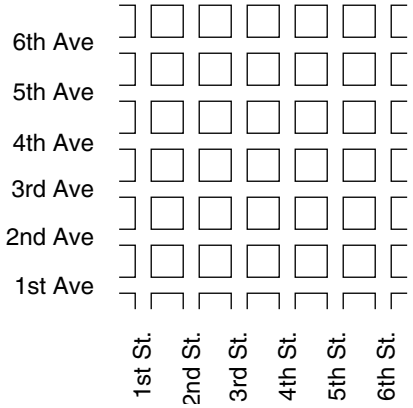
3. Many students have trouble remembering how to determine if a curve represents a distribution that is *positively skewed* or *negatively skewed*. What is an easy way to remember this?

NAME _____ DATE _____ PERIOD _____

Enrichment

Street Networks: Finding All Possible Routes

A section of a city is laid out in square blocks. Going north from the intersection of First Avenue and First Street, the avenues are 1st, 2nd, 3rd, and so on. Going east, the streets are numbered in the same way.



Factorials can be used to find the number, $r(e, n)$, of different routes between two intersections. The formula is shown below.

$$r(e, n) = \frac{[(e - 1) + (n - 1)]!}{(e - 1)!(n - 1)!}$$

The number of streets going east is e; the number of avenues going north is n.

The following problems examine the possible routes from one location to another. Assume that you never use a route that is unnecessarily long. Assume that $e \geq 1$ and $n \geq 1$.

Solve each problem.

1. List all the possible routes from 1st Street and 1st Avenue to 4th Street and 3rd Avenue. Use ordered pairs to show the routes, with street numbers first, and avenue numbers second. For example, each route starts at (1, 1) and ends at (4, 3).

2. Use the formula to compute the number of routes from (1, 1) to (4, 3). There are 4 streets going east and 3 avenues going north.

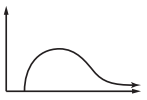
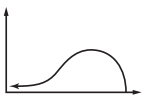
3. Find the number of routes from 1st Street and 1st Avenue to 7th Street and 6th Avenue.

NAME _____ DATE _____ PERIOD ____

Study Guide and Intervention

The Normal Distribution

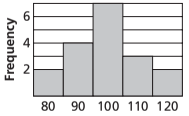
Normal and Skewed Distributions A **continuous probability** distribution is represented by a curve.

Types of Continuous Distributions	Normal	Positively Skewed	Negatively Skewed
			

Example **Determine whether the data below appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.**
{100, 120, 110, 100, 110, 80, 100, 90, 100, 120, 100, 90, 110, 100, 90, 80, 100, 90}

Make a frequency table for the data.

Value	80	90	100	110	120
Frequency	2	4	7	3	2



Then use the data to make a histogram.

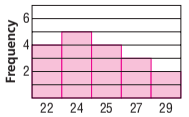
Since the histogram is roughly symmetric, the data appear to be normally distributed.

Exercises

Determine whether the data in each table appear to be *positively skewed*, *negatively skewed*, or *normally distributed*. Make a histogram of the data.

1. {27, 24, 29, 25, 27, 22, 24, 25, 29, 24, 25, 22, 27, 24, 22, 25, 24, 22}

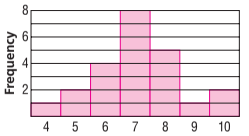
positively skewed



2.

Shoe Size	4	5	6	7	8	9	10
No. of Students	1	2	4	8	5	1	2

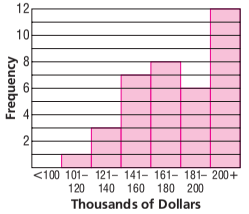
normally distributed



3.

Housing Price	No. of Houses Sold
less than $100,000	0
$100,00–$120,000	1
$121,00–$140,000	3
$141,00–$160,000	7
$161,00–$180,000	8
$181,00–$200,000	6
over $200,000	12

negatively skewed

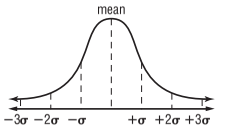


NAME _____ DATE _____ PERIOD ____

Study Guide and Intervention *(continued)*

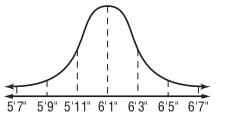
The Normal Distribution

Use Normal Distributions

Normal Distribution	Normal distributions have these properties.
	The graph is maximized at the mean. The mean, median, and mode are about equal. About 68% of the values are within one standard deviation of the mean. About 95% of the values are within two standard deviations of the mean. About 99% of the values are within three standard deviations of the mean.

Example **The heights of players in a basketball league are normally distributed with a mean of 6 feet 1 inch and a standard deviation of 2 inches.**

a. What is the probability that a player selected at random will be shorter than 5 feet 9 inches?



Draw a normal curve. Label the mean and the mean plus or minus multiples of the standard deviation.

The value of 5 feet 9 inches is 2 standard deviations below the mean, so approximately 2.5% of the players will be shorter than 5 feet 9 inches.

b. If there are 240 players in the league, about how many players are taller than 6 feet 3 inches?

The value of 6 feet 3 inches is one standard deviation above the mean. Approximately 16% of the players will be taller than this height.

$240 \times 0.16 \approx 38$

About 38 of the players are taller than 6 feet 3 inches.

Exercises

EGG PRODUCTION The number of eggs laid per year by a particular breed of chicken is normally distributed with a mean of 225 and a standard deviation of 10 eggs.

1. About what percent of the chickens will lay between 215 and 235 eggs per year? **68%**

2. In a flock of 400 chickens, about how many would you expect to lay more than 245 eggs per year? **10 chickens**

MANUFACTURING The diameter of bolts produced by a manufacturing plant is normally distributed with a mean of 18 mm and a standard deviation of 0.2 mm.

3. What percent of bolts coming off of the assembly line have a diameter greater than 18.4 mm? **2.5%**

4. What percent have a diameter between 17.8 and 18.2 mm? **68%**

NAME _____ DATE_____ PERIOD ____

Skills Practice

The Normal Distribution

Determine whether the data in each table appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

1.

Miles Run	Track Team Members
0–4	3
5–9	4
10–14	7
15–19	5
20–23	2

normally distributed

2.

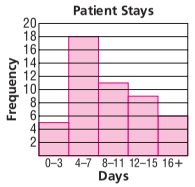
Speeches Given	Political Candidates
0–5	1
6–11	2
12–17	3
18–23	8
24–29	8

positively skewed

For Exercises 3 and 4, use the frequency table that shows the average number of days patients spent on the surgical ward of a hospital last year.

Days	Number of Patients
0–3	5
4–7	18
8–11	11
12–15	9
16+	6

3. Make a histogram of the data.



4. Do the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*? Explain.
Positively skewed; the histogram is high at the left and has a tail to the right.

DELIVERY For Exercises 5–7, use the following information.
The time it takes a bicycle courier to deliver a parcel to his farthest customer is normally distributed with a mean of 40 minutes and a standard deviation of 4 minutes.

5. About what percent of the courier's trips to this customer take between 36 and 44 minutes? 68%

6. About what percent of the courier's trips to this customer take between 40 and 48 minutes? 47.5%

7. About what percent of the courier's trips to this customer take less than 32 minutes? 2.5%

TESTING For Exercises 8–10, use the following information.
The average time it takes sophomores to complete a math test is normally distributed with a mean of 63.3 minutes and a standard deviation of 12.3 minutes.

8. About what percent of the sophomores take more than 75.6 minutes to complete the test? 16%

9. About what percent of the sophomores take between 51 and 63.3 minutes? 34%

10. About what percent of the sophomores take less than 63.3 minutes to complete the test? 50%

NAME _____ DATE_____ PERIOD ____

Practice (Average)

The Normal Distribution

Determine whether the data in each table appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

1.

Time Spent at a Museum Exhibit	
Minutes	Frequency
0–25	27
26–50	46
51–75	89
75–100	57
100+	24

normally distributed

2.

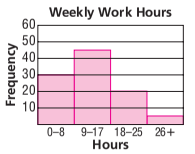
Average Age of High School Principals	
Age in Years	Number
31–35	3
36–40	8
41–45	15
46–50	32
51–55	40
56–60	38
60+	4

negatively skewed

For Exercises 3 and 4, use the frequency table that shows the number of hours worked per week by 100 high school seniors.

Hours	Number of Students
0–8	30
9–17	45
18–25	20
26+	5

3. Make a histogram of the data.



4. Do the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*? Explain.
Positively skewed; the histogram is high at the left and has a tail to the right.

TESTING For Exercises 5–10, use the following information.
The scores on a test administered to prospective employees are normally distributed with a mean of 100 and a standard deviation of 15.

5. About what percent of the scores are between 70 and 130? 95%

6. About what percent of the scores are between 85 and 130? 81.5%

7. About what percent of the scores are over 115? 16%

8. About what percent of the scores are lower than 85 or higher than 115? 32%

9. If 80 people take the test, how many would you expect to score higher than 130? 2

10. If 75 people take the test, how many would you expect to score lower than 85? 12

11. TEMPERATURE The daily July surface temperature of a lake at a resort has a mean of 82° and a standard deviation of 4.2°. If you prefer to swim when the temperature is at least 77.8°, about what percent of the days does the temperature meet your preference? 84%

Answers

Answers

NAME _____ DATE_____ PERIOD ____

Reading to Learn Mathematics

The Normal Distribution

Reading the Lesson

1. Indicate whether each of the following statements is *true* or *false*.

a. In a continuous probability distribution, there is a finite number of possible outcomes. **false**

b. Every normal distribution can be represented by a bell curve. **true**

c. A distribution that is represented by a curve that is high at the left and has a tail to the right is negatively skewed. **false**

d. A normal distribution is an example of a skewed distribution. **false**

2. Ms. Rose gave the same quiz to her two geometry classes. She recorded the following scores.

First-period class:

Score	0	1	2	3	4	5	6	7	8	9	10
Frequency	1	0	1	0	3	4	5	7	4	3	2

Fifth-period class:

Score	0	1	2	3	4	5	6	7	8	9	10
Frequency	0	0	0	0	3	4	9	7	6	1	0

In each class, 30 students took the quiz. The mean score for each class was 6.4. Which set of scores has the greater standard deviation? (Answer this question without doing any calculations.) Explain your answer.

First period class; sample answer: The scores are more spread out from the mean than for the fifth period class.

Helping You Remember

3. Many students have trouble remembering how to determine if a curve represents a distribution that is *positively skewed* or *negatively skewed*. What is an easy way to remember this?

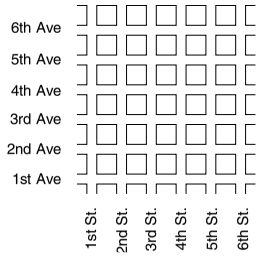
Sample answer: Follow the tail! If the tail is on the right (positive direction), the distribution is positively skewed. If the tail is on the left (negative direction), the distribution is negatively skewed.

NAME _____ DATE_____ PERIOD ____

Enrichment

Street Networks: Finding All Possible Routes

A section of a city is laid out in square blocks. Going north from the intersection of First Avenue and First Street, the avenues are 1st, 2nd, 3rd, and so on. Going east, the streets are numbered in the same way.



Factorials can be used to find the number, $r(e, n)$, of different routes between two intersections. The formula is shown below.

$$r(e, n) = \frac{[(e-1)+(n-1)]!}{(e-1)!(n-1)!}$$

The number of streets going east is e; the number of avenues going north is n.

The following problems examine the possible routes from one location to another. Assume that you never use a route that is unnecessarily long. Assume that $e \geq 1$ and $n \geq 1$.

Solve each problem.

1. List all the possible routes from 1st Street and 1st Avenue to 4th Street and 3rd Avenue. Use ordered pairs to show the routes, with street numbers first, and avenue numbers second. For example, each route starts at (1, 1) and ends at (4, 3).

(1, 1) – (2, 1) – (3, 1) – (4, 1) – (4, 2) – (4, 3)
(1, 1) – (2, 1) – (3, 1) – (3, 2) – (4, 2) – (4, 3)
(1, 1) – (2, 1) – (3, 1) – (3, 2) – (3, 3) – (4, 3)
(1, 1) – (2, 1) – (2, 2) – (3, 2) – (4, 2) – (4, 3)
(1, 1) – (2, 1) – (2, 2) – (3, 2) – (3, 3) – (4, 3)
(1, 1) – (2, 1) – (2, 2) – (2, 3) – (3, 3) – (4, 3)
(1, 1) – (1, 2) – (2, 2) – (3, 2) – (4, 2) – (4, 3)
(1, 1) – (1, 2) – (2, 2) – (3, 2) – (3, 3) – (4, 3)
(1, 1) – (1, 2) – (2, 2) – (2, 3) – (3, 3) – (4, 3)
(1, 1) – (1, 2) – (1, 3) – (2, 3) – (3, 3) – (4, 3)

2. Use the formula to compute the number of routes from (1, 1) to (4, 3). There are 4 streets going east and 3 avenues going north.

$$\frac{(3+2)!}{3!2!} = 10$$

3. Find the number of routes from 1st Street and 1st Avenue to 7th Street and 6th Avenue.

$$\frac{(6+5)!}{6!5!} = 462$$