

MAIN IDEA

Graph dilations on a coordinate plane.

New Vocabulary

dilation
center
enlargement
reduction

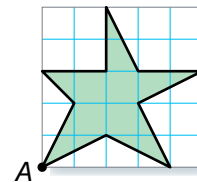
Math Online

glencoe.com

- Concepts in Motion
- Extra Examples
- Personal Tutor
- Self-Check Quiz

MINI Lab

The figure shown is drawn on 0.5-centimeter grid paper, so each square is 0.5-by-0.5 centimeter. Redraw the figure using squares that are 1-by-1 centimeter. Use point A as your starting point.



1. Measure and compare corresponding lengths on the original and new figure. Describe the relationship between these measurements. How does this relate to the change in grid size?
2. **MAKE A CONJECTURE** What size squares should you use to create a version of the original figure with dimensions that are four times the corresponding lengths on the original? Explain.

The image produced by enlarging or reducing a figure is called a **dilation**. A dilation image is similar to the original figure. This means that corresponding lengths on the two figures are proportional.

The **center** of the dilation is a fixed point used for measurement when altering the size of the figure. The ratio of a length on the image to a length on the original figure is the scale factor of the dilation.

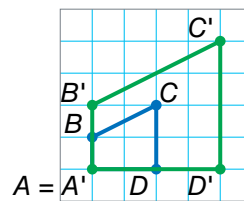
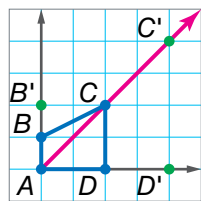
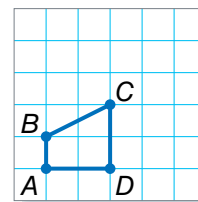
EXAMPLE Draw a Dilation

- 1 Copy polygon $ABCD$ shown on graph paper. Then draw the image of the figure after a dilation with center A by a scale factor of 2.

Step 1 Draw ray AB , or \overrightarrow{AB} , extending it to the edges of the grid.

Step 2 Use a ruler to locate point B' on \overrightarrow{AB} such that $AB' = 2(AB)$.

Step 3 Repeat Steps 1 and 2 for points C' and D' . Then draw polygon $A'B'C'D'$ where $A = A'$.

**CHECK Your Progress**

- a. Draw and label a large triangle XYZ on grid paper. Then draw the image of $\triangle XYZ$ after a dilation with center X and scale factor $\frac{1}{4}$.

In Example 1, if point A has coordinates $(0, 0)$, then the table below lists the coordinates of corresponding points on the original figure and its image. Notice that the coordinates of the image are (ax, ay) , where a is the scale factor.

Original Coordinates	Relationship	Image Coordinates
$D(2, 0)$	$(2 \cdot 2, 0 \cdot 2)$	$D'(4, 0)$
$C(2, 2)$	$(2 \cdot 2, 2 \cdot 2)$	$C'(4, 4)$
$B(0, 1)$	$(0 \cdot 2, 1 \cdot 2)$	$B'(0, 2)$
$A(0, 0)$	$(0 \cdot 2, 0 \cdot 2)$	$A'(0, 0)$

Study Tip

Dilations on a Coordinate Plane

The ratio of the x - and y -coordinates of the vertices of an image to the corresponding values of the coordinates of the vertices of the original figure is the same as the scale factor of the dilation.

To find the coordinates of the vertices of an image after a dilation with center $(0, 0)$, multiply the x - and y -coordinates by the scale factor.

EXAMPLE Graph a Dilation

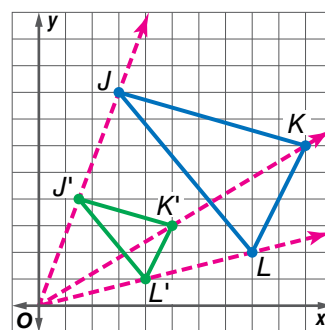
- 2 Graph $\triangle JKL$ with vertices $J(3, 8)$, $K(10, 6)$, and $L(8, 2)$. Then graph its image $\triangle J'K'L'$ after a dilation with a scale factor of $\frac{1}{2}$.

To find the vertices of the dilation, multiply each coordinate in the ordered pairs by $\frac{1}{2}$. Then graph both images on the same axes.

$$J(3, 8) \rightarrow \left(3 \cdot \frac{1}{2}, 8 \cdot \frac{1}{2}\right) \rightarrow J'\left(\frac{3}{2}, 4\right)$$

$$K(10, 6) \rightarrow \left(10 \cdot \frac{1}{2}, 6 \cdot \frac{1}{2}\right) \rightarrow K'(5, 3)$$

$$L(8, 2) \rightarrow \left(8 \cdot \frac{1}{2}, 2 \cdot \frac{1}{2}\right) \rightarrow L'(4, 1)$$



Check for Reasonableness

Draw lines through the origin and each of the vertices of the original figure. The vertices of the dilation should lie on those same lines.

CHECK Your Progress

Find the coordinates of the image of $\triangle JKL$ after a dilation with each scale factor. Then graph $\triangle JKL$ and $\triangle J'K'L'$.

b. scale factor: 3

c. scale factor: $\frac{1}{3}$

Examine the scale factors and the images produced after the dilations in Examples 1 and 2. These and other examples suggest the following.

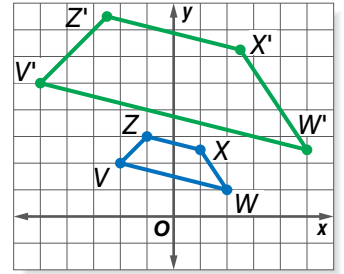
- A dilation with a scale factor greater than 1 produces an **enlargement**, an image that is larger than the original figure.
- A dilation with a scale factor between 0 and 1 produces a **reduction**, an image that is smaller than the original figure.

Study Tip

Alternate Form
Scale factors can also be written as decimals.

EXAMPLE Find and Classify a Scale Factor

- 3 Quadrilateral $V'Z'X'W'$ is a dilation of quadrilateral $VZXW$. Find the scale factor of the dilation, and classify it as an *enlargement* or a *reduction*.



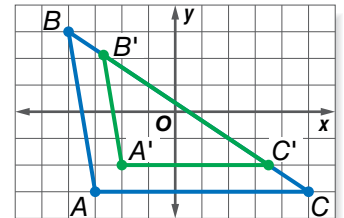
Write a ratio of the x - or y -coordinate of one vertex of the dilation to the x - or y -coordinate of the corresponding vertex of the original figure. Use the y -coordinates of $V(-2, 2)$ and $V'(-5, 5)$.

$$\frac{y\text{-coordinate of point } V'}{y\text{-coordinate of point } V} = \frac{5}{2} \quad \text{Verify by using other coordinates.}$$

The scale factor is $\frac{5}{2}$. Since $\frac{5}{2} > 1$, the dilation is an enlargement.

CHECK Your Progress

- d. Triangle $A'B'C'$ is a dilation of $\triangle ABC$. Find the scale factor of the dilation, and classify it as an *enlargement* or a *reduction*.



Real-World EXAMPLE

- 4 **EYES** An optometrist dilates a patient's pupils by a factor of $\frac{5}{3}$. If the pupil before dilation has a diameter of 5 millimeters, find the new diameter after the pupil is dilated.

Words	The size of the pupil after dilation is $\frac{5}{3}$ the size of the pupil before dilation.
Variable	Let a represent the size of the pupil after dilation.
Equation	$a = \frac{5}{3} \cdot 5$

$$a = \frac{5}{3}(5) \quad \text{Write the equation.}$$

$$a \approx 8.33 \quad \text{Multiply.}$$

The pupil will be about 8.3 millimeters in diameter after dilation.

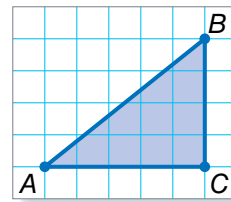
CHECK Your Progress

- e. **COMPUTERS** Dante uses an image of his dog as the wallpaper on his computer desktop. The original image is 5 inches high and 7 inches wide. If his computer scales the image by a factor of $\frac{5}{4}$, what are the dimensions of the dilated image?

CHECK Your Understanding

Example 1 (p. 225) Copy $\triangle ABC$ on graph paper. Then draw the image of the figure after the dilation with the given center and scale factor.

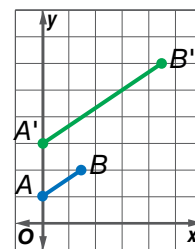
- center: A , scale factor: $\frac{1}{2}$
- center: C , scale factor: $\frac{3}{2}$



Example 2 (p. 226) Triangle JKL has vertices $J(-4, 2)$, $K(-2, -4)$, and $L(3, 6)$. Find the vertices of $\triangle J'K'L'$ after a dilation with the given scale factor. Then graph $\triangle JKL$ and $\triangle J'K'L'$.

- scale factor: 3
- scale factor: $\frac{1}{4}$

Example 3 (p. 227) 5. On the graph, $\overline{A'B'}$ is a dilation of \overline{AB} . Find the scale factor of the dilation, and classify it as an *enlargement* or as a *reduction*.



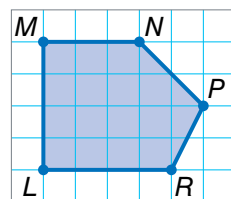
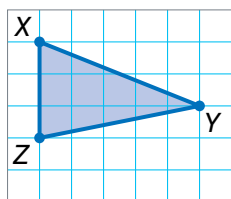
Example 4 (p. 227) 6. **GRAPHIC DESIGN** Jacqui designed a 6-inch by $7\frac{1}{2}$ -inch logo for her school. The logo is to be reduced by a scale factor of $\frac{1}{3}$ and used to make face paintings. What are the dimensions of the dilated image?

Practice and Problem Solving

HOMEWORK HELP

For Exercises	See Examples
7–10	1
11–14	2
15–18	3
19–20	4

Copy each figure on graph paper. Then draw the image of the figure after the dilation with the given center and scale factor.

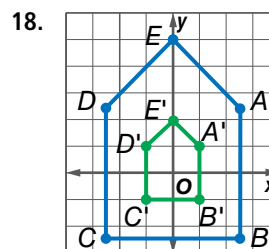
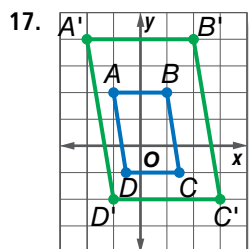
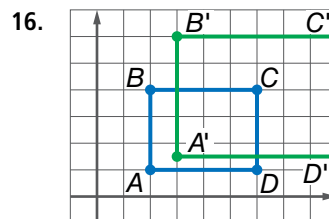
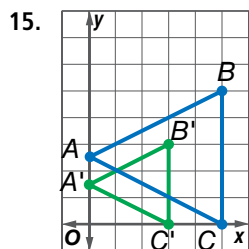


- center: X , scale factor: $\frac{7}{3}$
- center: Z , scale factor: $\frac{2}{3}$
- center: L , scale factor: $\frac{3}{4}$
- center: N , scale factor: 2

Find the vertices of polygon $H'J'K'L'$ after polygon $HJKL$ is dilated using the given scale factor. Then graph polygon $HJKL$ and polygon $H'J'K'L'$.

- $H(-1, 3)$, $J(3, 2)$, $K(2, -3)$, $L(-2, -2)$; scale factor 2
- $H(0, 2)$, $J(3, 1)$, $K(0, -4)$, $L(-2, -3)$; scale factor 3
- $H(-6, 2)$, $J(4, 4)$, $K(7, -2)$, $L(-2, -4)$; scale factor $\frac{1}{2}$
- $H(-8, 4)$, $J(6, 4)$, $K(6, -4)$, $L(-8, -4)$; scale factor $\frac{3}{4}$

On each graph, one figure is a dilation of the other. Find the scale factor of each dilation and classify it as an enlargement or as a reduction.

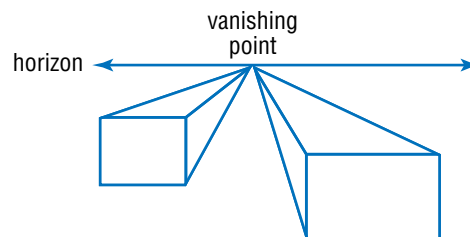


19. **PUBLISHING** To place a picture in his class newsletter, Joquin must reduce the picture by a scale factor of $\frac{3}{10}$. Find the dimensions of the reduced picture if the original is 15 centimeters wide and 10 centimeters high.
20. **PROJECTION** An overhead projector transforms the image on a transparency so that it is shown enlarged by a scale factor of 3.5 on a screen. If the original image is 3 inches long by 4 inches wide, find the dimensions of the projected image.
21. **BARN ART** Scott Hagan painted the Ohio bicentennial logo on one barn in each of Ohio's 88 counties. Each logo measured about 20 feet by 20 feet. Although Hagan drew each logo freehand, they are amazingly similar. If the original logo on which each painting was based measured 5 inches by 5 inches, what is the scale factor from the original logo to one of Hagan's paintings? Justify your answer.

DRAWING For Exercises 22 and 23, use the following information.

Artists use dilations to create the illusion of distance and depth. If you stand on a sidewalk and look in the distance, the parallel sides appear to converge and meet at a point. This is called the vanishing point.

22. Which figure appears to be closer? Explain your reasoning.
23. Draw a figure similar to the one shown at the right. Measure the larger rectangle. Above the horizon, draw a similar figure that is $\frac{7}{5}$ the size of that rectangle.



EXTRA PRACTICE
See pages 679, 703.

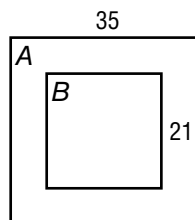
H.O.T. Problems

24. **OPEN ENDED** Graph a triangle and its image after a dilation with a scale factor greater than 1. Graph the resulting image after a dilation with a scale factor between 0 and 1. Predict the scale factor from the original to the final image. Explain your reasoning and verify your prediction.
25. **CHALLENGE** Describe the image of a figure after a dilation with a scale factor of -2 .
26. **WRITING IN MATH** Write a general rule for finding the new coordinates of any ordered pair (x, y) after a dilation with a scale factor of k .



TEST PRACTICE

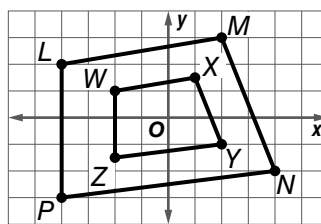
27. Square A is similar to square B .



What scale factor was used to dilate square A to square B ?

- A $\frac{1}{7}$
 B $\frac{3}{5}$
 C $\frac{5}{3}$
 D 7

28. Quadrilateral $LMNP$ was dilated to form quadrilateral $WXYZ$.

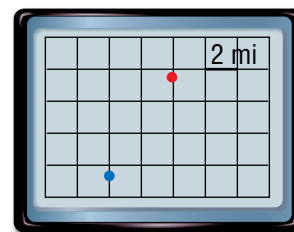
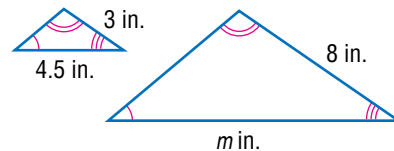


Which number best represents the scale factor used to change quadrilateral $LMNP$ into quadrilateral $WXYZ$?

- F 3 H 2
 G $\frac{1}{2}$ J $\frac{1}{3}$

Spiral Review

29. The triangles at the right are similar. Write and solve a proportion to find the missing measure. (Lesson 4-7)
30. **GEOMETRY** A rectangle is 12 meters by 7 meters. To the nearest tenth, what is the length of one of its diagonals? (Lesson 3-6)
31. **TECHNOLOGY** A backpacker uses her GPS (Global Positioning System) receiver to find how much farther she needs to go to get to her stopping point. She is at the red dot on her GPS receiver screen, and the blue dot shows her destination. How much farther does she need to go? (Lesson 3-7)



GET READY for the Next Lesson

PREREQUISITE SKILL Write a proportion and solve for x . (Lesson 4-5)

32. 3 cm is to 5 ft as x cm is to 9 ft 33. 4 in. is to 5 mi as 5 in. is to x mi